Let us now look at implementing graph algorithms in MapReduce.

Why Graphs?

- Discussion is based on the book and slides by Jimmy Lin and Chris Dyer
- Analyze hyperlink structure of the Web
- Social networks
  - Facebook friendships, Twitter followers, email flows, phone call patterns
- Transportation networks
  - Roads, bus routes, flights
- Interactions between genes, proteins, etc.

What is a Graph?

- G = (V, E)
  - V: set of vertices (nodes)
  - E: set of edges (links), $E \subseteq V \times V$
- Edges can be directed or undirected
- Graph might have cycles or not (acyclic graph)
- Nodes and edges can be annotated
  - E.g., social network: node has demographic information like age; edge has type of relationship like friend or family

Graph Problems

- Graph search and path planning
  - Find driving directions from A to B
  - Recommend possible friends in social network
  - How to route IP packets or delivery trucks
- Graph clustering
  - Identify communities in social networks
  - Partition large graph to parallelize graph processing
- Minimum spanning trees
  - Connected graph of minimum total edge weight

More Graph Problems

- Bipartite graph matching
  - Match nodes on “left” with nodes on “right” side
  - E.g., match job seekers and employers, singles looking for dates, papers with reviewers
- Maximum flow
  - Maximum traffic between source and sink
  - E.g., optimize transportation networks
- Finding “special” nodes
  - E.g., disease hubs, leader of a community, people with influence

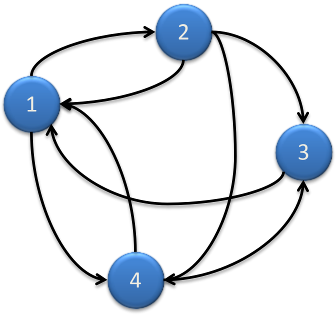
Graph Representations

- Usually one of these two:
  - Adjacency matrix
  - Adjacency list

Adjacency Matrix

- Matrix M of size |N| by |N|
  - Entry M(i,j) contains weight of edge from node i to node j; 0 if no edge

	1	**2**	**3**	**4**
1	0	1	0	1
2	1	0	1	1
3	1	0	0	0
4	1	0	1	0



Example source: Jimmy Lin

Properties

- Advantages
  - Easy to manipulate with linear algebra
  - Operation on outlinks and inlinks corresponds to iteration over rows and columns
- Disadvantage
  - Huge space overhead for sparse matrix
  - E.g., Facebook friendship graph

Adjacency List

- Compact row-wise representation of matrix

	1	**2**	**3**	**4**
1	0	1	0	1
2	1	0	1	1
3	1	0	0	0
4	1	0	1	0

1: 2, 4
2: 1, 3, 4
3: 1
4: 1, 3

Properties

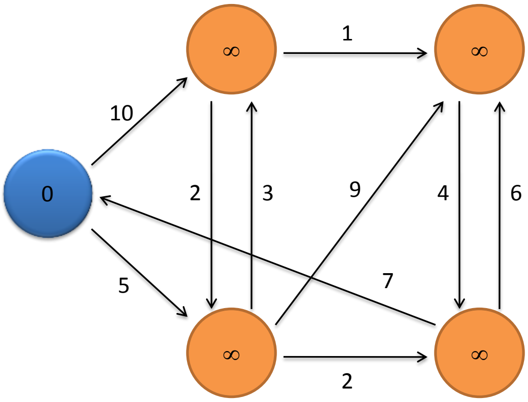
- Advantages
  - More space-efficient
  - Still easy to compute over outlinks for each node
- Disadvantage
  - Difficult to compute over inlinks for each node

- Note: remember inverse Web graph discussion

Parallel Breadth-First Search

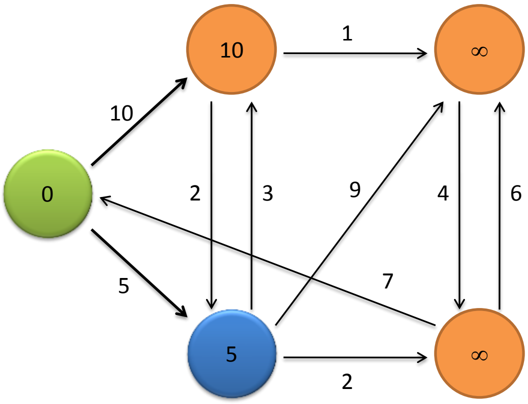
- Case study: single-source shortest path problem
  - Find the shortest path from a source node s to all other nodes in the graph
- For non-negative edge weights, Dijkstra’s algorithm is the classic sequential solution
  - Initialize distance d[s]=0, all others to ∞
  - Maintain priority queue of nodes sorted by distance
  - Remove first node u from queue and update d[v] for each node v in adjacency list of u if (1) v is in queue and (2) d[v] > d[u]+weight(u,v)

Dijkstra’s Algorithm Example

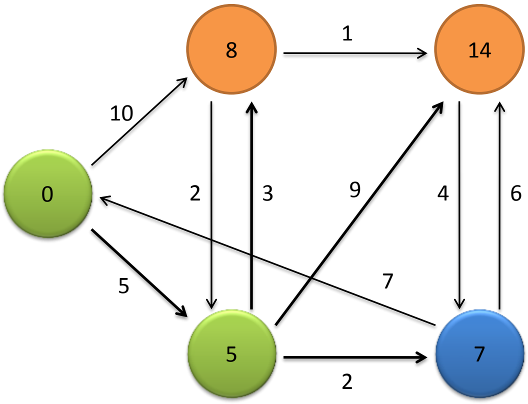


Example from Jimmy Lin’s presentation

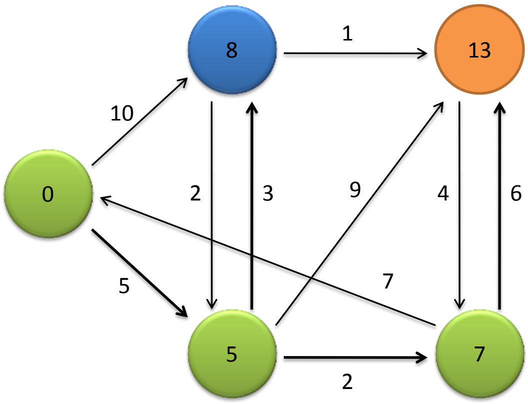
Dijkstra’s Algorithm Example



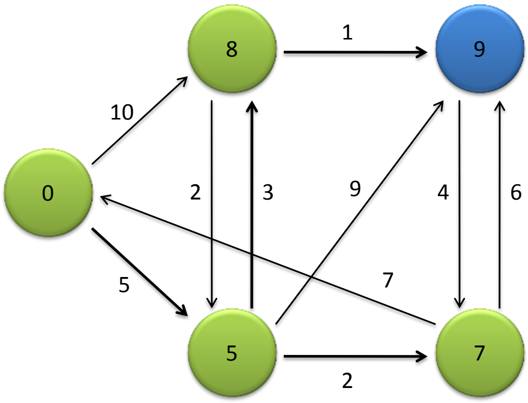
Dijkstra’s Algorithm Example



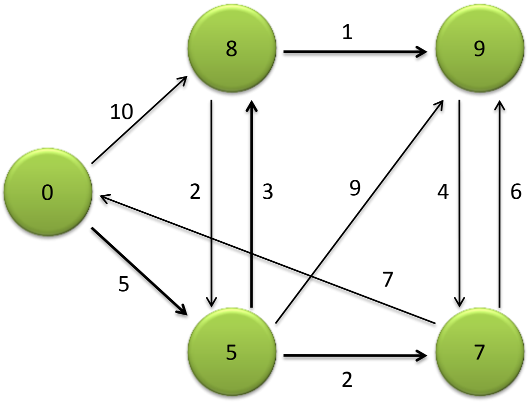
Dijkstra’s Algorithm Example



Dijkstra’s Algorithm Example



Dijkstra’s Algorithm Example



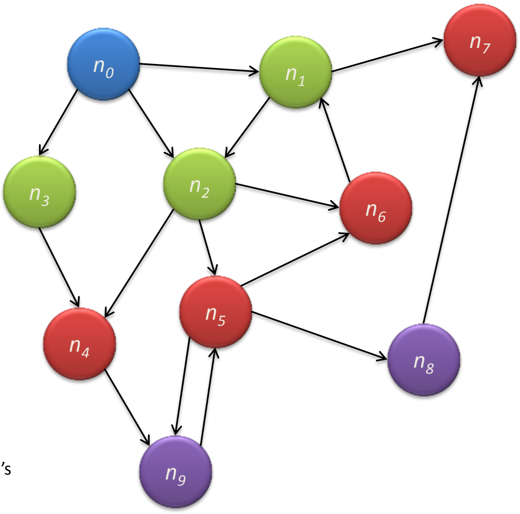
Parallel Single-Source Shortest Path

- Priority queue is core element of Dijkstra’s algorithm
  - No global shared data structure in MapReduce
- Dijkstra’s algorithm proceeds sequentially, node by node
  - Taking non-min node could affect correctness of algorithm
- Solution: perform parallel breadth-first search

Parallel Breadth-First Search

- Start at source s
- In first round, find all nodes reachable in one hop from s
- In second round, find all nodes reachable in two hops from s, and so on
- Keep track of min distance for each node
  - Also record corresponding path
- Iterations stop when no shorter path possible

BFS Visualization



Example from Jimmy Lin's presentation

MapReduce Code: Single Iteration

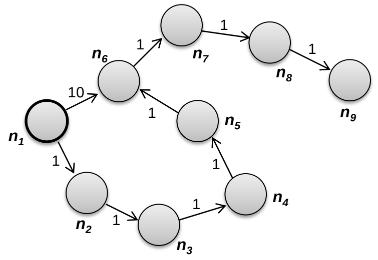
```
map(nid n, node N)            // N stores node's current min distance and adjacency list
  d = N.distance
  emit(nid n, N)              // Pass along graph structure
  for all nid m in N.adjacencyList do
    emit(nid m, d + w(n,m))   // Emit distances to reachable nodes


reduce(nid m, [d1,d2,…])
  dMin = ∞; M = ∅
  for all d in [d1,d2,…] do
    if isNode(d) then
      M = d                   // Recover graph structure
    else if d < dMin then     // Look for min distance in list
      dMin = d
  M.distance = dMin           // Update node's shortest distance
  emit(nid m, node M)
```

Overall Algorithm

- Need driver program to control the iterations
- Initialization: SourceNode.distance = 0, all others have distance=∞
- When to stop iterating?
- If all edges have weight 1, can stop as soon as no node has ∞ distance any more
  - Can detect this with Hadoop counter
- Number of iterations depends on graph diameter
  - In practice, many networks show the small-world phenomenon, e.g., six degrees of separation

Dealing With Diverse Edge Weights

- "Detour" path can be shorter than "direct" connection, hence cannot stop as soon as all node distances are finite
- Stop when no node's shortest distance changes any more
  - Can be detected with Hadoop counter
  - Worst case: |N| iterations



Example from Jimmy Lin's presentation

MapReduce Algorithm Analysis

- Brute-force approach that performs many irrelevant computations
  - Computes distances for nodes that still have infinity distance
  - Repeats previous computations inside "search frontier"
- Dijkstra's algorithm only explores the search frontier, but needs the priority queue

Typical Graph Processing in MapReduce

- Graph represented by adjacency list per node, plus extra node data
- Map works on a single node u
  - Node u's local state and links only
- Node v in u's adjacency list is intermediate key
  - Passes results of computation along outgoing edges
- Reduce combines partial results for each destination node
- Map also passes graph itself to reducers
- Driver program controls execution of iterations

PageRank Introduction

- Popularized by Google for evaluating the quality of a Web page
- Based on random Web surfer model
  - Web surfer can reach a page by jumping to it or by following the link from another page pointing to it
  - Modeled as random process
- Intuition: important pages are linked from many other (important) pages
  - Goal: find pages with greatest probability of access

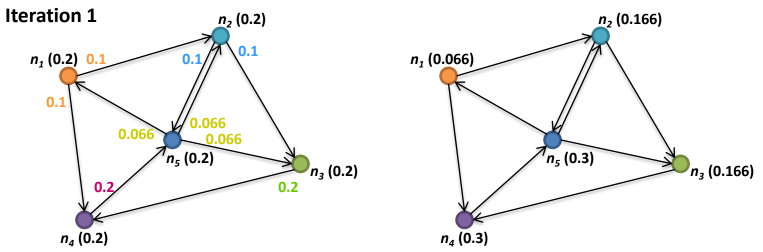
PageRank Definition

- PageRank of page n:
  - $P(n) = \alpha \frac{1}{|V|} + (1-\alpha) \sum_{m \in L(n)} \frac{P(m)}{C(m)}$
  - |V| is number of pages (nodes)
  - α is probability of random jump
  - L(n) is the set of pages linking to n
  - P(m) is m’s PageRank
  - C(m) is m’s out-degree
- Definition is recursive
  - Compute by iterating until convergence (fixpoint)

Computing PageRank

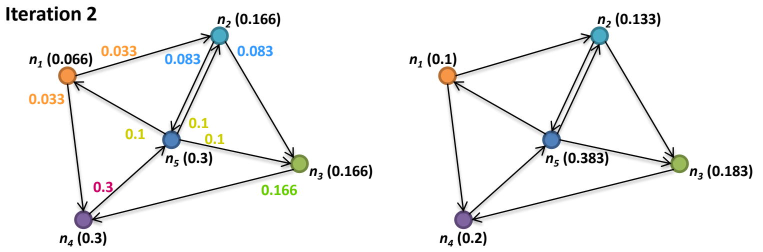
- Similar to BFS for shortest path
- Computing P(n) only requires P(m) and C(m) for all pages linking to n
  - During iteration, distribute P(m) evenly over outlinks
  - Then add contributions over all of n’s inlinks
- Initialization: any probability distribution over the nodes

PageRank Example

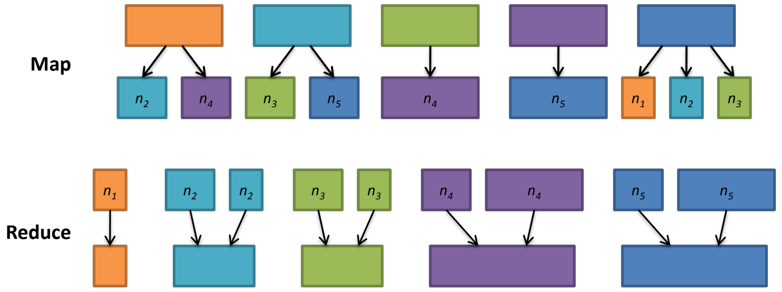


Source: Jimmy Lin's presentation

PageRank Example



PageRank in MapReduce



MapReduce Code

```
map(nid n, node N)              // N stores node's current PageRank and adjacency list
  p = N.pageRank / |N.adjacencyList|
  emit(nid n, N)                // Pass along graph structure
  for all nid m in N.adjacencyList do
    emit(nid m, p)              // Pass PageRank mass to neighbors


reduce(nid m, [p1,p2,…])
  s=0; M = $\emptyset$
  for all p in [p1,p2,…] do
    if isNode(p) then
      M = p                     // Recover graph structure
    else
      s += p                    // Sum incoming PageRank contributions
  M.pageRank = $\alpha/|V| + (1-\alpha)\cdot s$
  emit(nid m, node M)
```

Dangling Nodes

- Consider node x with no outgoing links
  - P(x) is not passed to any other node, hence gets “lost” in the Map phase
- Need to correct for the missing probability mass
  - Model: assume dangling page links to all pages
  - Mathematically equivalent to

$$P(n) = \alpha \frac{1}{|V|} + (1-\alpha)\left(\frac{\delta}{|V|} + \sum_{m \in L(n)} \frac{P(m)}{C(m)}\right)$$

  - δ: missing PageRank mass due to dangling nodes

PageRank with Dangling Nodes

- Challenge: need δ, which is the sum over the current page ranks of dangling nodes
  - MR-job1: compute δ
  - MR-job2: compute new PageRank using δ
- Alternative computations?
  - Order inversion pattern to make sure δ is available in all reduce tasks

Number of Iterations

- PageRank computation iterates until convergence
  - PageRank of all nodes no longer changes (or is within small tolerance)
  - Needs to be checked by driver
- Original PageRank paper: 52 iterations until convergence on graph with 322 million edges
  - Highly dependent on data properties

General Graph Processing Issues

- Sequential algorithms often use global data structure for efficiency
- In MapReduce with adjacency list representation, information can only be passed locally to or from direct neighbors
  - But can pre-compute other data structures, e.g., two-hop neighbors
- Presented algorithms have Map output of O(#edges), which works well for sparse graphs

General Graph Processing Issues

- Partitioning of graph into chunks strongly affects effectiveness of combiners
  - Often best to keep well-connected components together
- Numerical stability for large graphs
  - PageRank of individual page might be so small that it underflows standard floating point representation
  - Can work with logarithm-transformed numbers instead