Boolean Identities to Know – CSU290

(You don't *need* to know the names on the right.)

Simplifications (always replace instances of the left formula with the right)

$\neg\neg p$	=	p	(Double negation)
$p \wedge p$	=	p	(Contraction of AND)
$p \vee p$	=	p	(Contraction of OR)
$p \wedge \mathit{true}$	=	p	(AND identity)
$p \vee \mathit{false}$	=	p	(OR identity)
$p \wedge \mathit{false}$	=	false	(AND preclusion)
$p \vee \mathit{true}$	=	true	(OR satisfaction)
$p \wedge \neg p$	=	false	(AND contradiction)
$p \vee \neg p$	=	true	(OR tautology)
$\mathit{true} \rightarrow p$	=	p	(Unconditional statement)
$\mathit{false} \rightarrow p$	=	true	(Inapplicable statement)
$p \rightarrow \mathit{true}$	=	true	(Foregone conclusion)
$p \rightarrow \mathit{false}$	=	$\neg p$	(Reductio ad absurdum)
$p \leftrightarrow \mathit{true}$	=	p	(Equivalent to true)
$p \leftrightarrow \mathit{false}$	=	$\neg p$	(Equivalent to false)
$p \leftrightarrow p$	=	true	(Reflexivity of IFF)
$p \leftrightarrow \neg p$	=	false	(IFF contradiction)

Other identities (these could be used in either direction depending on the context)

$p \land q$	=	$q \land p$	(Commutativity of AND)
$p \lor q$	=	$q \lor p$	(Commutativity of OR)
$p \land (q \land r)$	=	$(p \land q) \land r$	(Associativity of AND)
$p \lor (q \lor r)$	=	$(p \lor q) \lor r$	(Associativity of OR)
$\neg(p \land q)$	=	$\neg p \lor \neg q$	(De Morgan's law 1)
$\neg(p \lor q)$	=	$\neg p \land \neg q$	(De Morgan's law 2)
$p \land (q \lor r)$	=	$(p \land q) \lor (p \land r)$	(Distribute AND over OR)
$p \lor (q \land r)$	=	$(p \lor q) \land (p \lor r)$	(Distribute OR over AND)
$p \rightarrow q$	=	$\neg p \lor q$	(Disjunctive form of implication)
$p \rightarrow q$	=	$\neg q \rightarrow \neg p$	(Contrapositive)
$\neg(p \rightarrow q)$	=	$p \land \neg q$	(Case of false implication)
$p \rightarrow (q \rightarrow r)$	=	$(p \land q) \rightarrow r$	(Implication (un)chaining)
$(p \rightarrow q) \land (\neg p \rightarrow q)$	=	q	(Case analysis)
$p \leftrightarrow q$	=	$(p \land q) \lor (\neg p \land \neg q)$	(Cases of equivalence)
$p \leftrightarrow q$	=	$q \leftrightarrow p$	(Commutativity of IFF)
$p \leftrightarrow (q \leftrightarrow r)$	=	$(p \leftrightarrow q) \leftrightarrow r$	(Associativity of IFF)

Inferences (the formula on the left implies the formula on the right)

p	→	$p \lor q$	(Expansion)
$p \land q$	→	p	(Assumption)
$p \land (p \rightarrow q)$	→	q	(Modus ponens)
$(p \rightarrow q) \land (q \rightarrow r)$	→	$p \rightarrow r$	(Hypothetical syllogism)
$\neg q \land (p \rightarrow q)$	→	$\neg p$	(Modus tollens)
$\neg p \land (p \lor q)$	→	q	(Disjunctive syllogism)

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