Decentralized Network Design

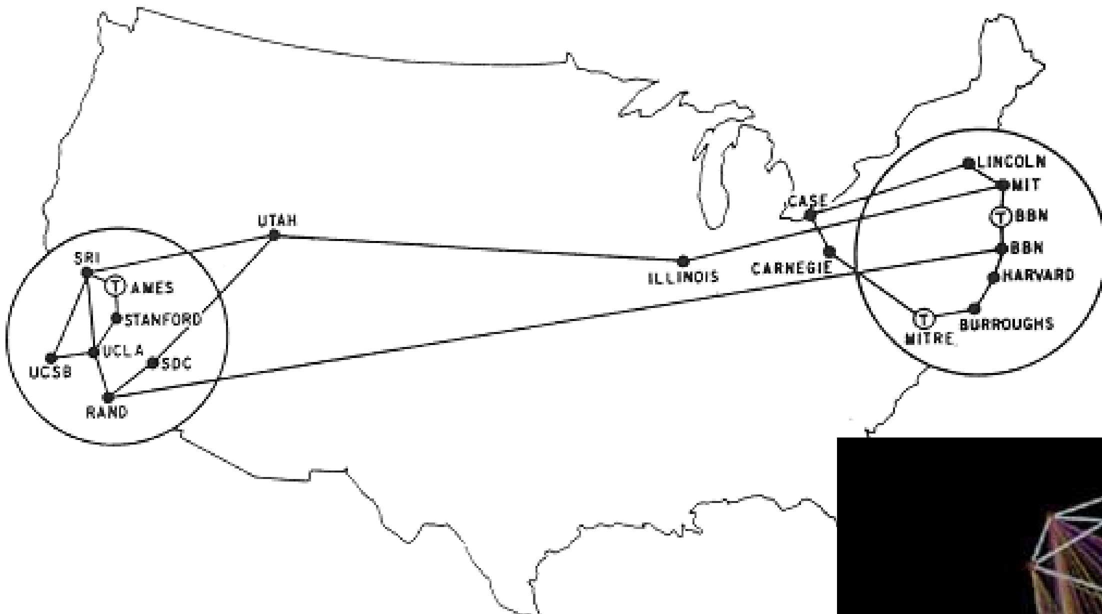
PhD Thesis Proposal
Laura Poplawski
2/4/09

Results Mentioned are from...

- Nikolaos Laoutaris, Laura J. Poplawski, Rajmohan Rajaraman, Ravi Sundaram, Shang-Hua Teng. *Bounded Budget Connection (BBC) Games or How to Make Friends and Influence People, on a Budget*.
  In PODC ’08, pages 165–174, 2008.
- Nikolaos Laoutaris, Laura J. Poplawski, Rajmohan Rajaraman, Ravi Sundaram, Shang-Hua Teng. *Bounded Budget Connection (BBC) Games or How to make friends and influence people, on a budget*.
  arXiv:0806.1727v1 [cs.GT]
- Laura J. Poplawski, Rajmohan Rajaraman, Ravi Sundaram, Shang-Hua Teng. *Preference Games and Personalized Equilibria, with Applications to Fractional BGP*.
  arXiv:0812.0598v2 [cs.GT]

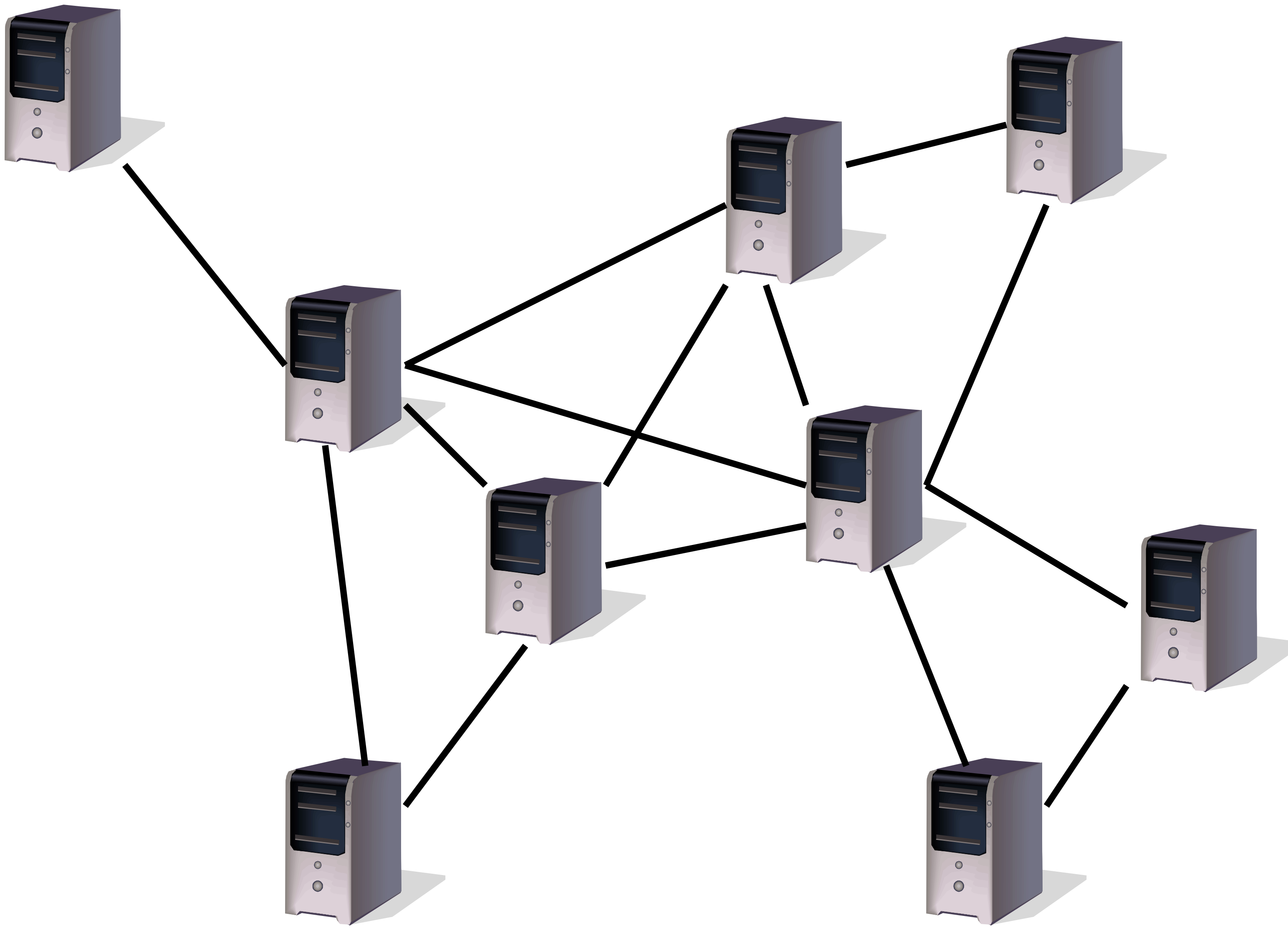
Decentralization

Decentralization

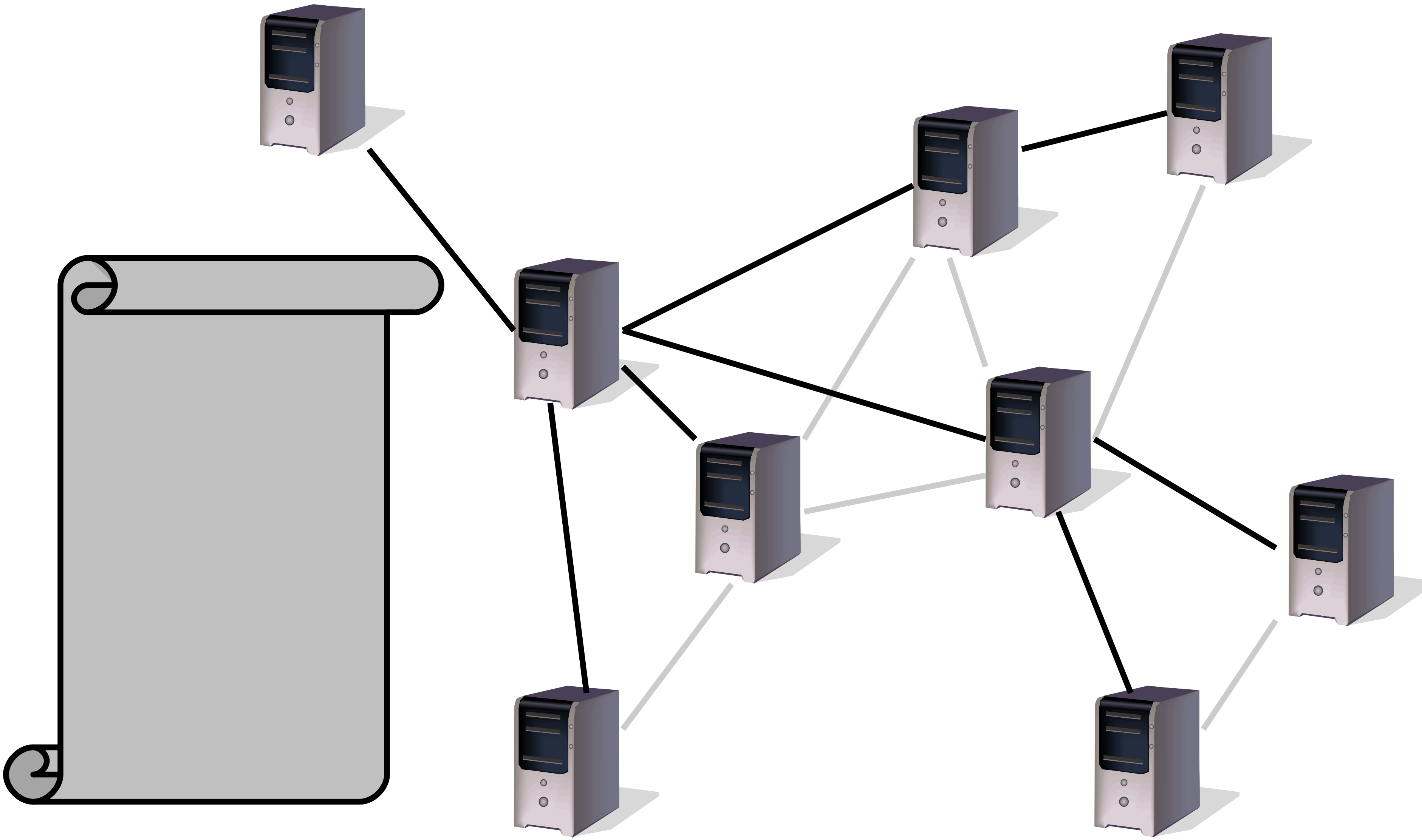


MAP 4 September 1971

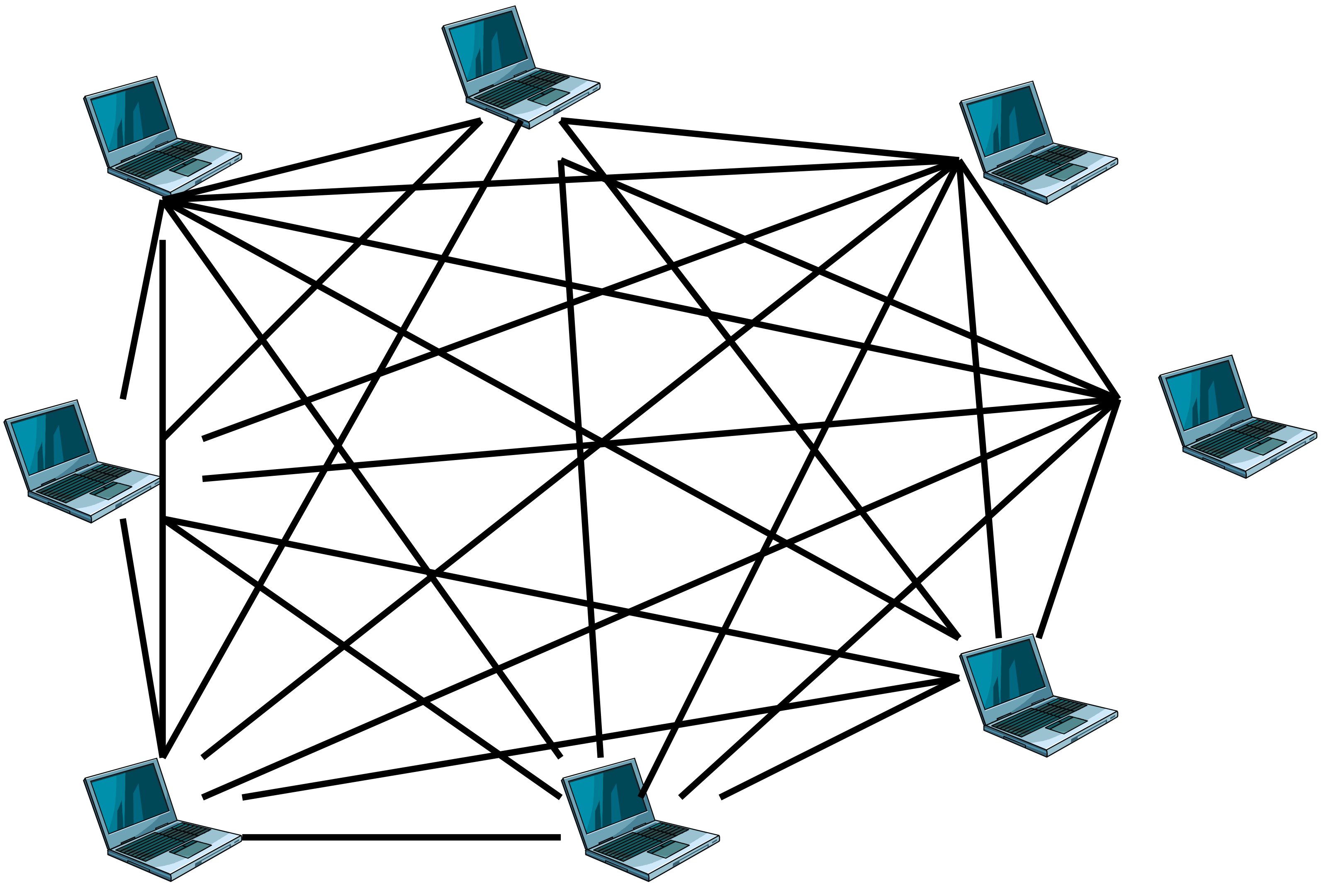
Decentralizing Overlay Networks



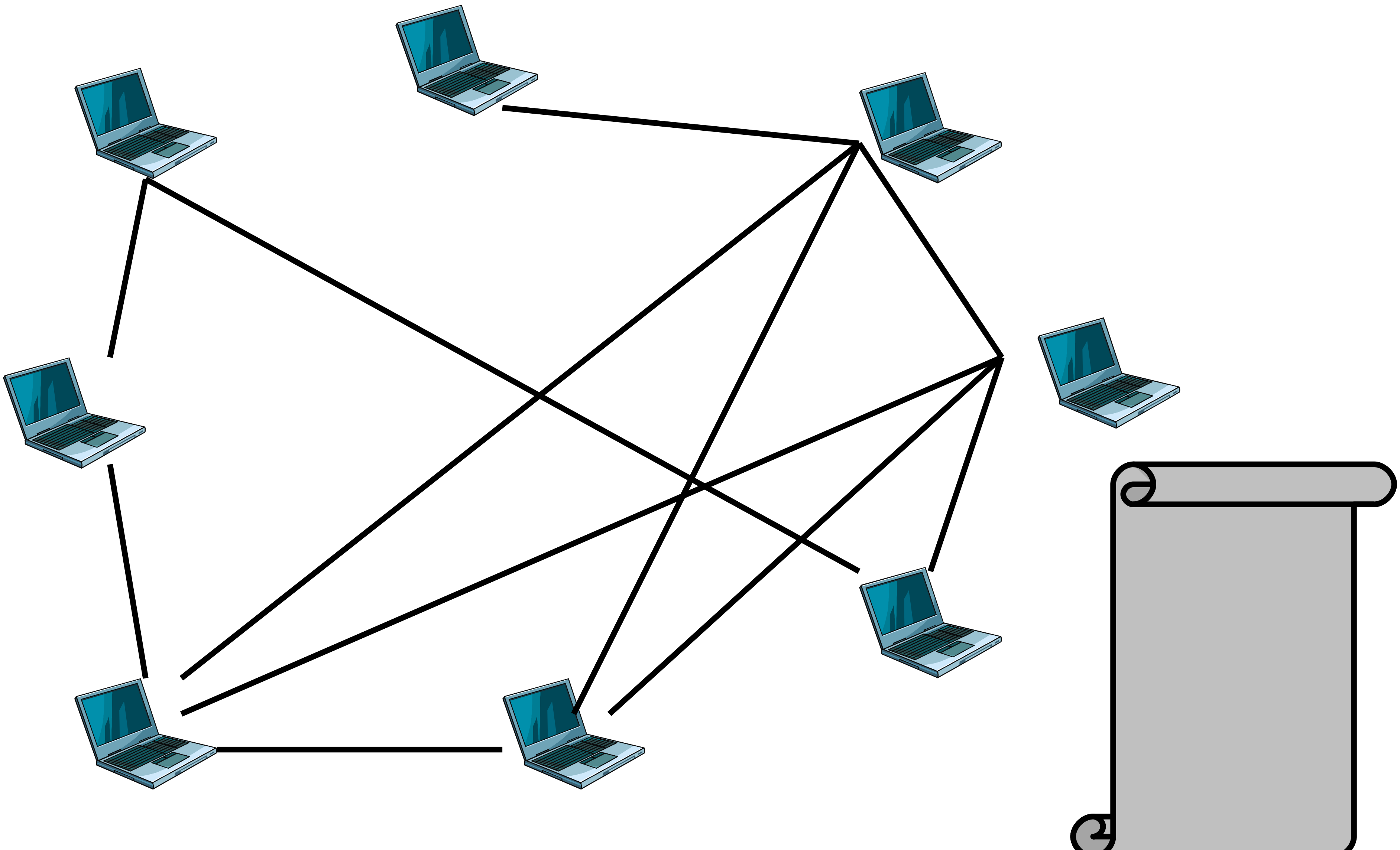
Decentralizing Overlay Networks



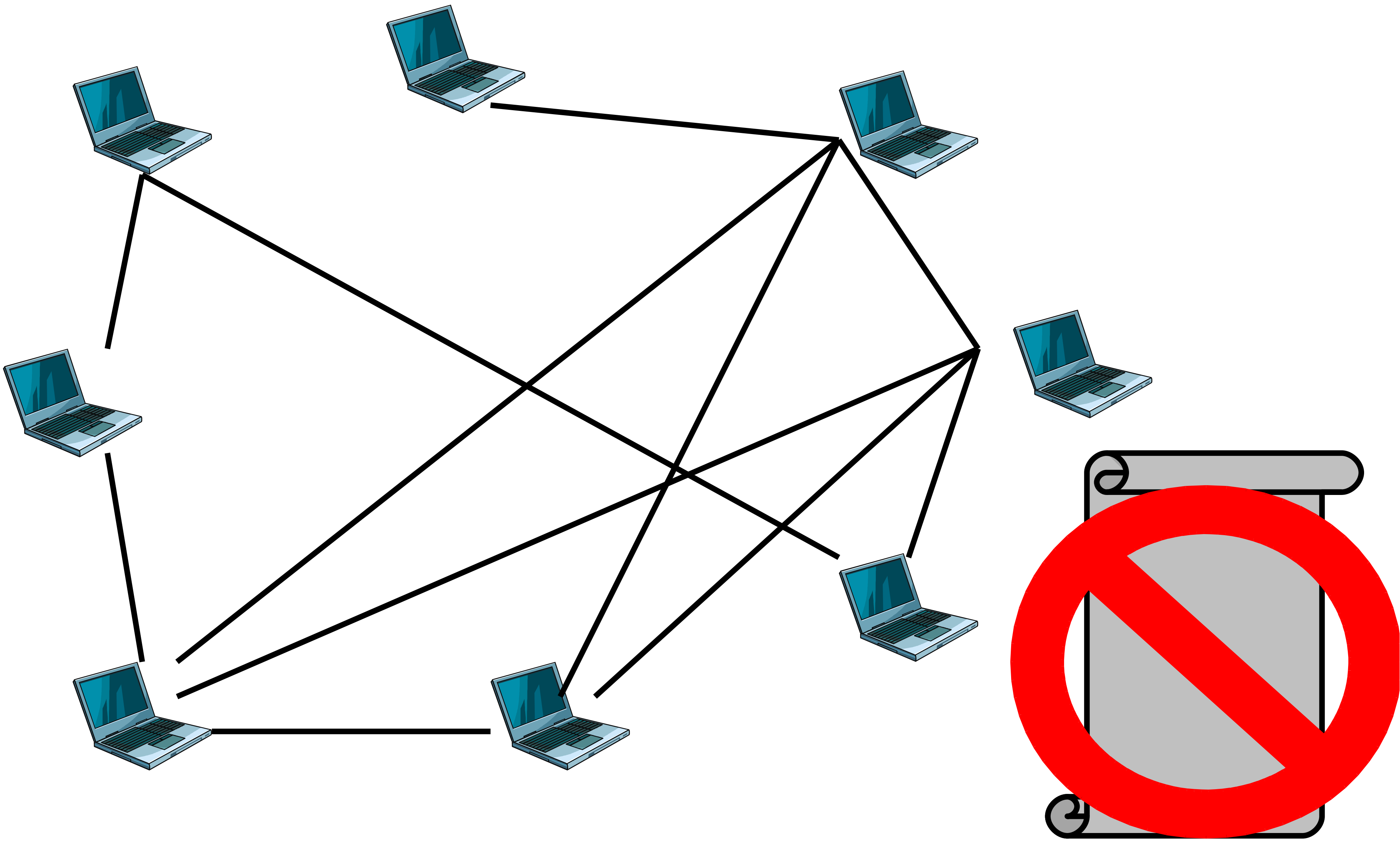
Decentralizing Overlay Networks



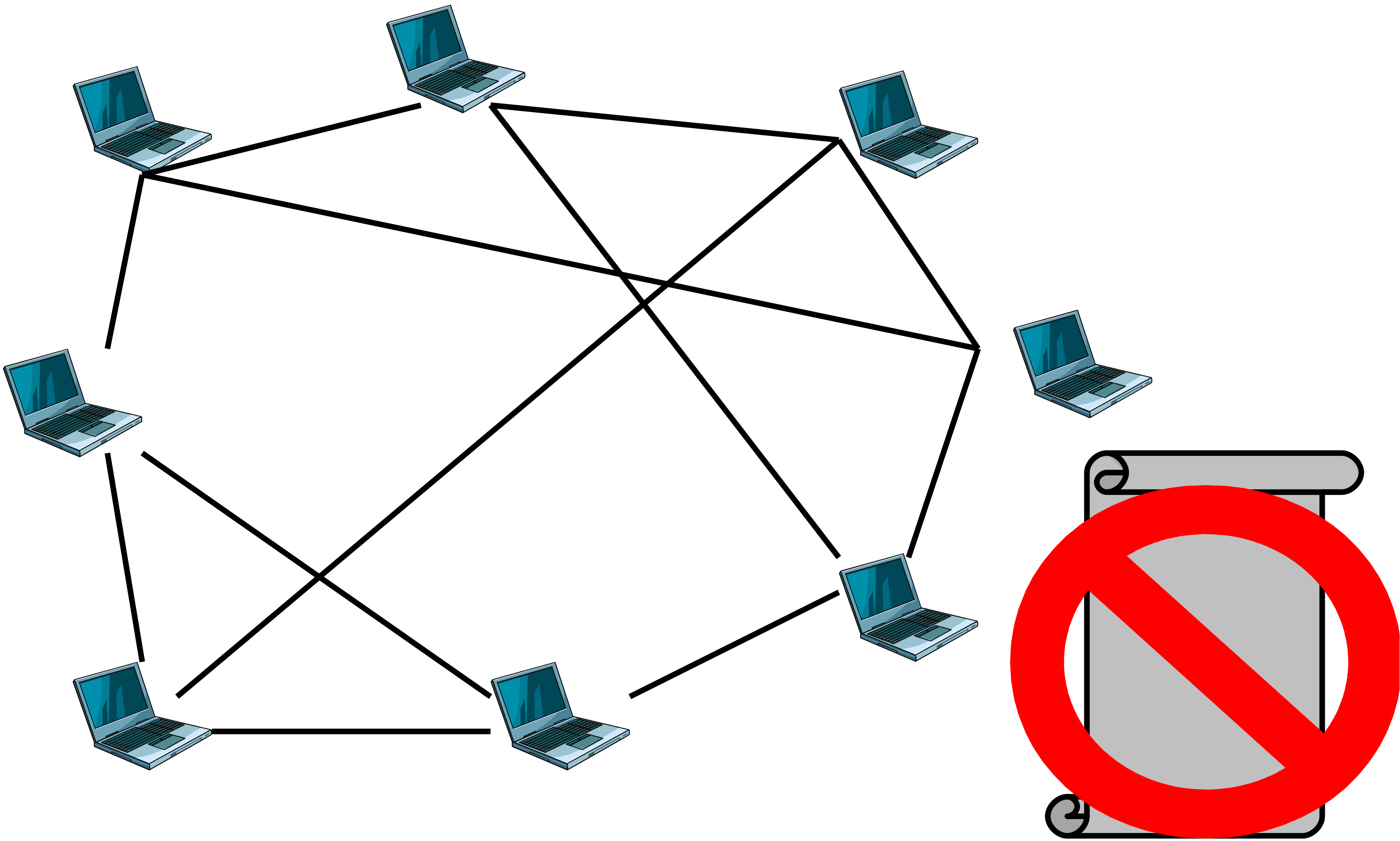
Decentralizing Overlay Networks



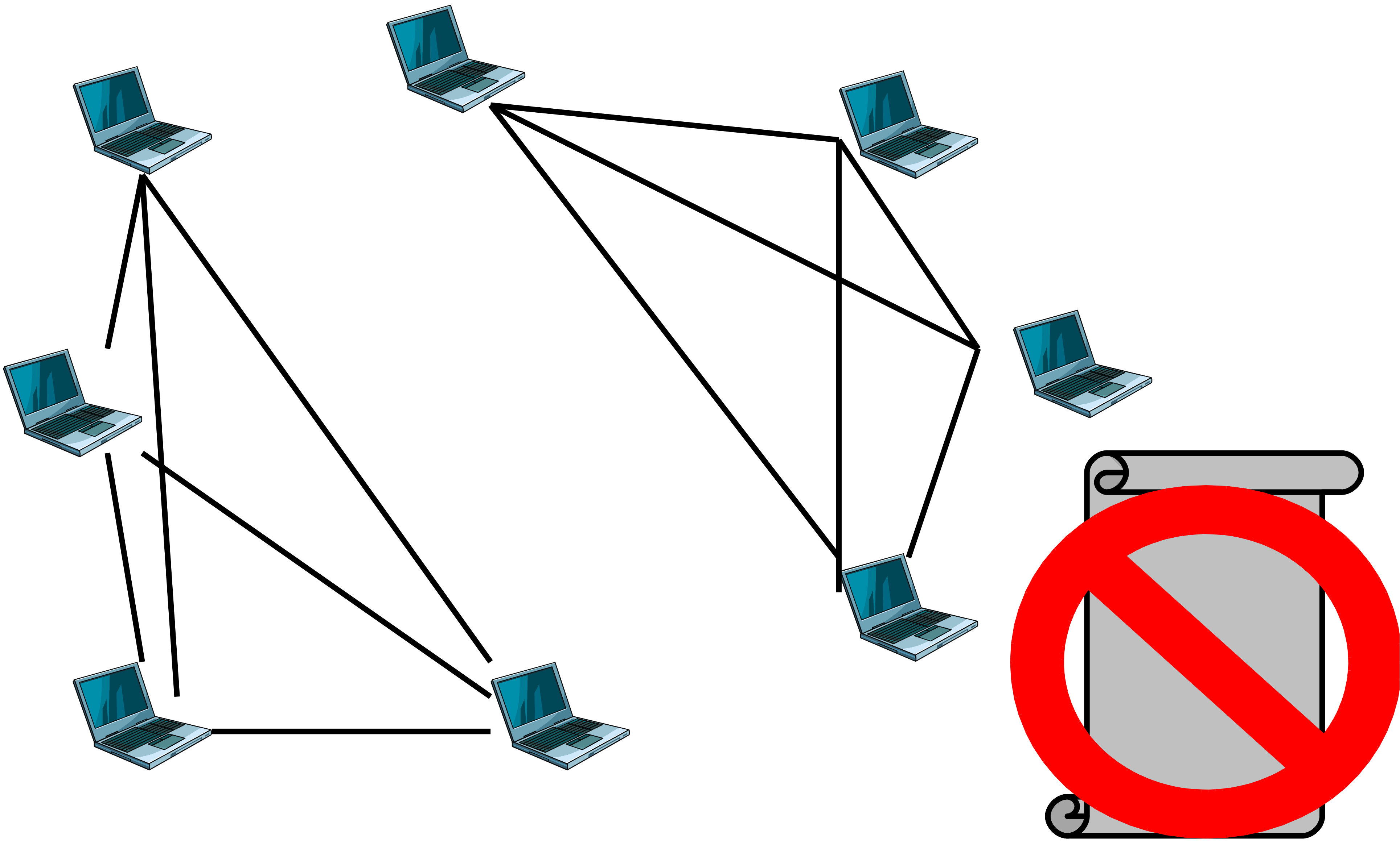
Decentralizing Overlay Networks



Decentralizing Overlay Networks



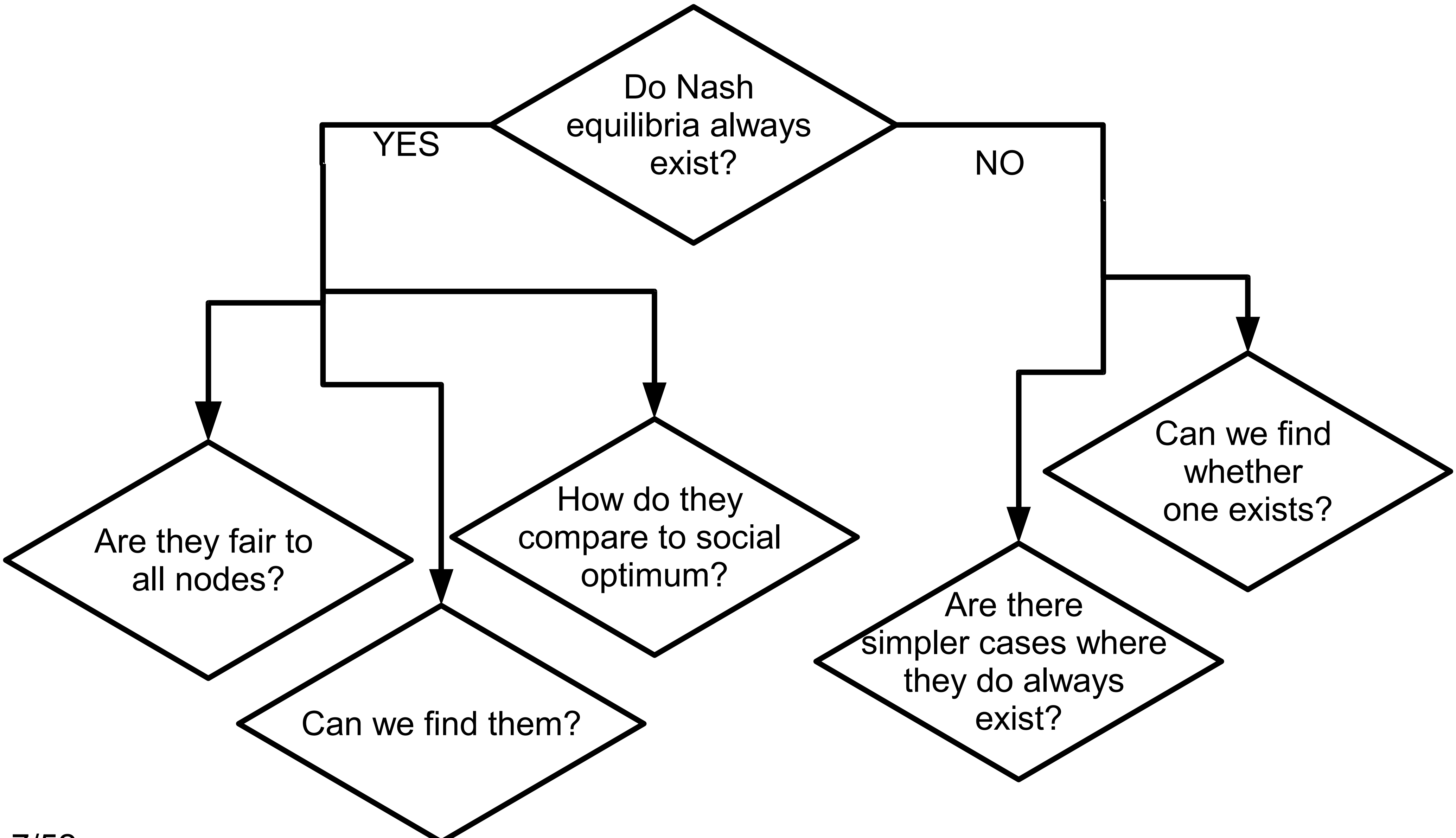
Decentralizing Overlay Networks



Techniques

- Algorithmic Game Theory
- First define a game:
  - Players = Nodes in the network
  - Actions = Connections they can make
  - Costs/Payoffs = How close am I to the other nodes?
- Then, study pure Nash equilibria
  - Do they always exist? Can we find them? What do they look like?
  - Only studying **pure** Nash equilibria

Techniques

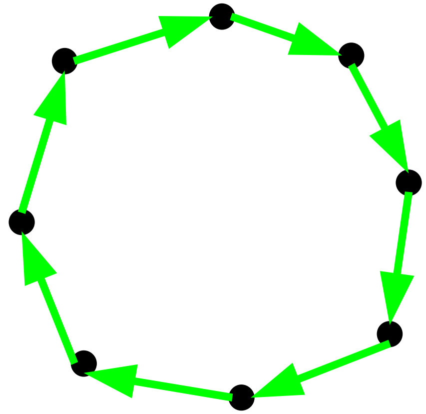

Do Nash equilibria always exist?
YES
NO
Are they fair to all nodes?
Can we find them?
How do they compare to social optimum?
Are there simpler cases where they do always exist?
Can we find whether one exists?

Simplest Game

- Players = Nodes in the network
- Actions = Connections they can make
  - One edge to any other node
- Costs/Payoffs = How close am I to the other nodes?
  - Average hop-count distance to all other nodes

Simplest Game

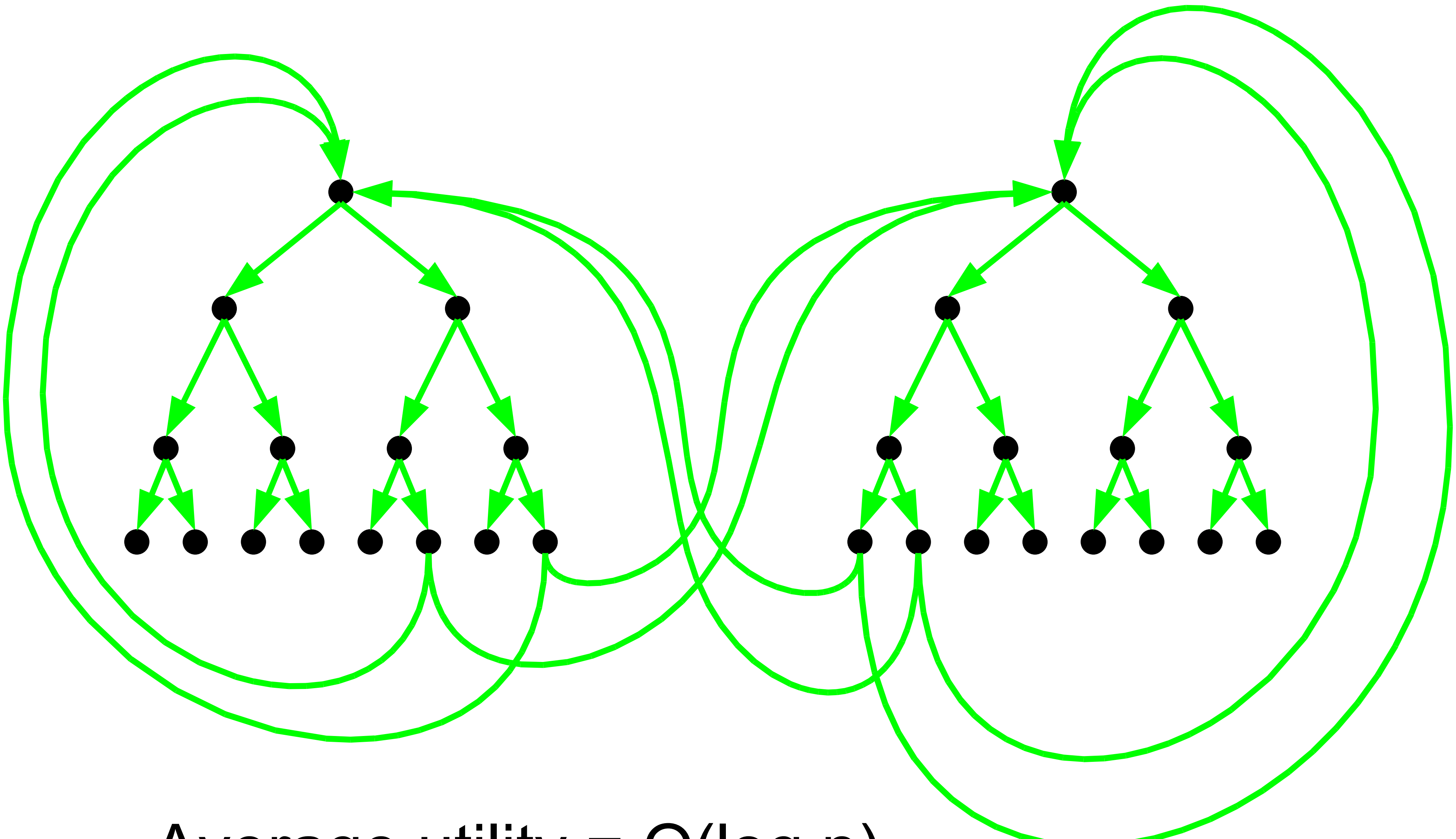
- Players = Nodes in the network
- Actions = Connections they can make
  - One edge to any other node
- Costs/Payoffs = How close am I to the other nodes?
  - Average hop-count distance to all other nodes



2 Connection Game

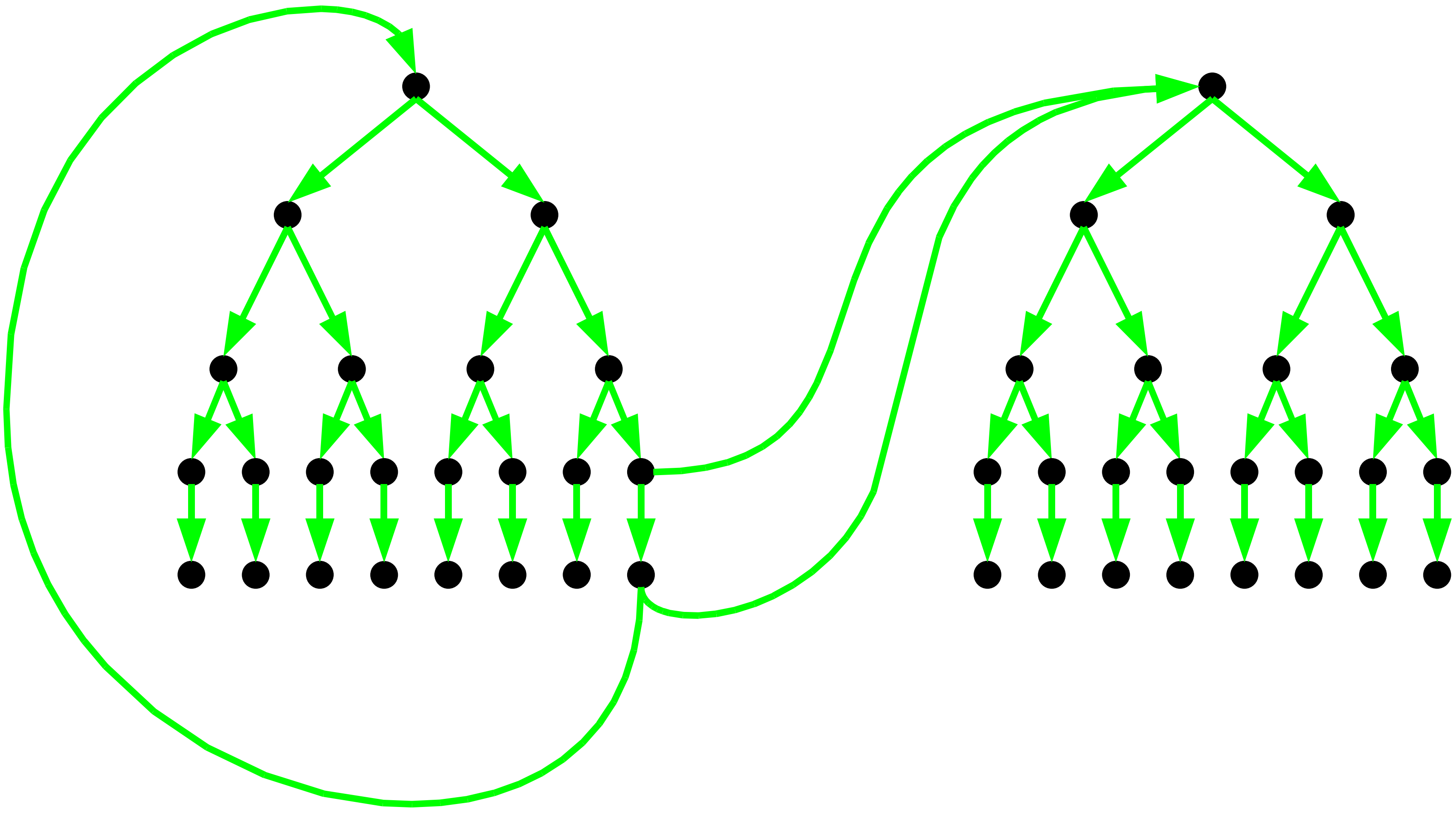
- Players = Nodes in the network
- Actions = Connections they can make
  - Two edges to any other node
- Costs/Payoffs = How close am I to the other nodes?
  - Average hop-count distance to all other nodes

2 Connection Game

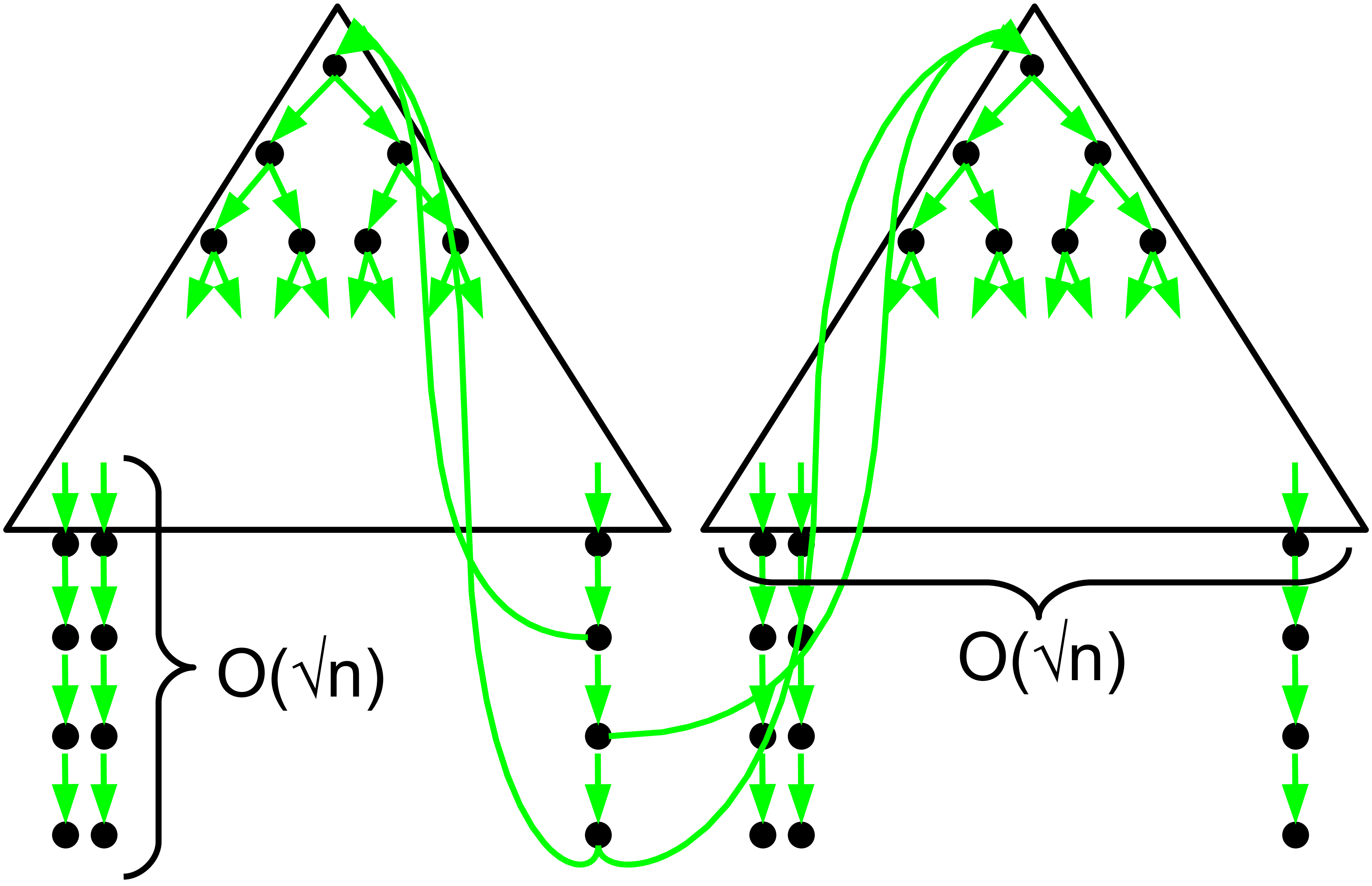


Average utility = O(log n)

2 Connection Game

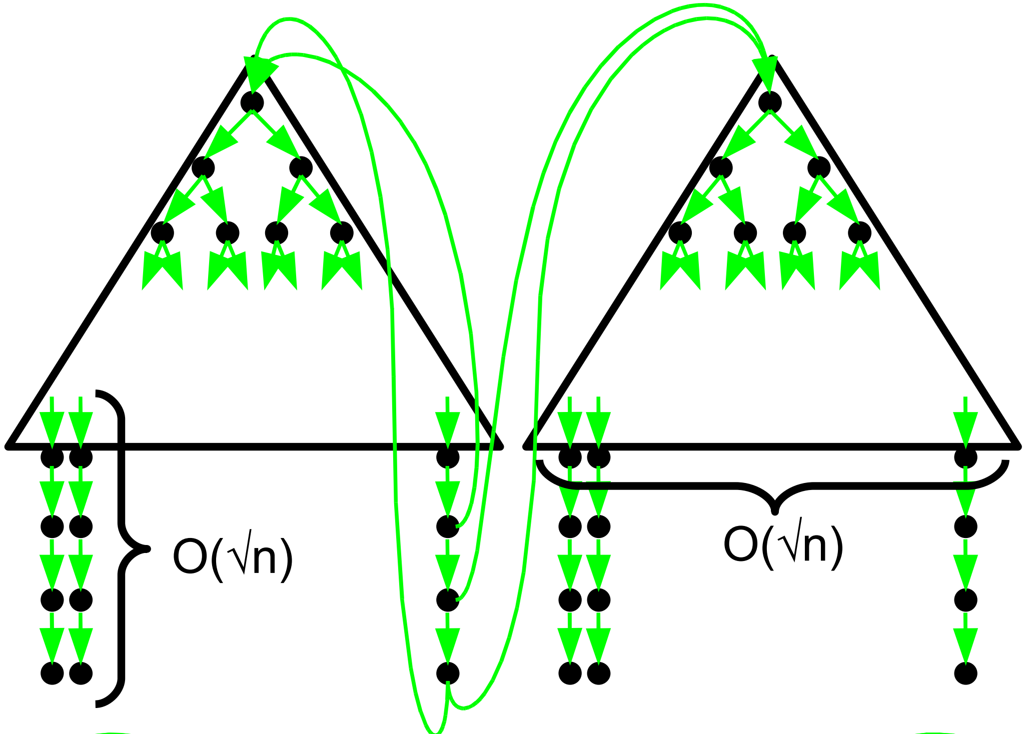


2 Connection Game

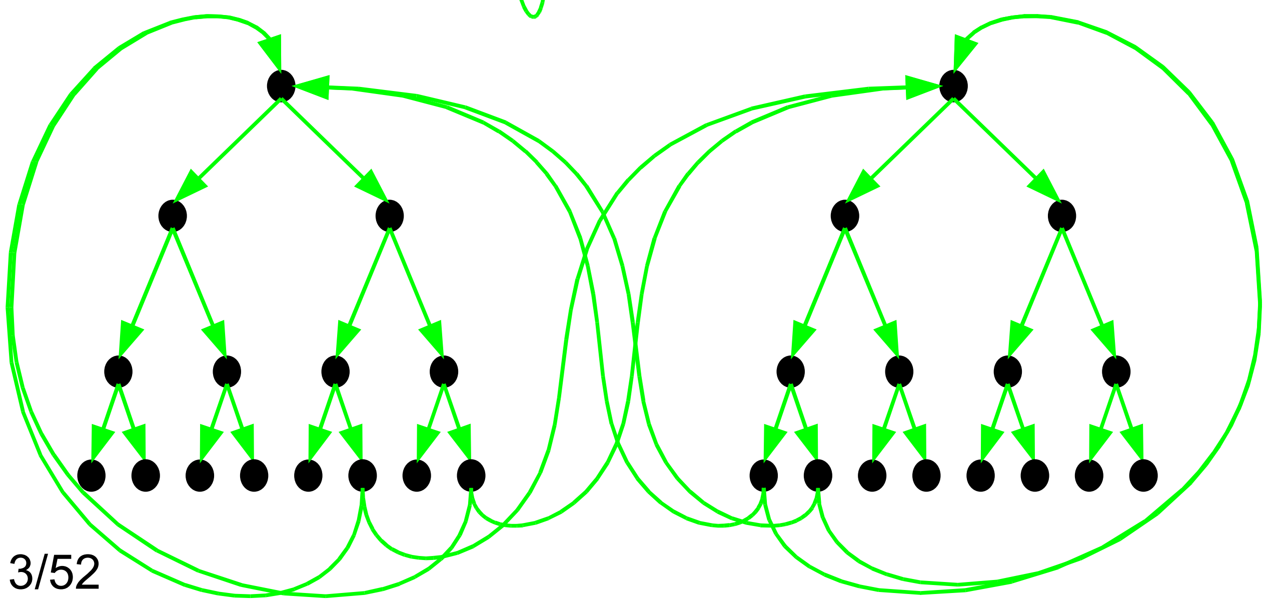


Average utility = O(√n)

2 Connection Game

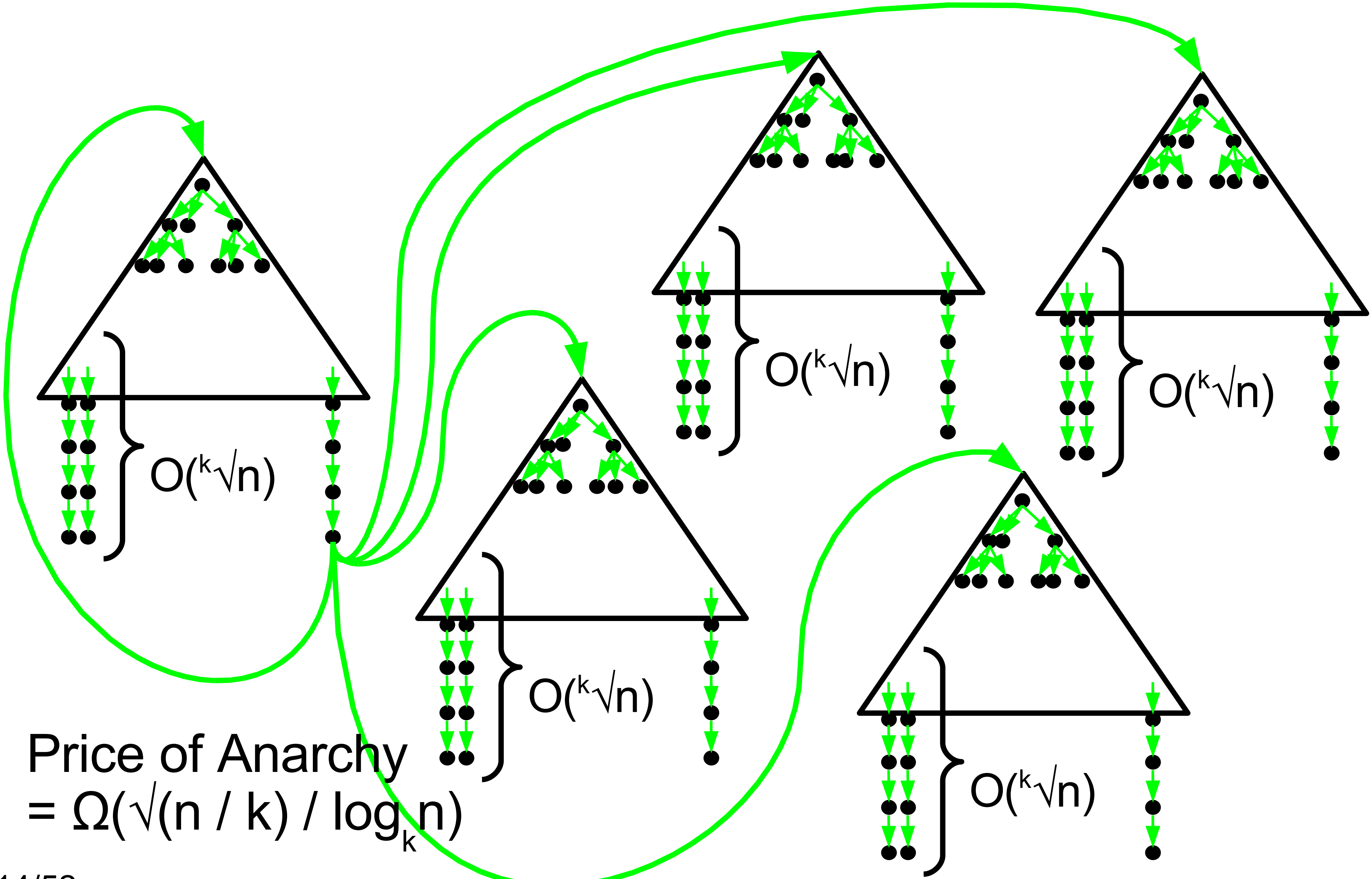


Average utility = O(√n)



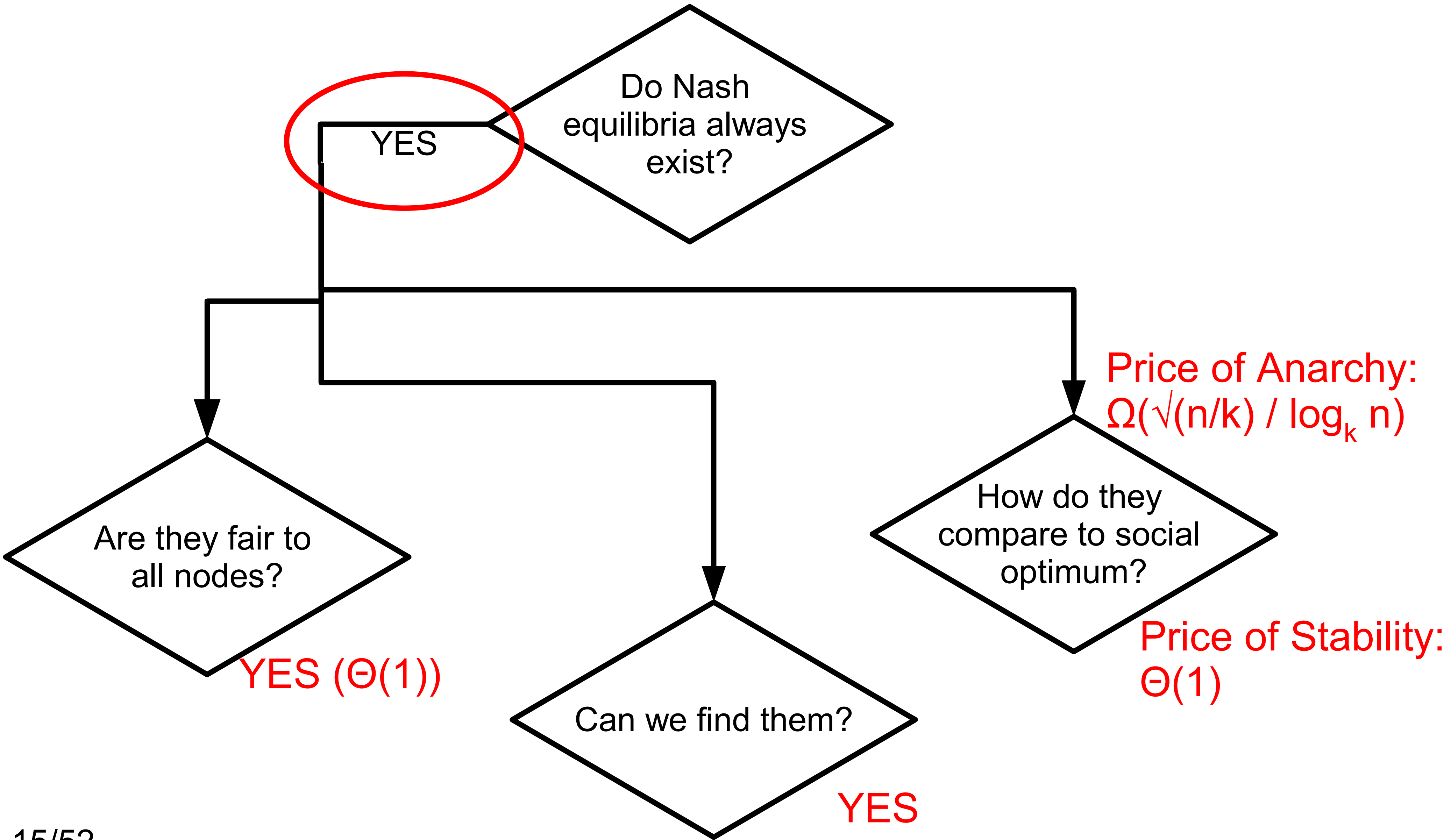
Average utility = O(log n)

k-Connection Game



Price of Anarchy
$= \Omega(\sqrt{(n / k)} / \log_k n)$

k-Connection Game



Bounded Budget Connection (BBC) Games

- Players = Nodes in the network
- Actions = Connections they can make
  - Budget to spent on edges
  - Cost for each edge
- Costs/Payoffs = How close am I to the other nodes?
  - Length on each edge
  - Affinity for each other node
  - Average affinity-weighted shortest path distance to all other nodes

The Model

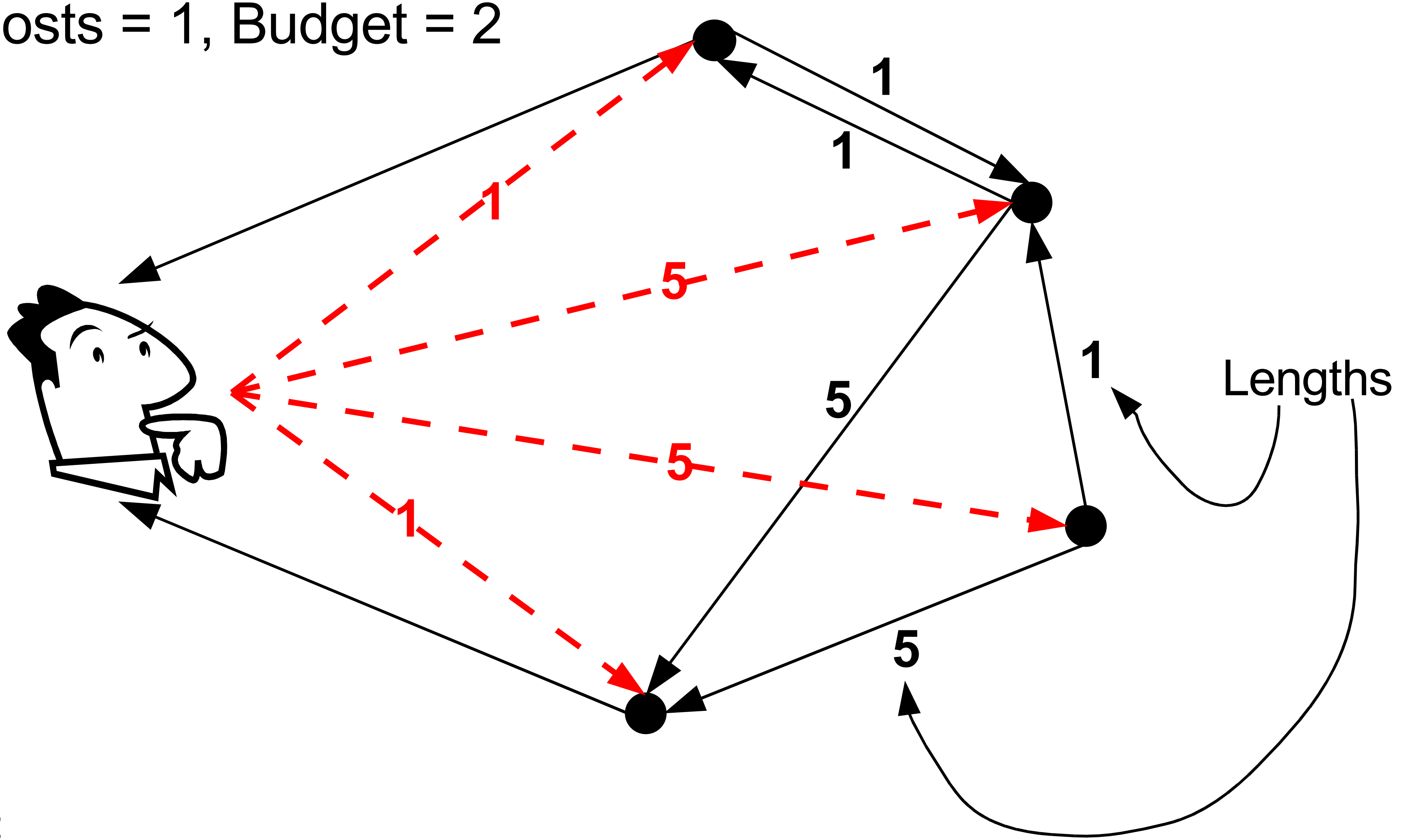
- Number of nodes
- Affinity for each directed pair of nodes
- Link cost for each directed pair of nodes
- Budget of allowed link cost per node, *k(v)*
- Length metric from the perspective of each node
- Each node *v* spends ≤ *k(v)* on links to minimize

$$\sum_{\text{other nodes}} (\text{affinity} * \text{shortest path distance})$$

↑ or disconnection penalty if no path exists.

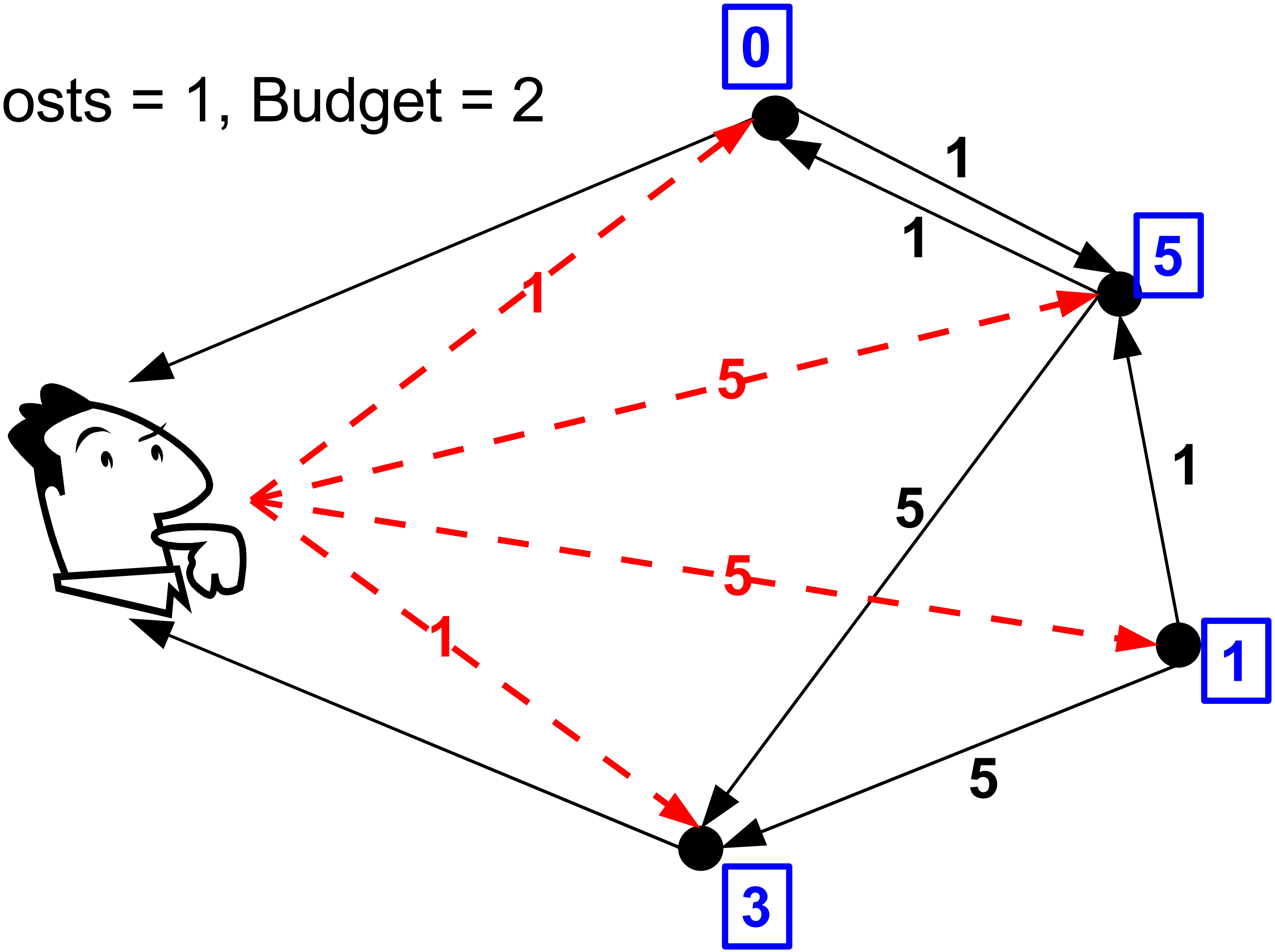
Example

Costs = 1, Budget = 2



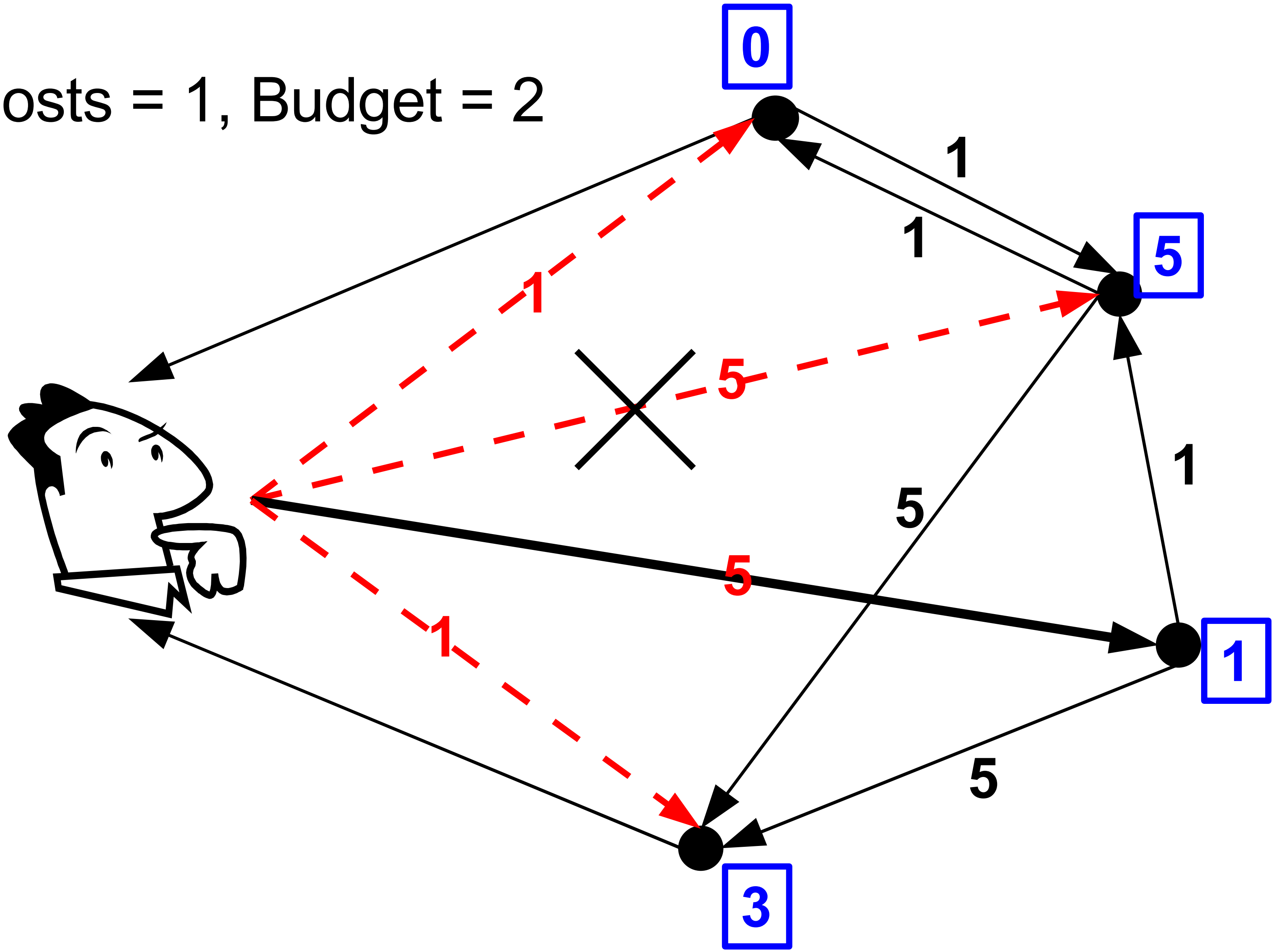
Example

Costs = 1, Budget = 2



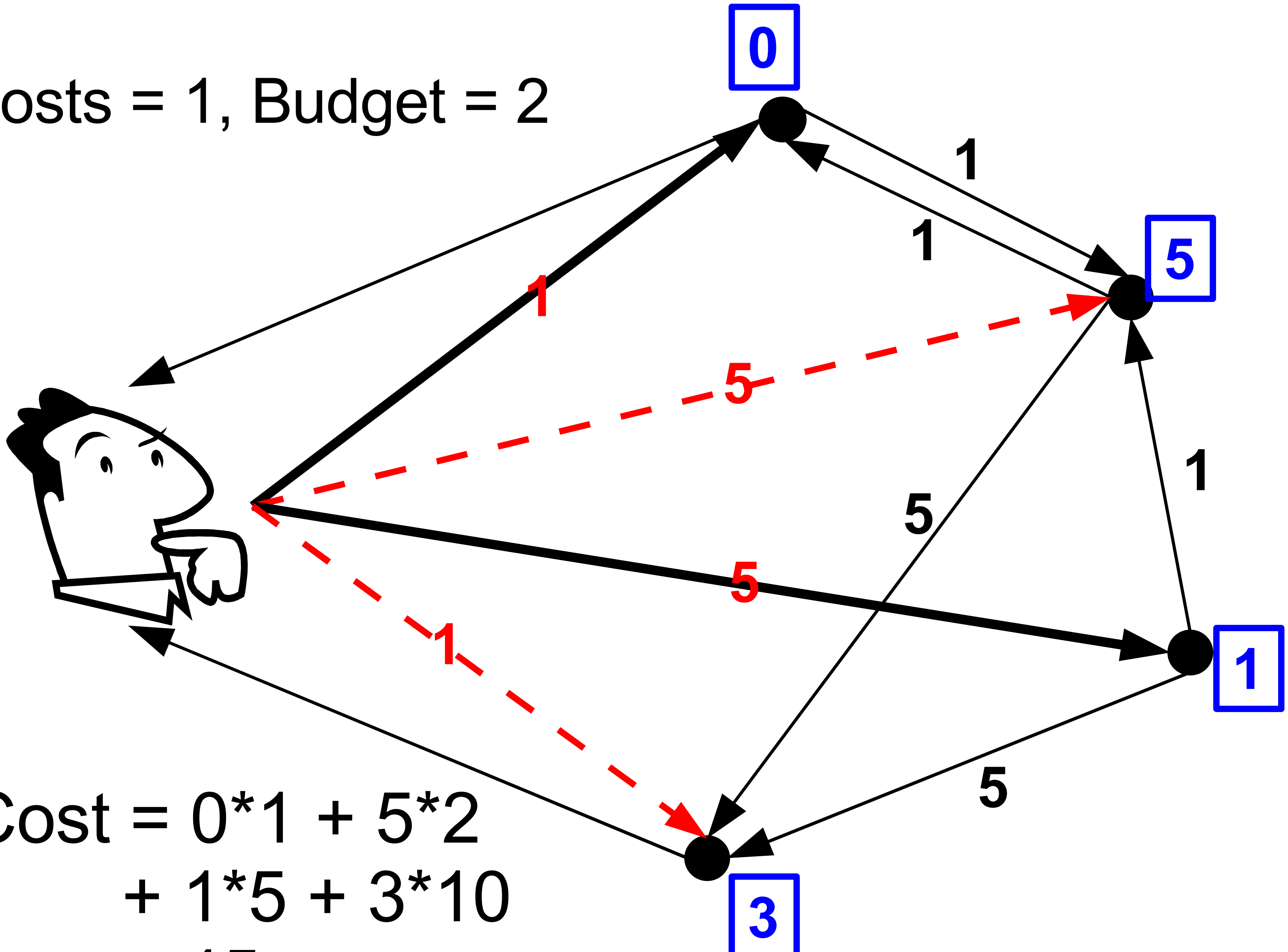
Example

Costs = 1, Budget = 2



Example

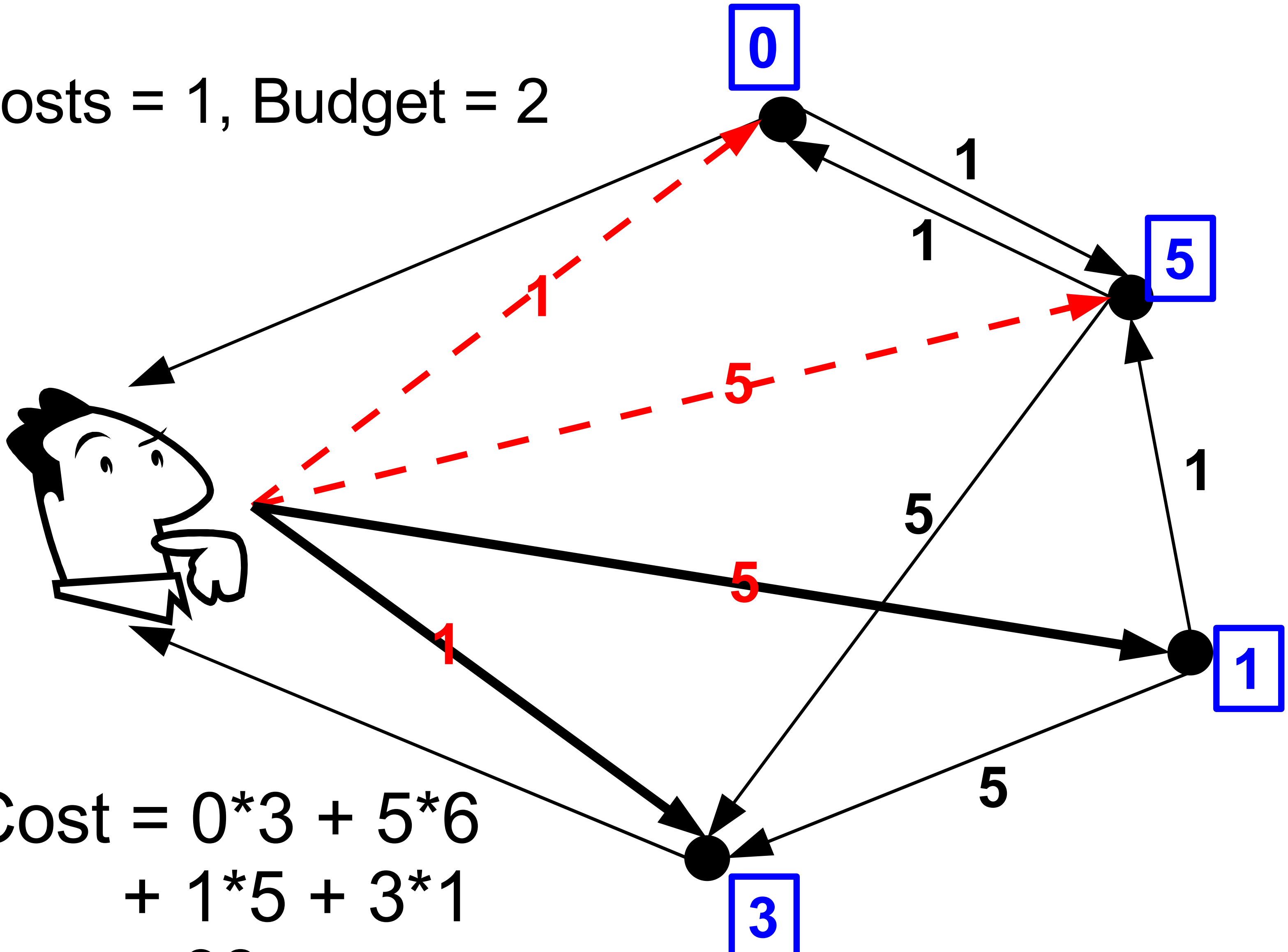
Costs = 1, Budget = 2



Cost = 0*1 + 5*2
+ 1*5 + 3*10
= 45

Example

Costs = 1, Budget = 2



Cost = 0*3 + 5*6
+ 1*5 + 3*1
= 38

Related Work on Network Connection Games

- Cost per edge built into utility instead of a budget built into actions.
  - Fabrikant, Luthra, Maneva, Papadimitriou, and Shenker. On a network creation game. PODC, 2003.
  - Albers, Eilts, Even-Dar, Mansour, and Roditty. On Nash equilibria for a network creation game. SODA, 2006.
  - Demaine, Hajiaghavi, and Mahini. The Price of Anarchy in Network Creation Games. PODC, 2007.
  - Halevi and Mansour. A Network Creation Game with Nonuniform Interests. WINE, 2007.

Related Work
on Network Connection Games

- Experimental results on very similar game.
  - Chun, Fonseca, Stoica, and Kubiatowicz. Characterizing selfishly constructed overlay routing networks. INFOCOM, 2004.
  - Laoutaris, Smaragdakis, Bestavros, John Byers. Implications of selfish neighbor selection in overlay networks. INFOCOM, 2007.

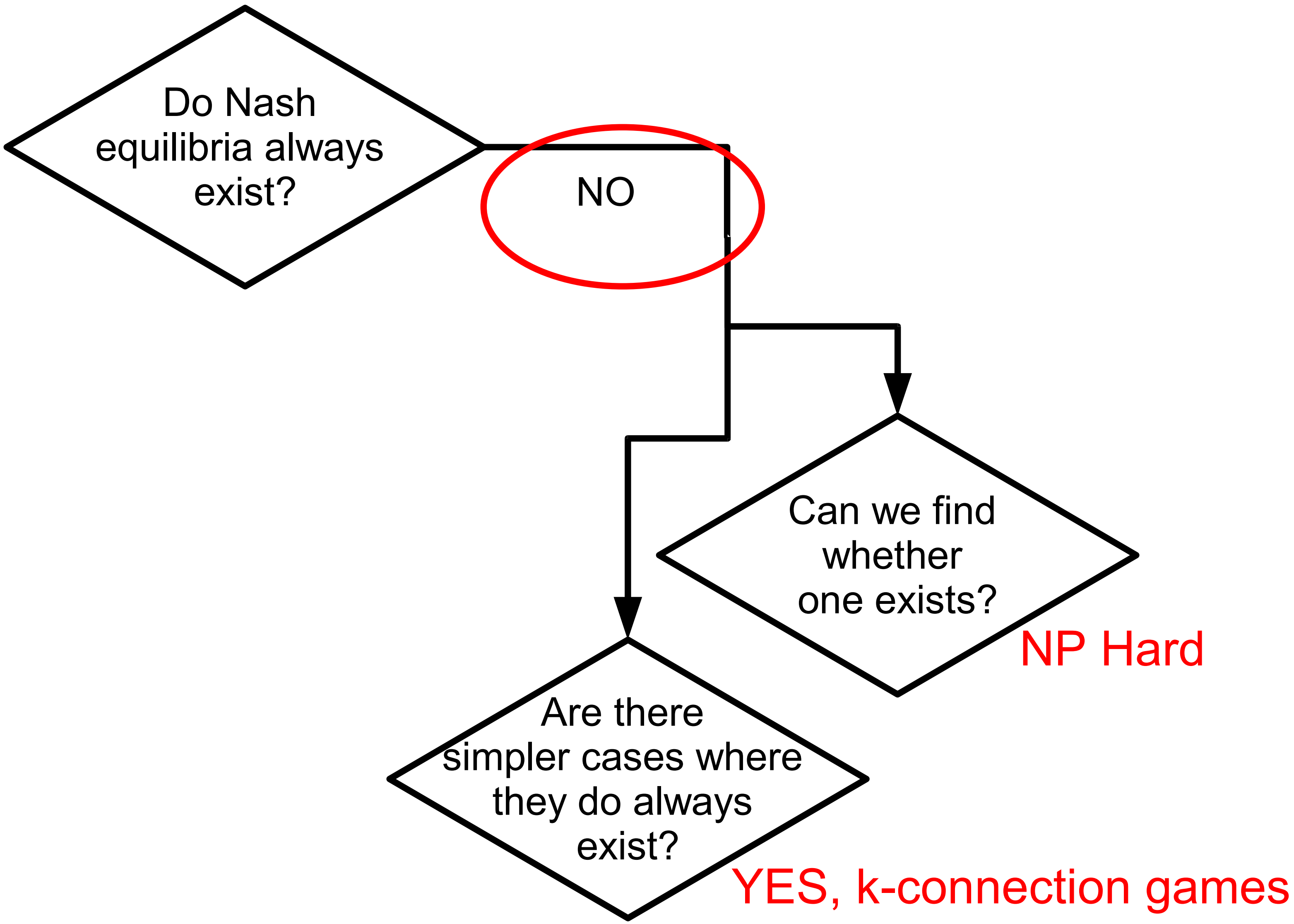
May be no Nash equilibrium

- Number of nodes
- **Affinity for each directed pair of nodes**
- **Link cost for each directed pair of nodes**
- Budget of allowed link cost per node, *k(v)*
- Length metric from the perspective of each node
- Each node *v* spends ≤ *k(v)* on links to minimize

$$\sum_{\text{other nodes}} (\text{affinity} * \text{shortest path distance})$$

↑ or disconnection penalty if no path exists.

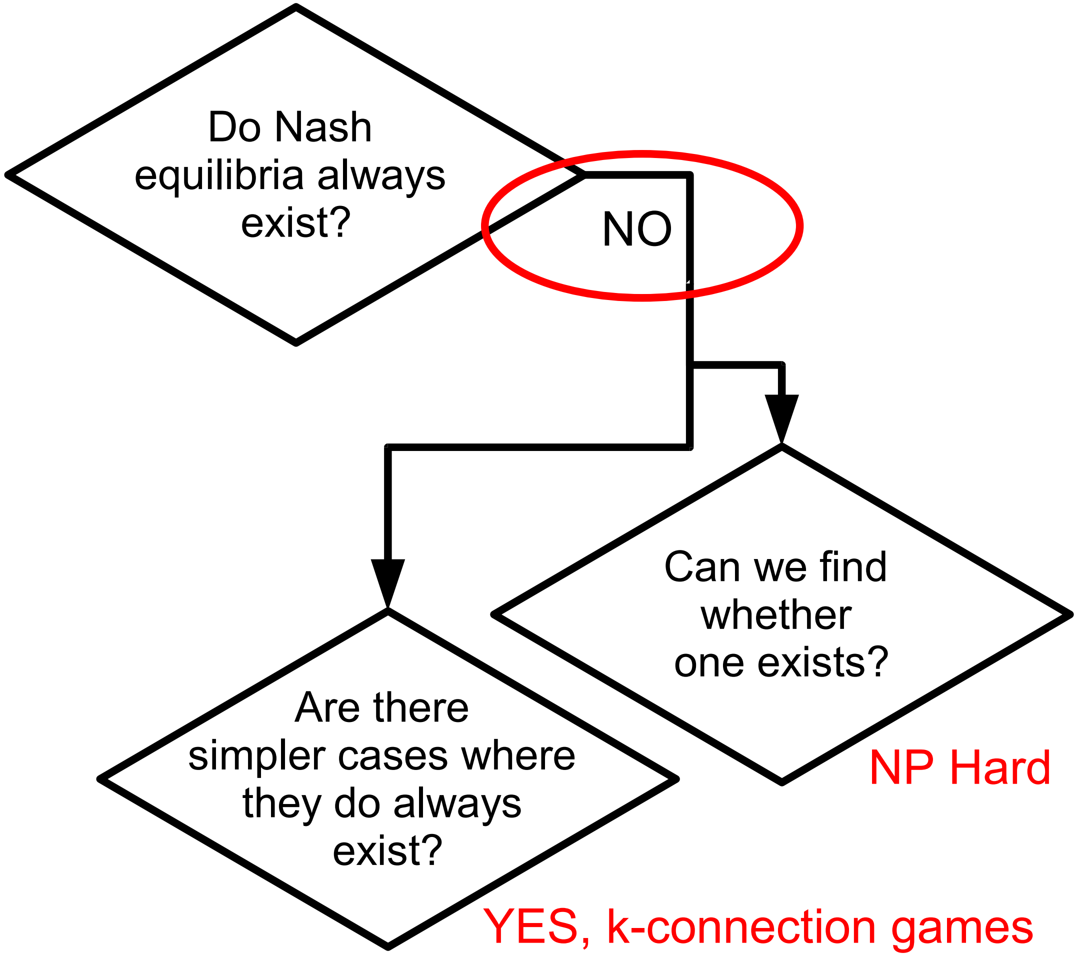
BBC Games


Do Nash equilibria always exist?
NO
Can we find whether one exists?
NP Hard
Are there simpler cases where they do always exist?
YES, k-connection games

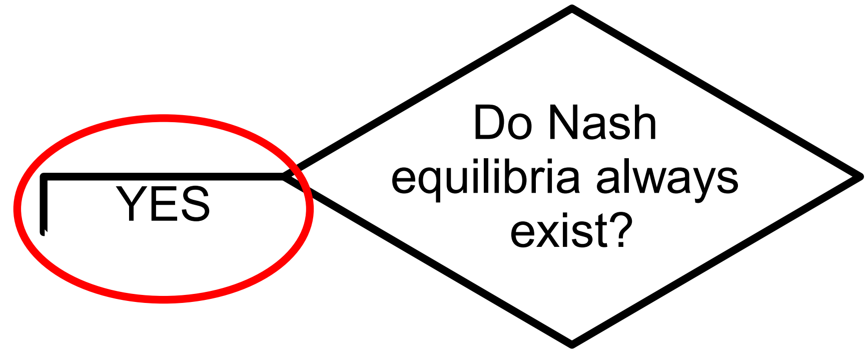
Open Questions

- Only budgets or lengths are non-uniform
- All nodes have same affinity function

BBC Games



Fractional Games



Fractional BBC Game

- Players = Nodes in the network
- Actions = Connections they can make
  - Budget to spend on edges, cost per edge
  - Fractionally purchase adjacent edges, spending up to the budget
- Costs/Payoffs = How close am I to the other nodes?
  - Affinities for other nodes, lengths for each edge
  - Affinity-weighted average cost of 1-unit minimum cost flow (capacity = purchased amount)

Fractional BBC Game

- Number of nodes
- Affinity for each directed pair of nodes
- Link cost for each directed pair of nodes
- Budget of allowed link cost per node, *k(v)*
- Length metric from the perspective of each node
- Each node *v* spends ≤ *k(v)* on links to minimize

$$\sum_{\text{other nodes}} (\text{affinity} * \text{cost of min cost 1 unit flow})$$

↑ or disconnection penalty if no path exists.

Fractional BBC Game

- Number of nodes
- ~~Affinity for each directed pair of nodes~~ Specified universal destination node
- Link cost for each directed pair of nodes
- Budget of allowed link cost per node, *k(v)*
- Length metric from the perspective of each node
- Each node *v* spends ≤ *k(v)* on links to minimize

cost of min cost 1 unit flow to destination

or disconnection penalty if no path exists.

Fractional BBC Game

- Number of nodes
- ~~Affinity for each directed pair of nodes~~ Specified universal destination node
- Link cost ~~for each directed pair of nodes~~ =1
- Budget ~~of allowed link cost per node~~, *k(v)* =1
- Length metric from the perspective of each node
- Each node *v* spends ≤ *k(v)* on links to minimize

cost of min cost 1 unit flow to destination

↑ or disconnection penalty if no path exists.

Fractional BBC Game

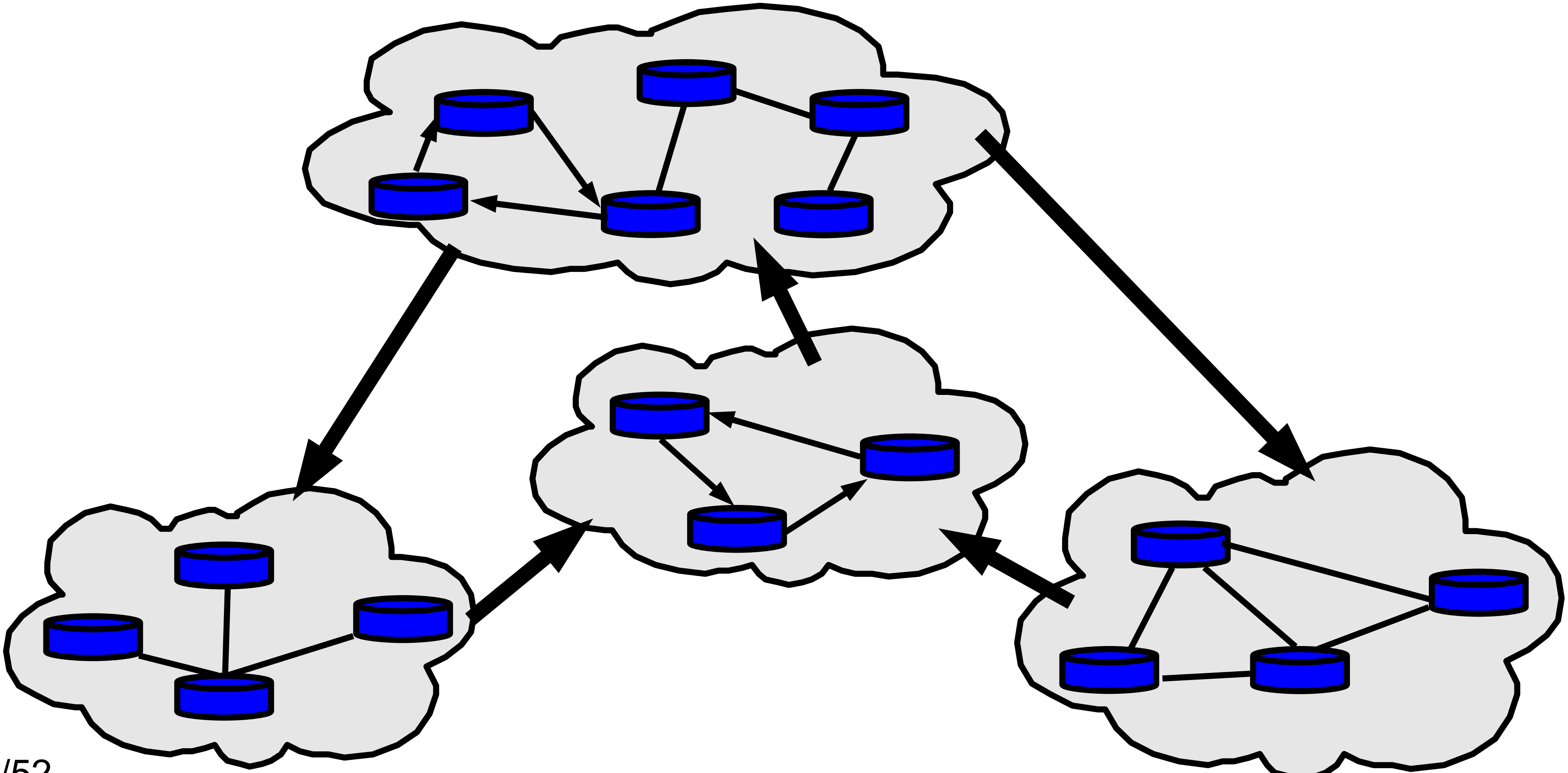
- Universal destination node
- Link cost = 1
- Budget = 1
- Length metric from the perspective of each node
- Each node *v* spends 1 on links to minimize cost of min cost 1 unit flow to destination

Fractional BGP Game

- Universal destination node
- Weight 1 to spread across paths
- Preference list across paths to the destination
- Cannot use a path more than the next node along the path
- Best Response: Take as much as possible of highest preference paths.

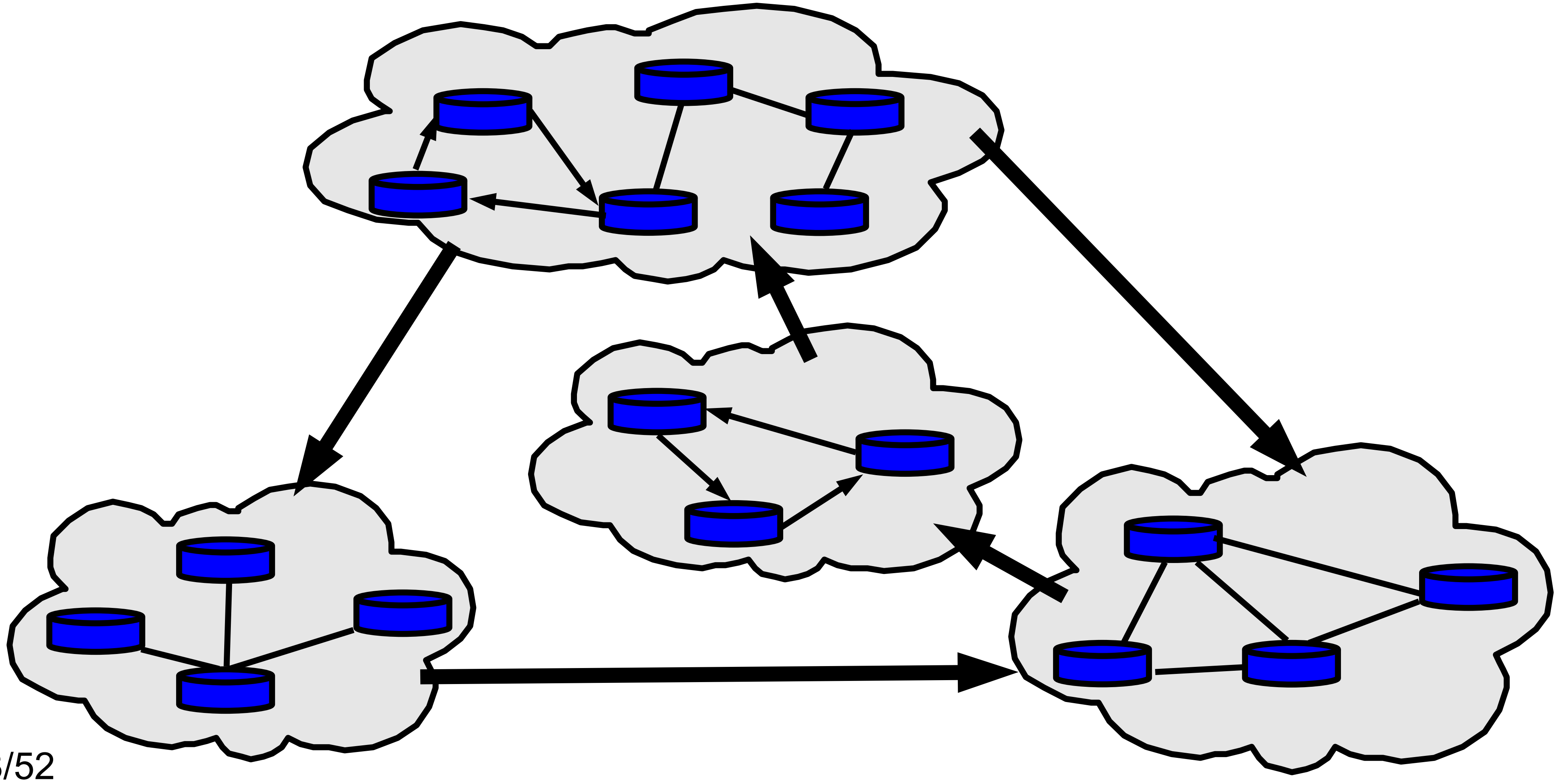
Border Gateway Protocol

- Rehkter, Li. A Border Gateway Protocol (BGP version 4). RFC 1771, 1995.



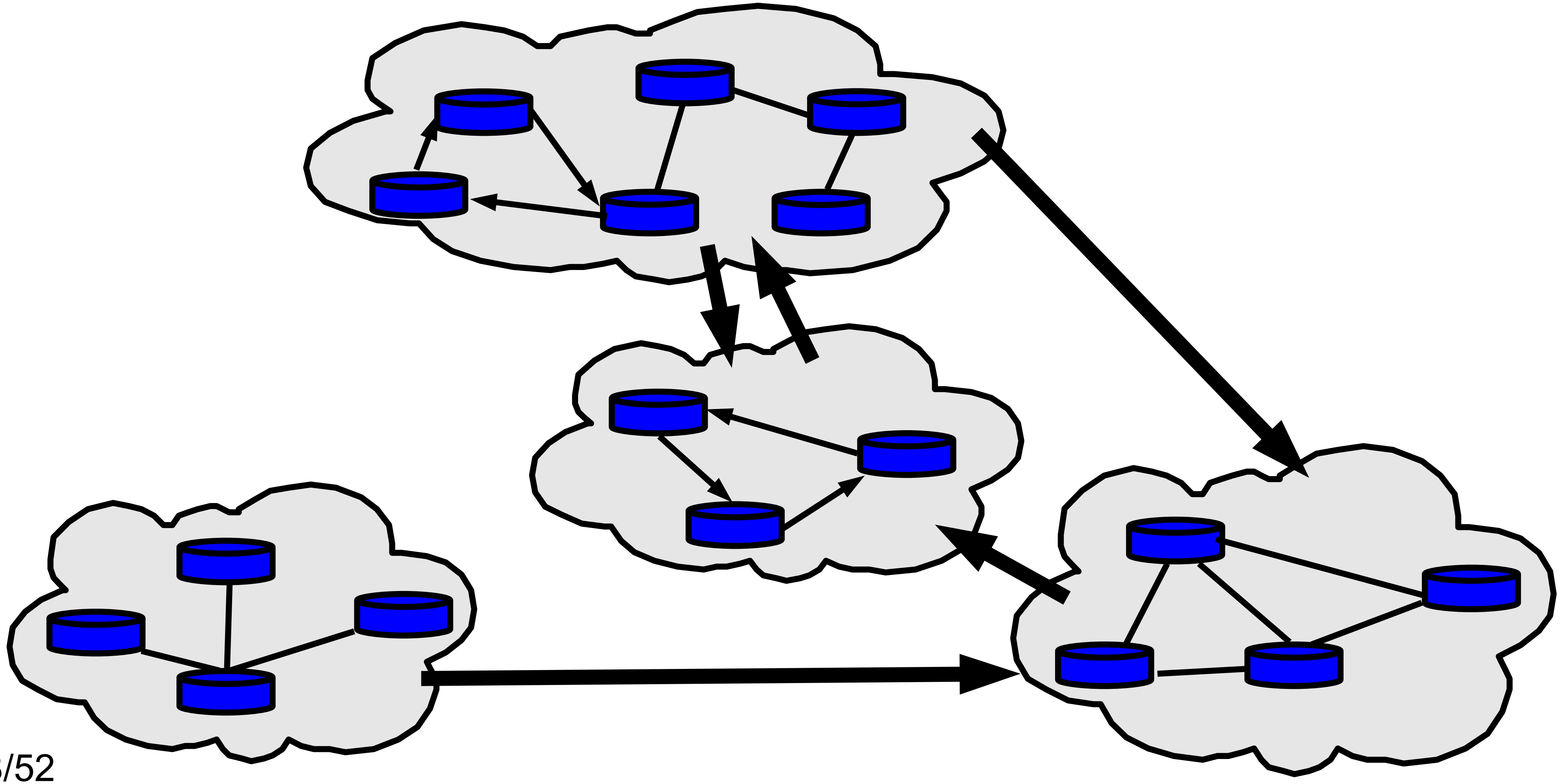
Border Gateway Protocol

- Varadhan, Govindan, and Estrin. Persistent Route Oscillations in Inter-Domain Routing. Technical Report USC CS TR 96-631, Dept of Computer Science, USC, 1996.



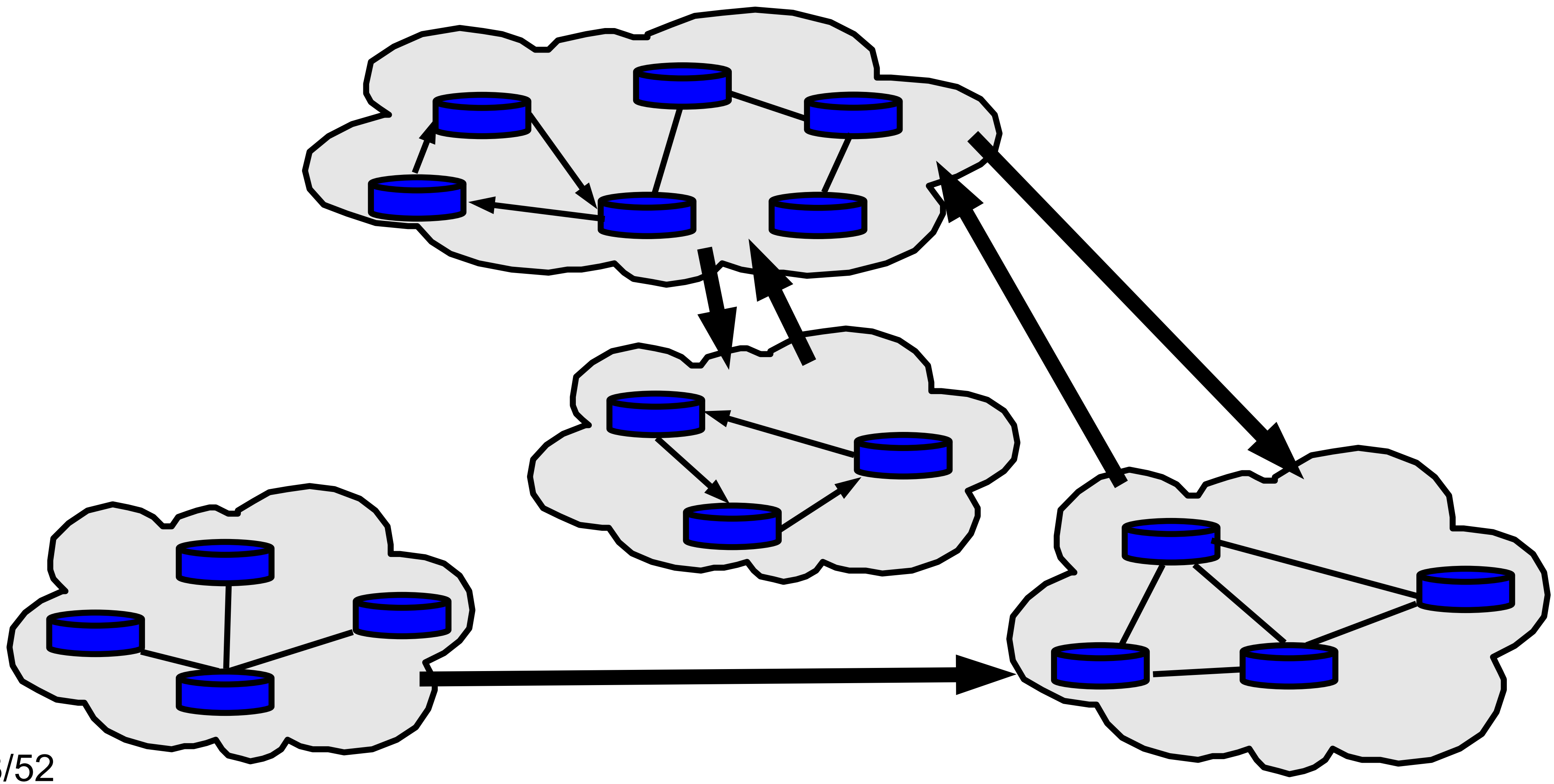
Border Gateway Protocol

- Varadhan, Govindan, and Estrin. Persistent Route Oscillations in Inter-Domain Routing. Technical Report USC CS TR 96-631, Dept of Computer Science, USC, 1996.



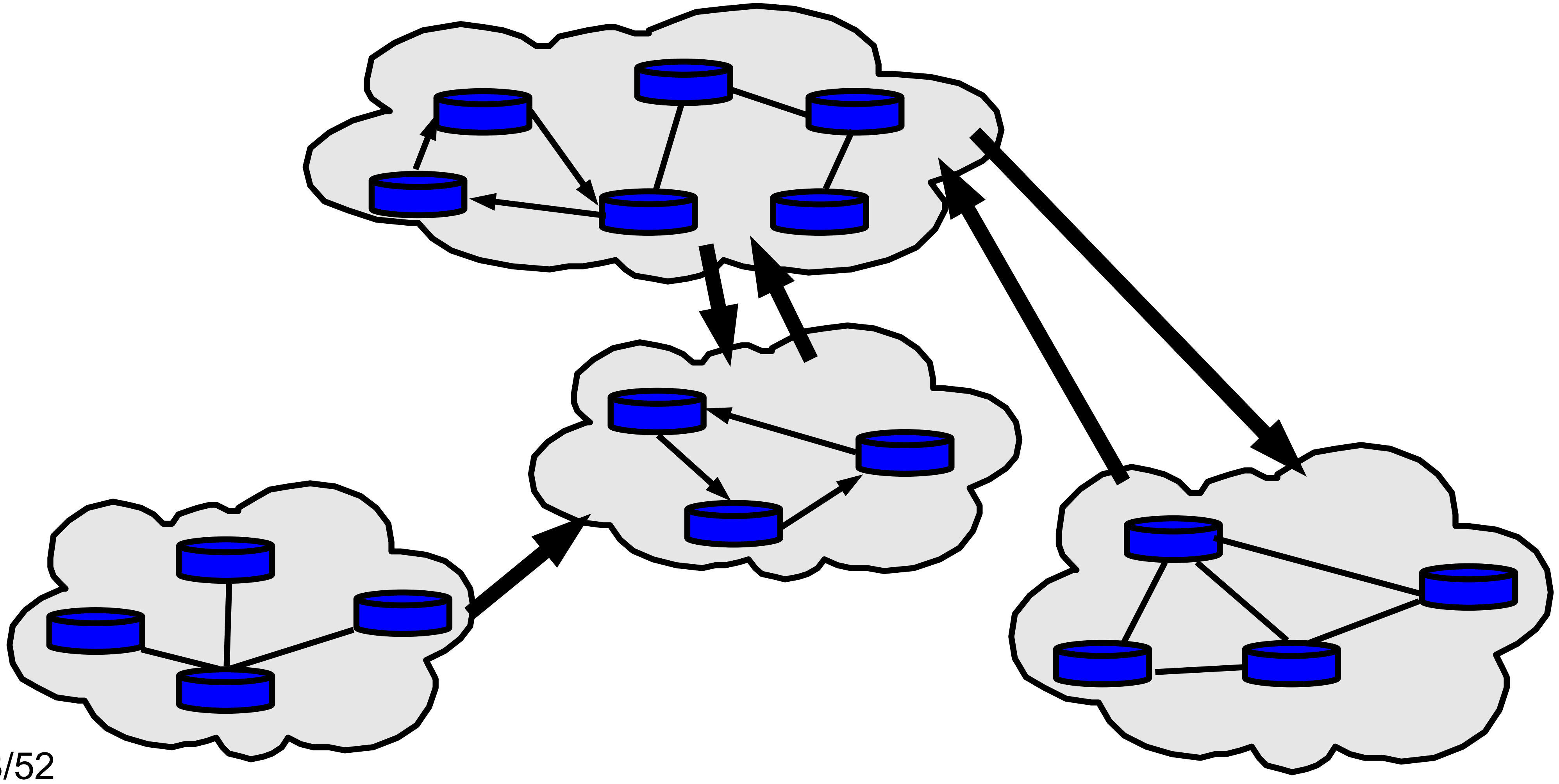
Border Gateway Protocol

- Varadhan, Govindan, and Estrin. Persistent Route Oscillations in Inter-Domain Routing. Technical Report USC CS TR 96-631, Dept of Computer Science, USC, 1996.



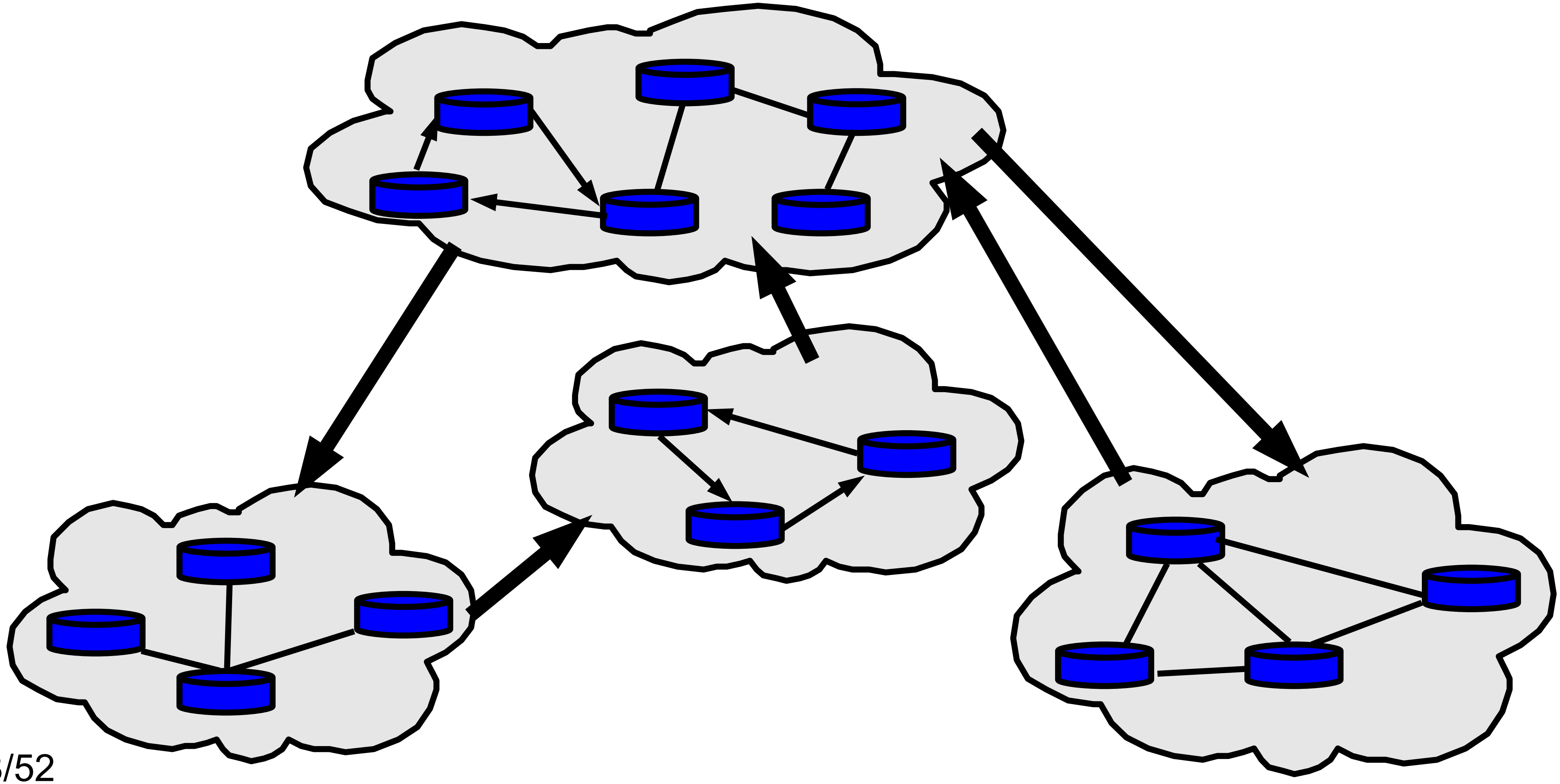
Border Gateway Protocol

- Varadhan, Govindan, and Estrin. Persistent Route Oscillations in Inter-Domain Routing. Technical Report USC CS TR 96-631, Dept of Computer Science, USC, 1996.



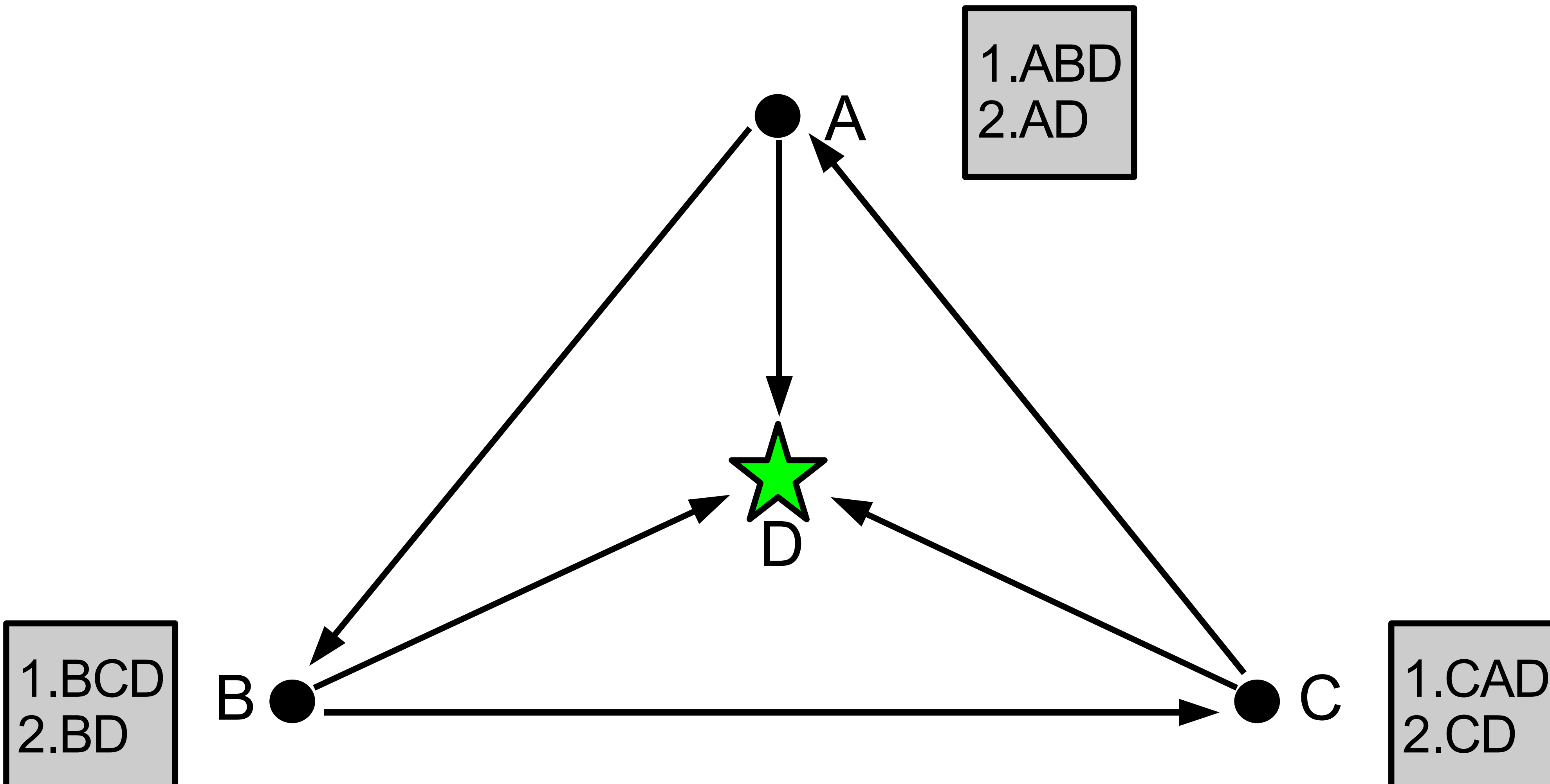
Border Gateway Protocol

- Varadhan, Govindan, and Estrin. Persistent Route Oscillations in Inter-Domain Routing. Technical Report USC CS TR 96-631, Dept of Computer Science, USC, 1996.



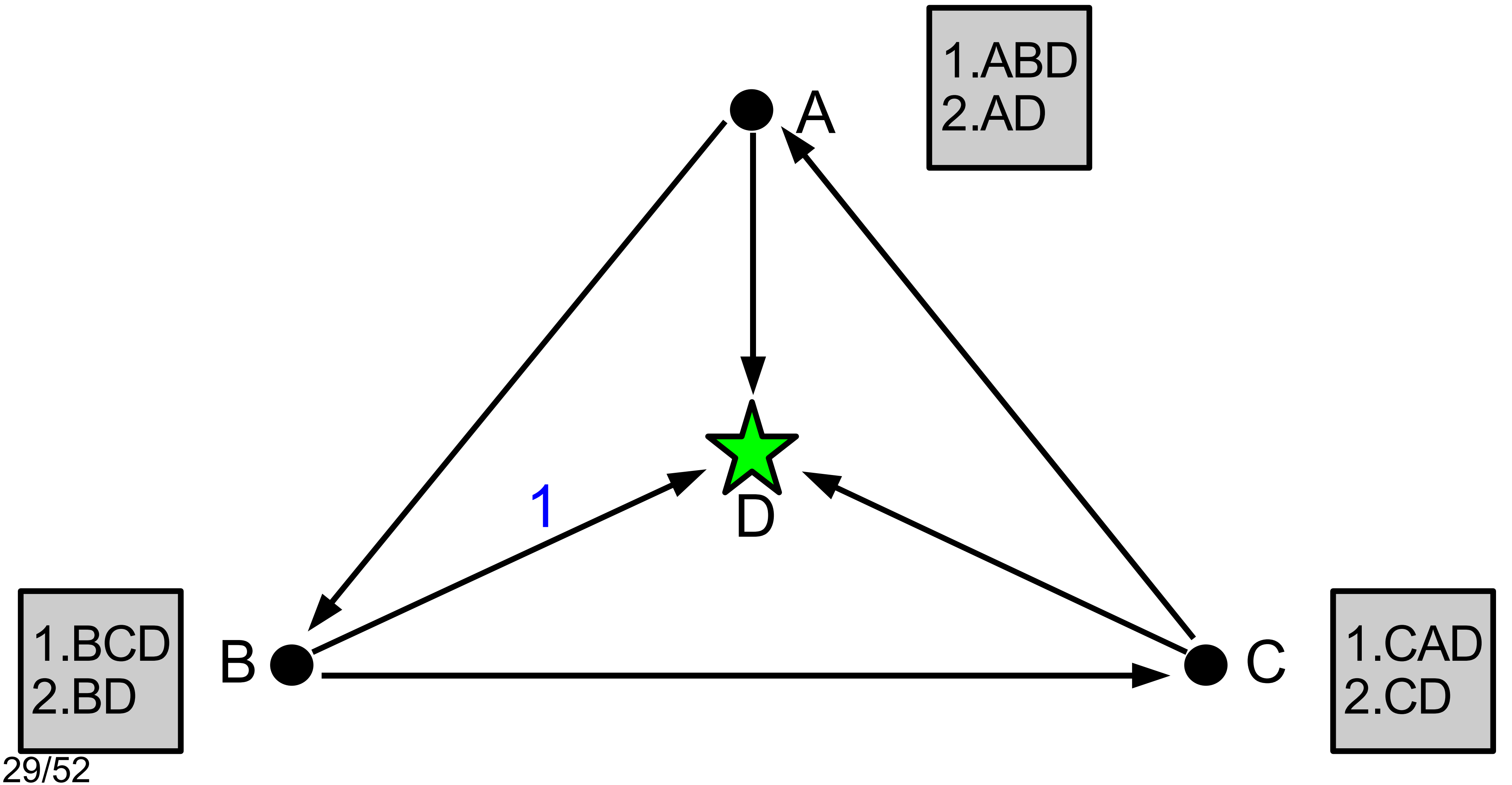
Stable Paths Problem

- Griffin, Shepherd, and Wilfong. The stable paths problem and interdomain routing. Transactions on Networking, 2002.



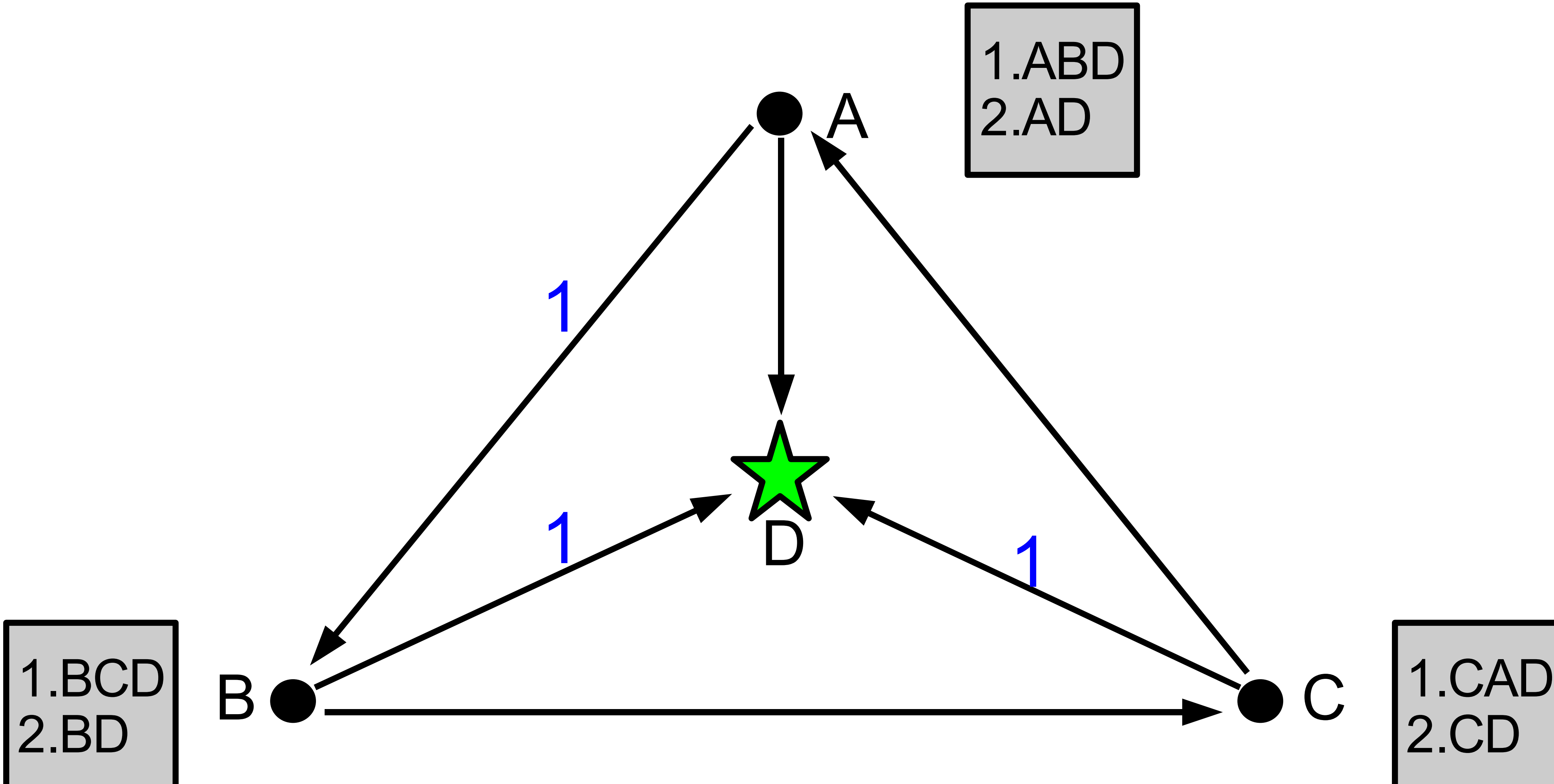
Stable Paths Problem

- Griffin, Shepherd, and Wilfong. The stable paths problem and interdomain routing. Transactions on Networking, 2002.



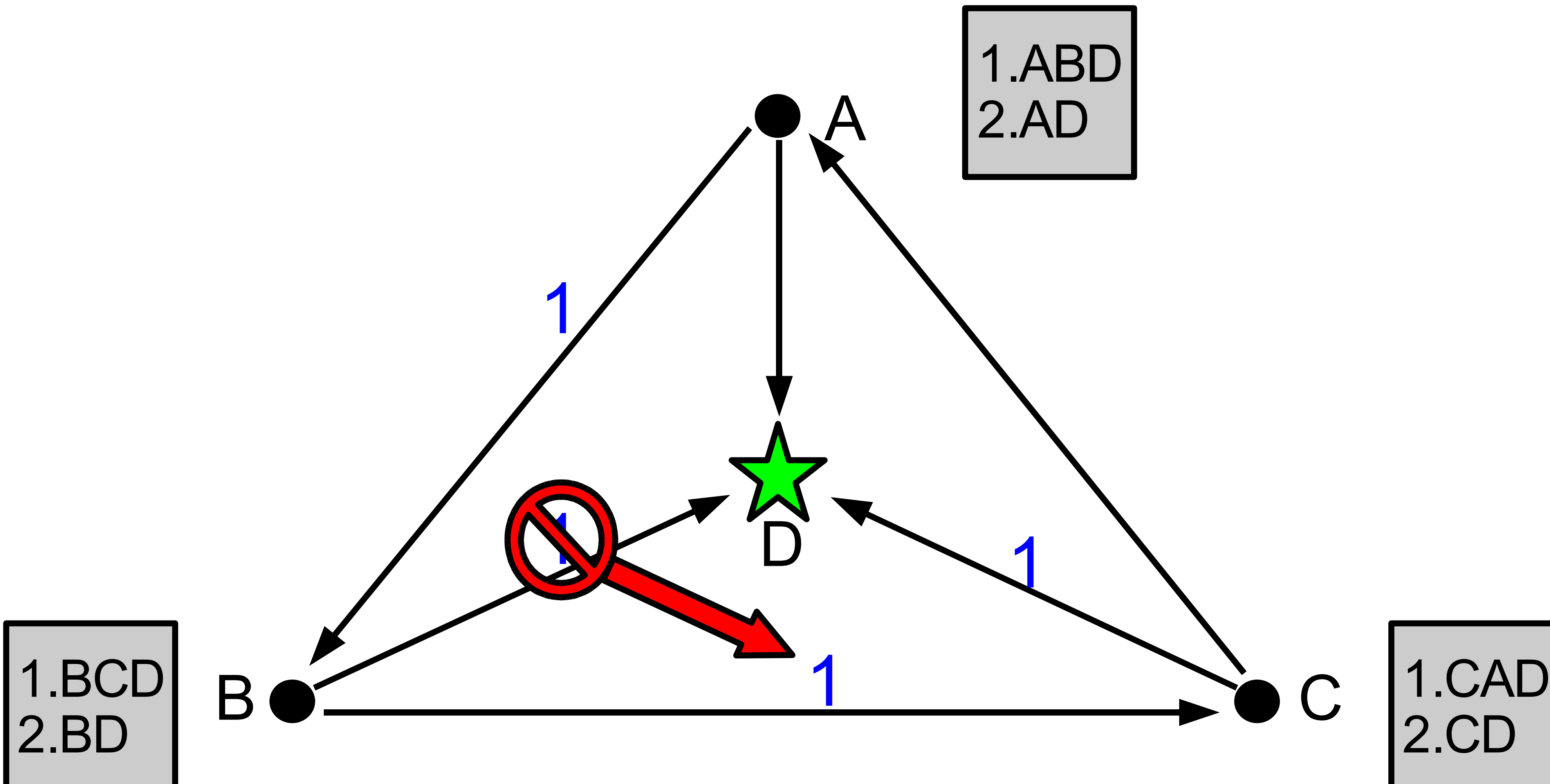
Stable Paths Problem

- Griffin, Shepherd, and Wilfong. The stable paths problem and interdomain routing. Transactions on Networking, 2002.



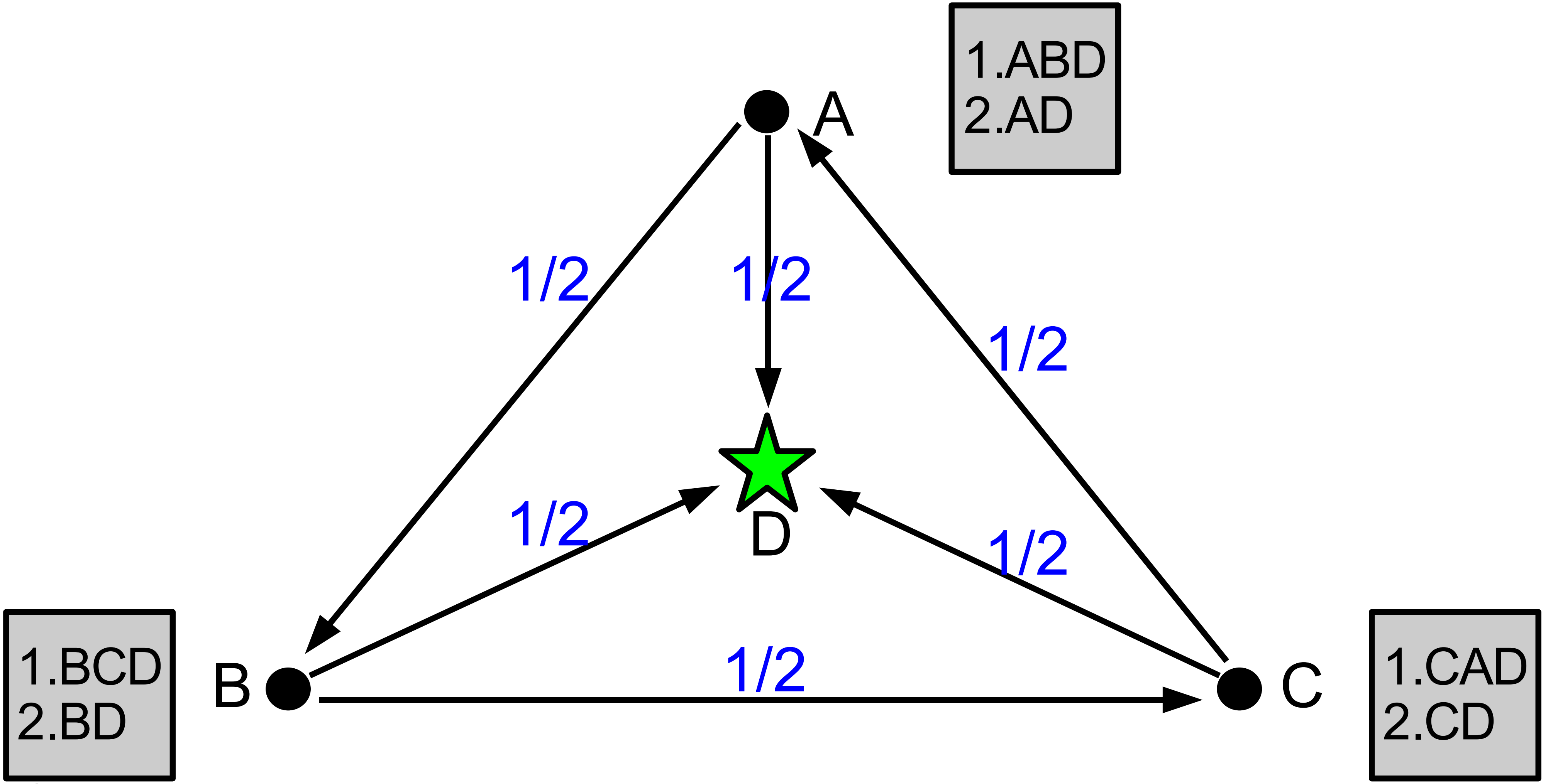
Stable Paths Problem

- Griffin, Shepherd, and Wilfong. The stable paths problem and interdomain routing. Transactions on Networking, 2002.

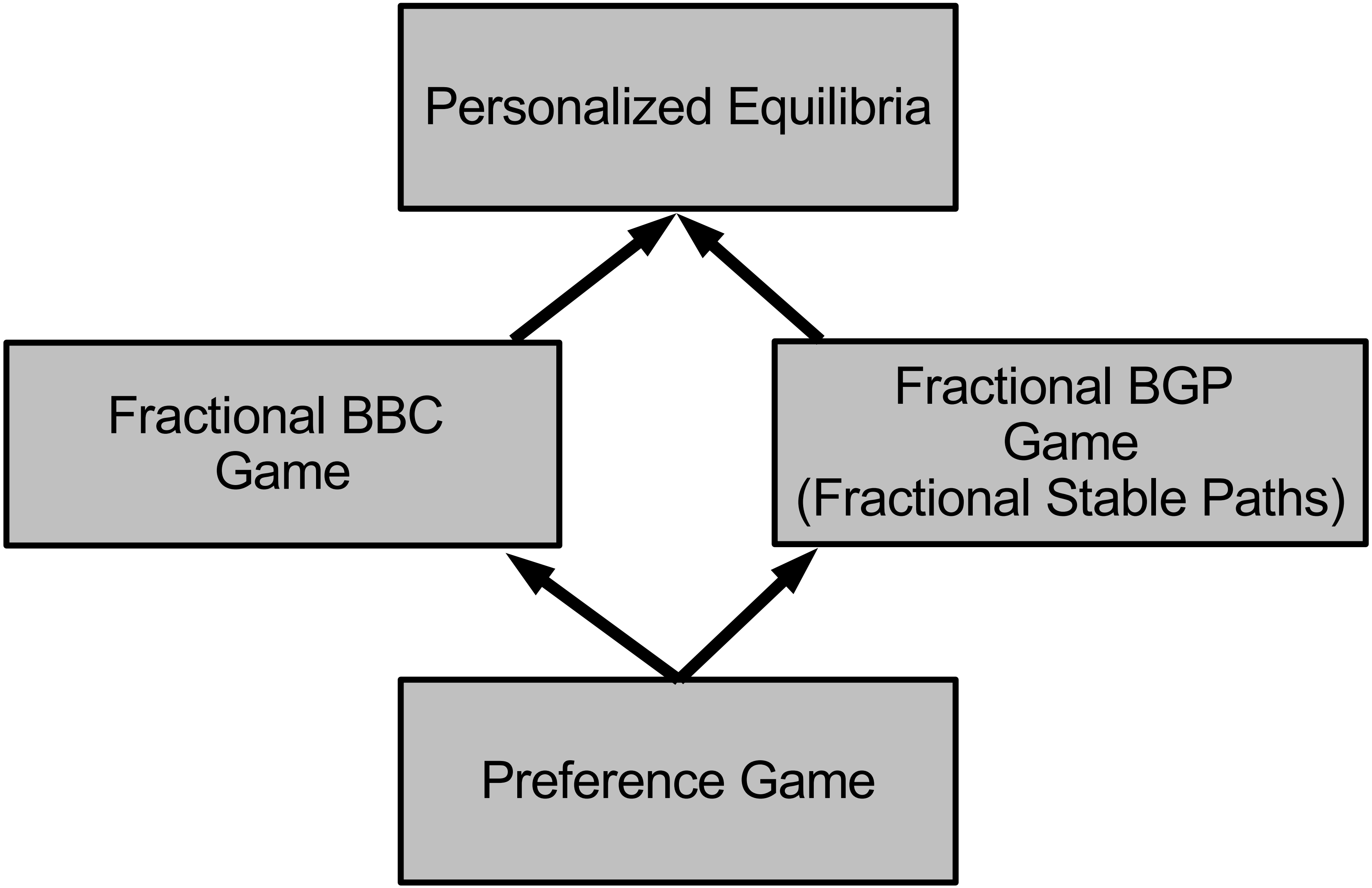


Fractional Stable Paths Problem

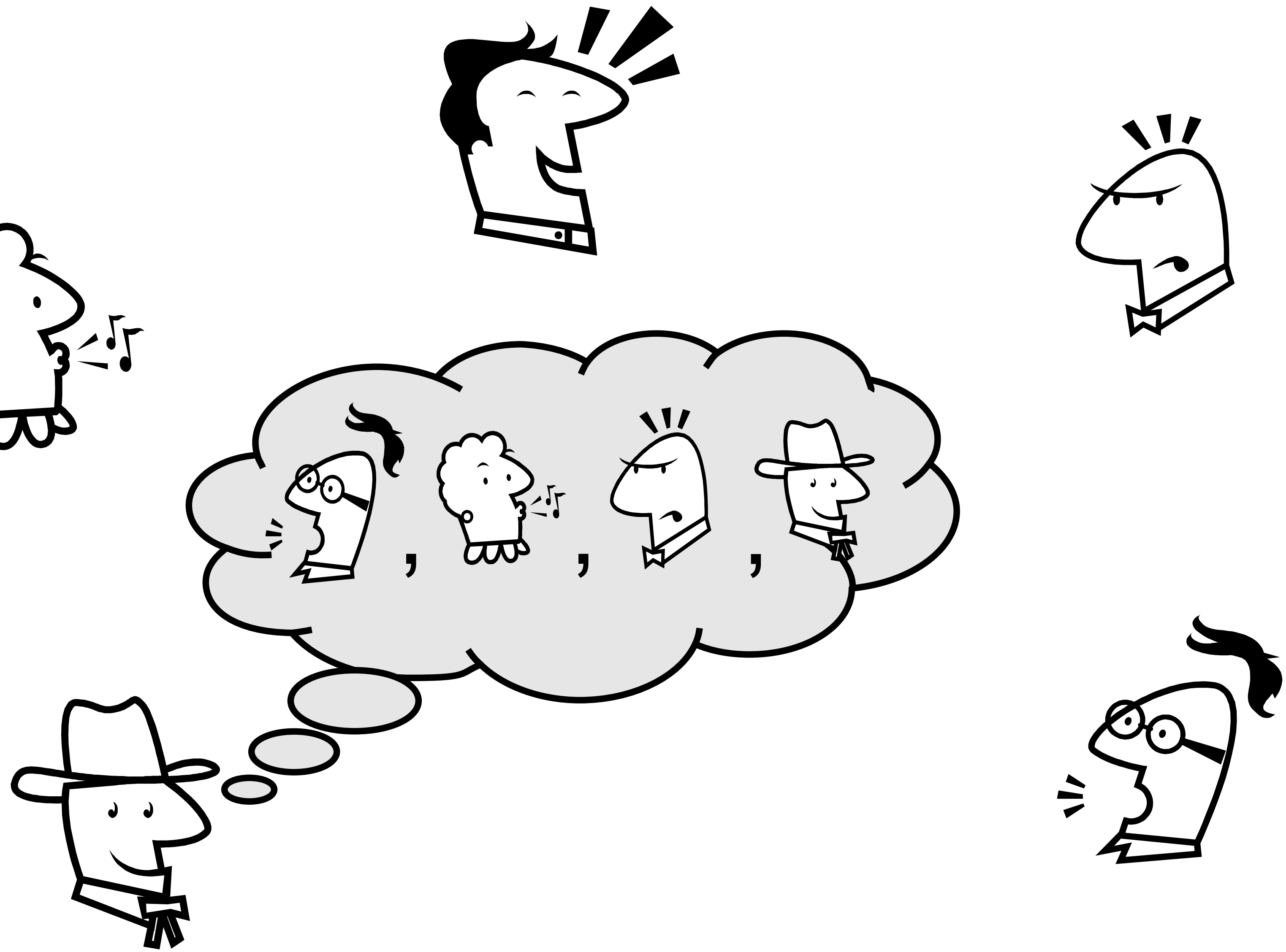
- Haxell and Wilfong. A fractional model of the border gateway protocol (BGP). SODA, 2008.



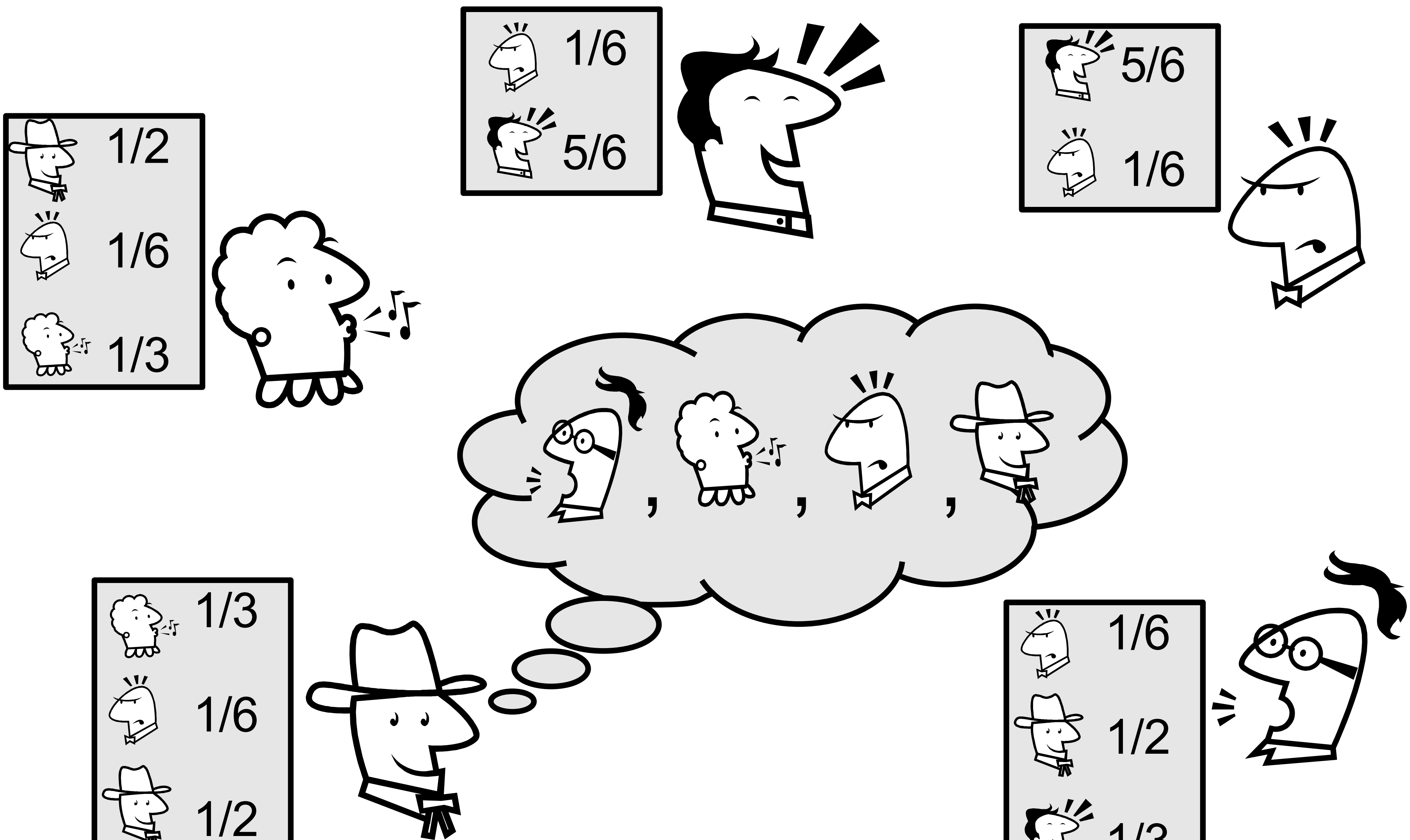
Fractional Games



The Preference Game



The Preference Game



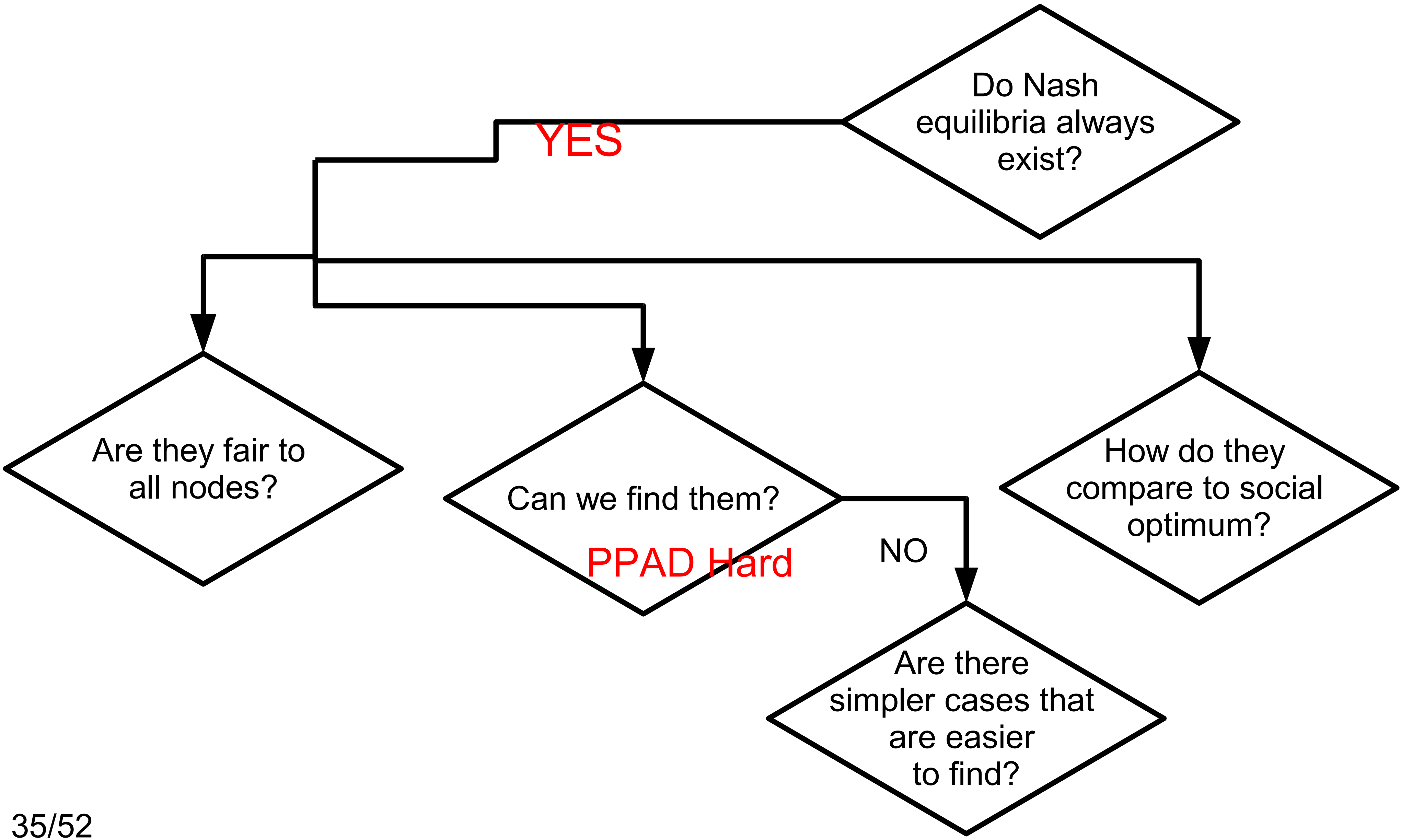
The Preference Game

- Reduces to both fractional BBC and fractional BGP games.
- A pure Nash equilibrium always exists.
- In fact, a *rational* pure Nash equilibrium always exists.
- Seems like it should be easy to “solve”
- If preferences follow some rules, it is easy to solve.

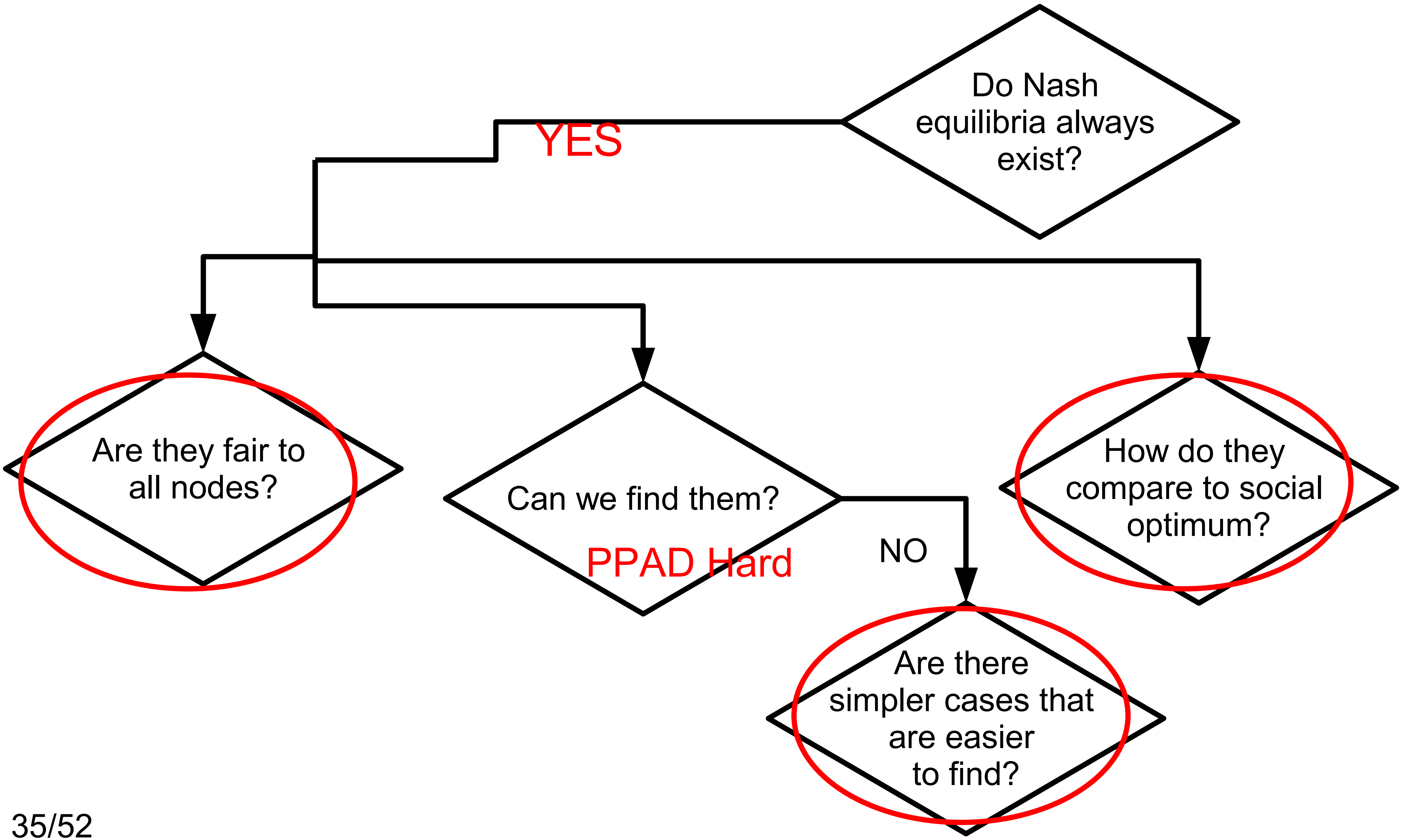
The Preference Game

- In general: PPAD hard to find an equilibrium (even an approximate equilibrium)
  - PPAD = Same as “end of the line”
    - Papadimitriou. On the Complexity of the Parity Argument and Other Inefficient Proofs of Existence. JCSS 48(3), 1994.
  - As hard as finding mixed Nash in general games
    - Daskalakis, Goldberg, Papadimitriou. The Complexity of Computing a Nash Equilibrium In STOC, 2006.
    - Goldberg, Papadimitiou. Reducibility Among Equilibrium Problems. STOC, 2006.
    - Chen, Deng, and Teng. Computing Nash Equilibria: Approximation and Smoothed Complexity. FOCS, 2006.

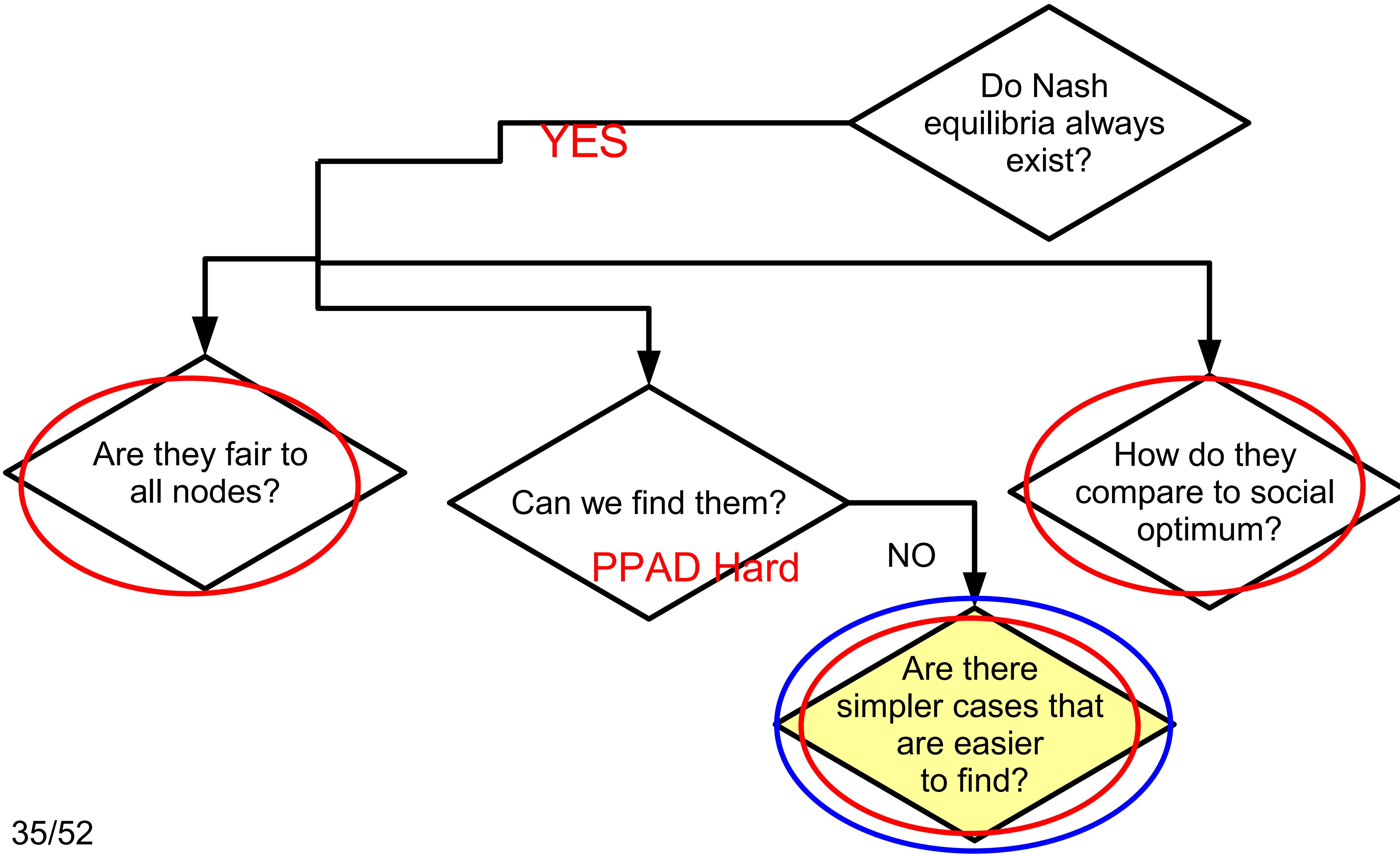
Fractional Games



Open questions on fractional games



Open questions on fractional games



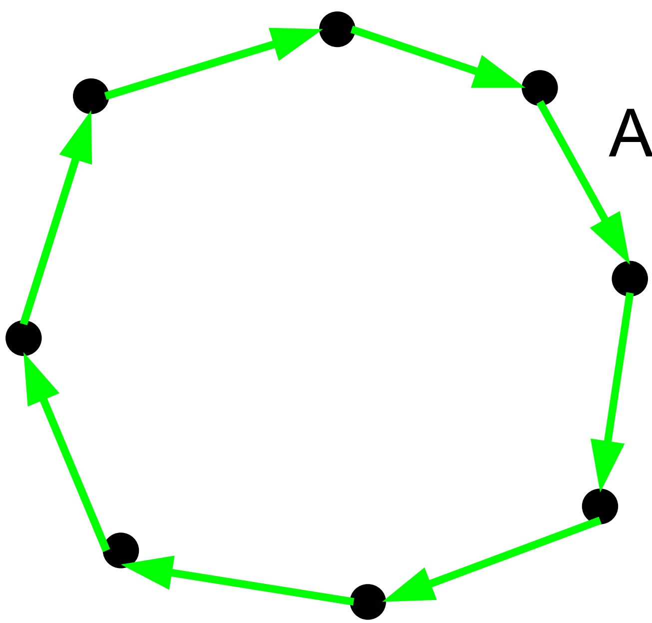
Fractional BBC Games, k=1

- Number of nodes = n
- Affinity for each directed pair of nodes = 1
- Link cost for each directed pair of nodes = 1
- Budget of allowed link cost per node, *k(v)* = 1
- Length metric = [1]
- Each node *v* spends ≤ *k(v)* on links to minimize

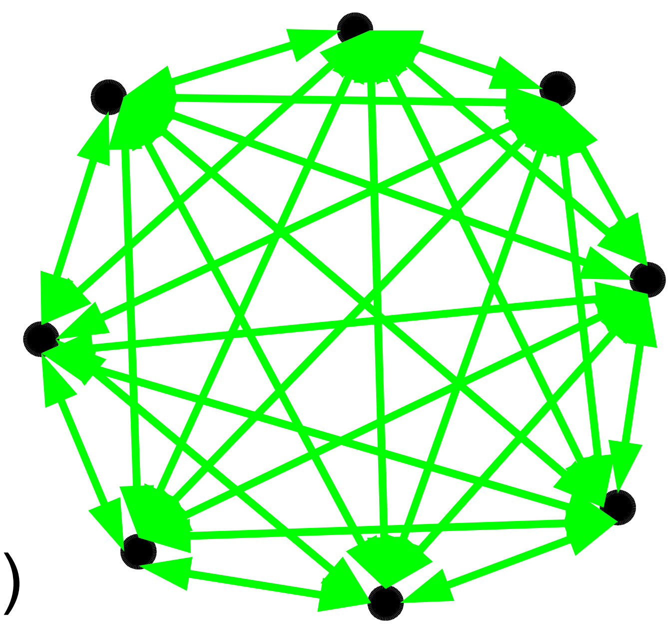
$$\sum_{\text{other nodes}} (\text{cost of min cost 1 unit flow})$$

↑ or disconnection penalty if no path exists.

Fractional BBC Games, k=1



Each edge weight = 1
Average distance per node = (n-1) / 2



Each edge weight = 1/(n-1)
Average distance per node = 2 - 1/(n-1)

Fractional BBC, k=1

- If each node can get 1 unit of flow to each other, it is an equilibrium.
- Is this condition necessary?

Characterizing Fractional Games

Mixed Nash for the integral version:

- If you play an action 1/3, you play must this 1/3 of the time against each set of opponents' actions.
- If two opponents play actions 1/4 each, you may only play 1/16 against that combination.

Pure Nash in the fractional version:

- If you play an action 1/3, you may use this with any legal 1/3 of the opponents' actions.
- If two opponents play actions 1/4 each, you may play 1/4 against this combination.

Personalized Equilibrium

Mixed Nash for the integral version:

- If you play an action 1/3, you play must this 1/3 of the time against each set of opponents' actions.
- If two opponents play actions 1/4 each, you may only play 1/16 against that combination.

Pure Nash in the fractional version:

- If you play an action 1/3, you may use this with any legal 1/3 of the opponents' actions.
- If two opponents play actions 1/4 each, you may play 1/4 against this combination.

same rules in general matrix games.

Personalized Equilibrium

	R	P	S
R	0, 0	-1, 1	1, -1
P	1, -1	0, 0	-1, 1
S	-1, 1	1, -1	0, 0

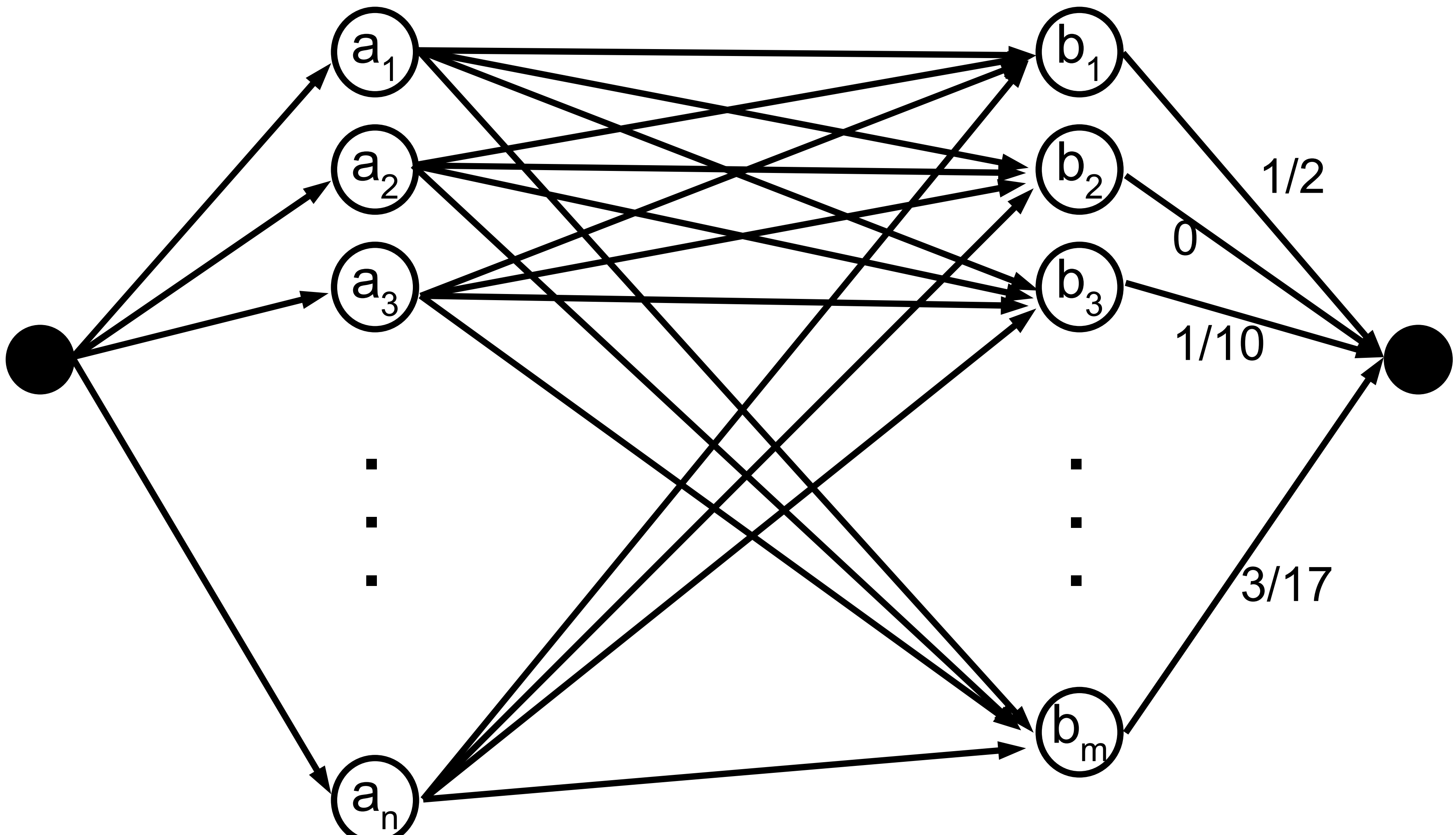
Mixed Nash: 1/3 on each

Payoff = 1/3 * ((1/3)*0 + (1/3)*-1 + (1/3)*1)
+ 1/3 * ((1/3)*1 + (1/3)*0 + (1/3)*-1)
+ 1/3 * ((1/3)*-1 + (1/3)*1 + (1/3)*0)
= 0

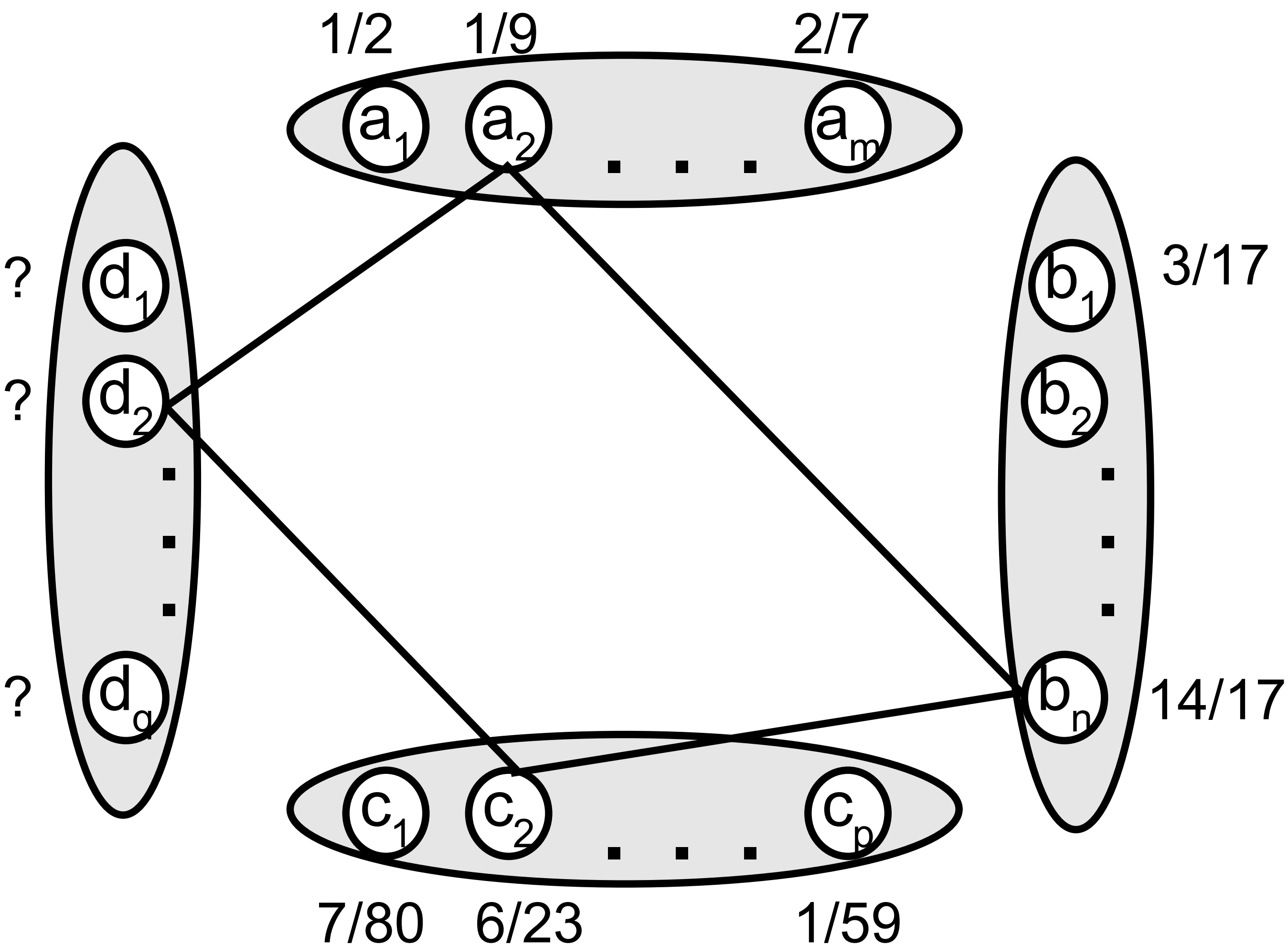
Personalized, 1/3 each –
can match up actions to best personal advantage

Payoff = (1/3)*1 + (1/3)*1 + (1/3)*1 = 1

Personalized Equilibrium



Personalized Equilibrium



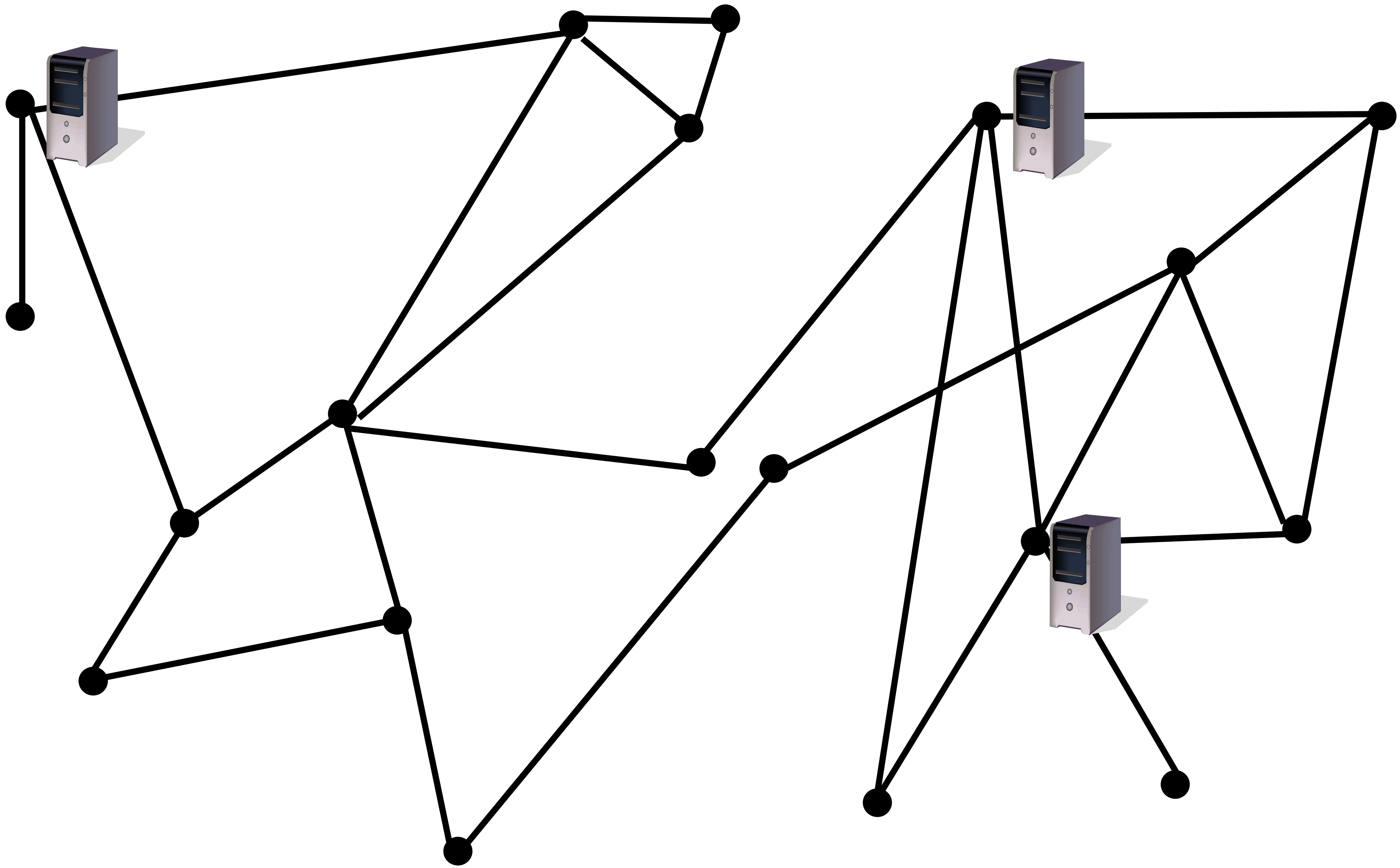
Personalized Equilibrium

- Rational Equilibrium always exists (solution to union of many linear programs)
- 2-player: fully characterized (can be represented by linear program)
- 3-player: ???
- 4-player: PPAD hard to compute
- 5+-player: PPAD hard to approximate

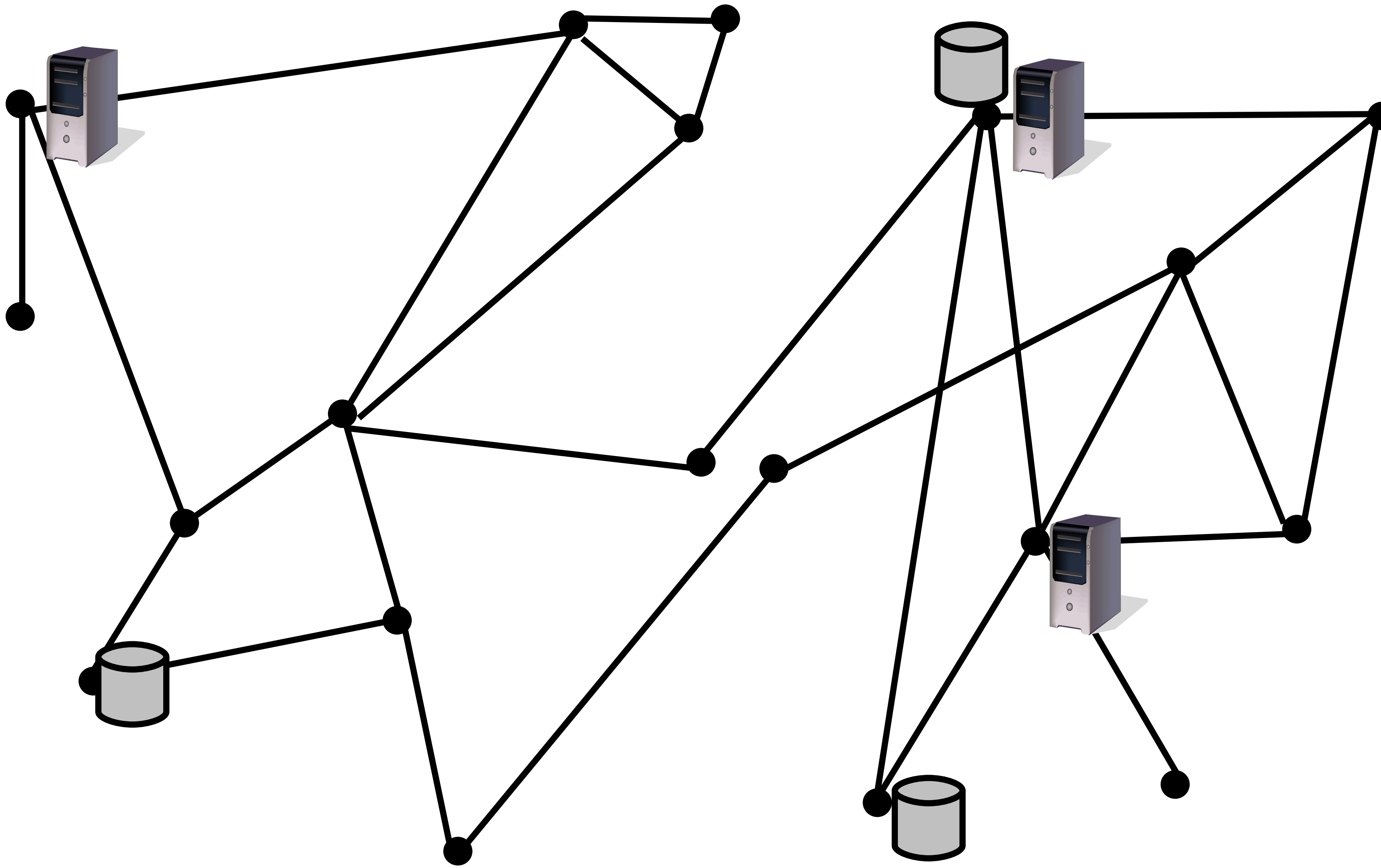
Segue Slide

- Games between overlay network nodes
  - BBC Games
  - Fractional BBC Games
  - Fractional BGP Games
- Characterizing Fractional Games
  - Preference Games
  - Personalized Equilibria
- Interaction Between Decentrally Designed Network and Centralized Algorithm

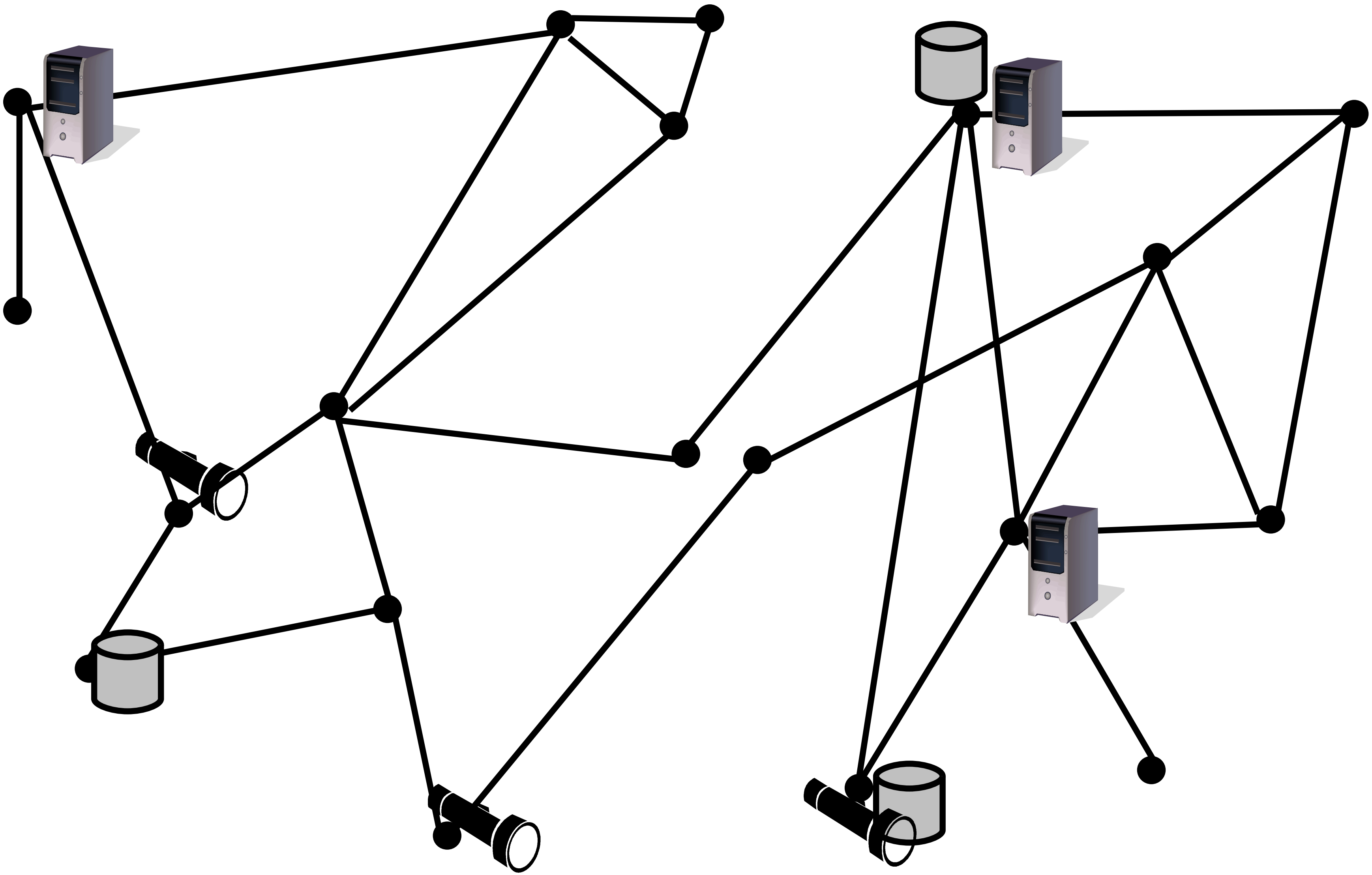
Centralized Algorithm



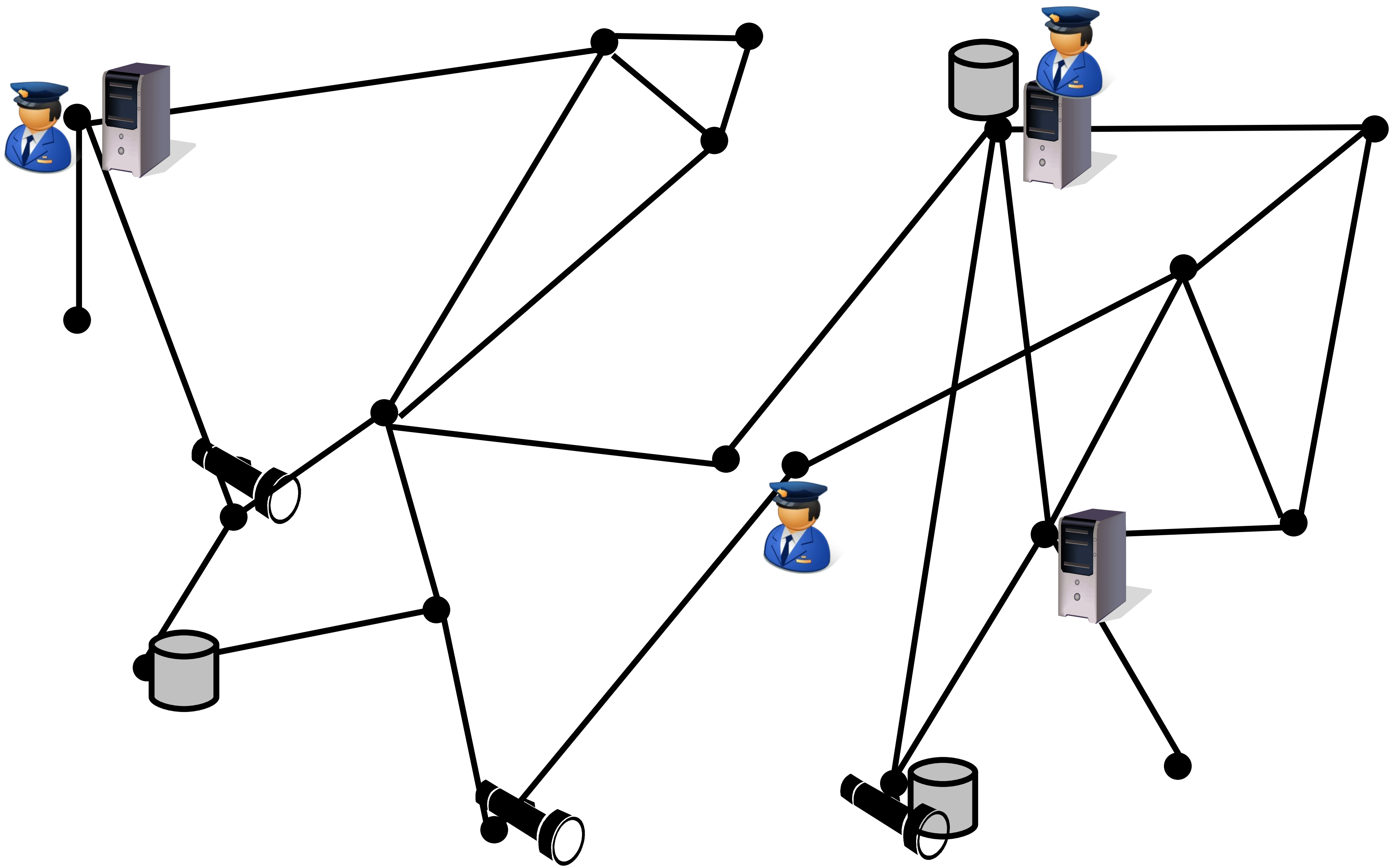
Centralized Algorithm



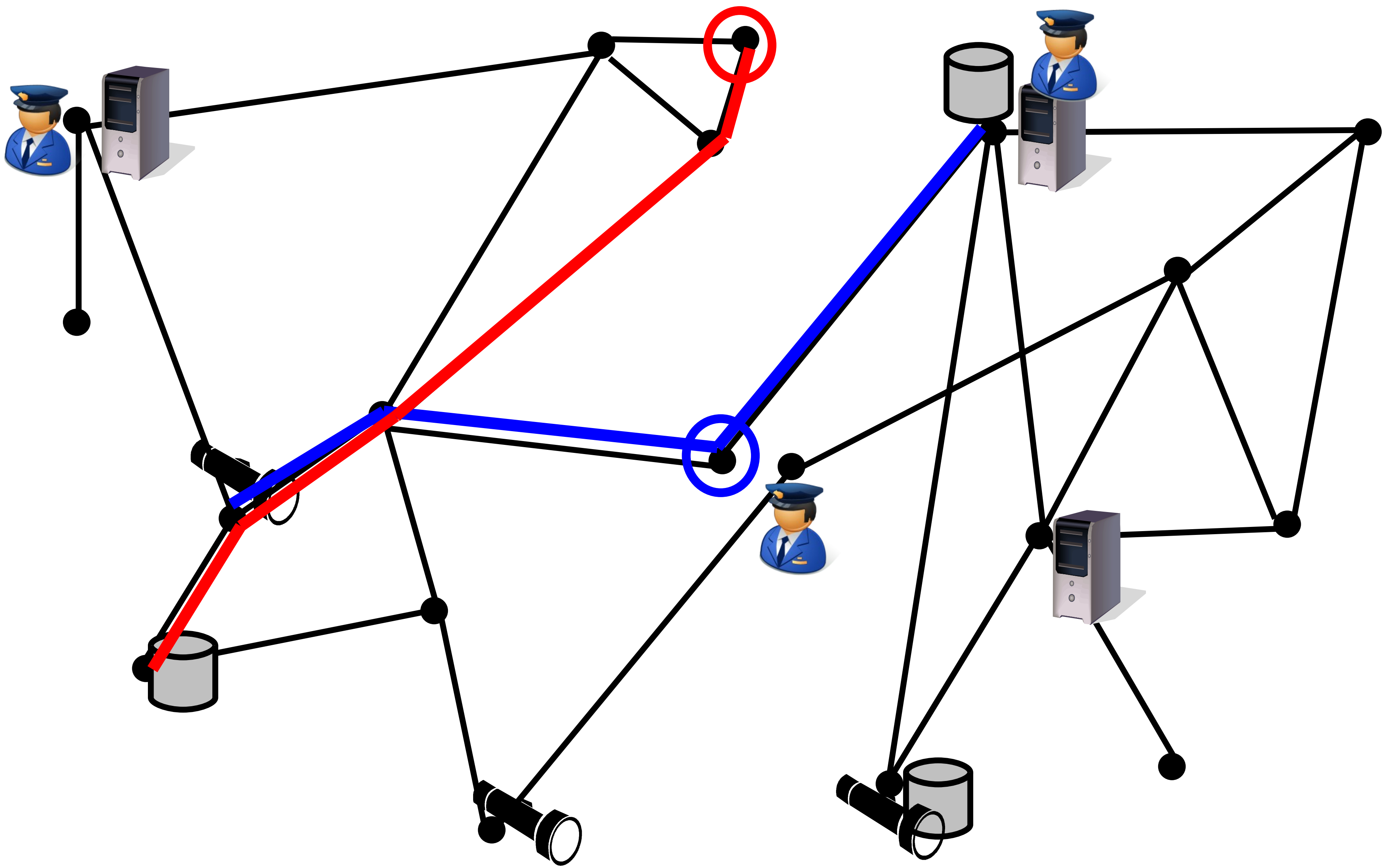
Centralized Algorithm



Centralized Algorithm



Centralized Algorithm



Steiner Tree Facility Location

- Network is result of a BBC game
- Each node will pick cheapest subgraph connecting it to commodities of interest

Steiner Tree Facility Location

- G = (V,E) with edge lengths
- Set T of types
- Interest sets: I(v) = Subset of T
- Cost to build type t at node v: c(t,v)
- Budget per type k(t)
- Want to find sets L(t) ≤ V (for each t) to minimize:

$$\sum_{v \text{ in } V} x(v) + \sum_{t \text{ in } T} \sum_{v \text{ in } L(t)} c(t,v)$$

x(v) is the cost of the minimum Steiner tree connecting v to at least one node in L(t) for each t in I(v)

Related Work on Steiner Tree and Facility Location

- R. Ravi , A. Sinha. Multicommodity facility location. SODA, 2004.
- Naveen Garg , Goran Konjevod , R. Ravi, A polylogarithmic approximation algorithm for the Steiner group tree problem, SODA, 1998.

Steiner Tree Facility Location

- Most general version: would also solve Group Steiner problem, which is NP hard to approximate to better than $O(\log^2 n)$, even on trees.
- Simplifications:
  - Set $c(t,v) = c(t,u)$ for all u,v (9-approximation on trees)
  - Also set $k(t) = 1$ for all t (solve optimally on trees)

What if commodity building were part of the game?

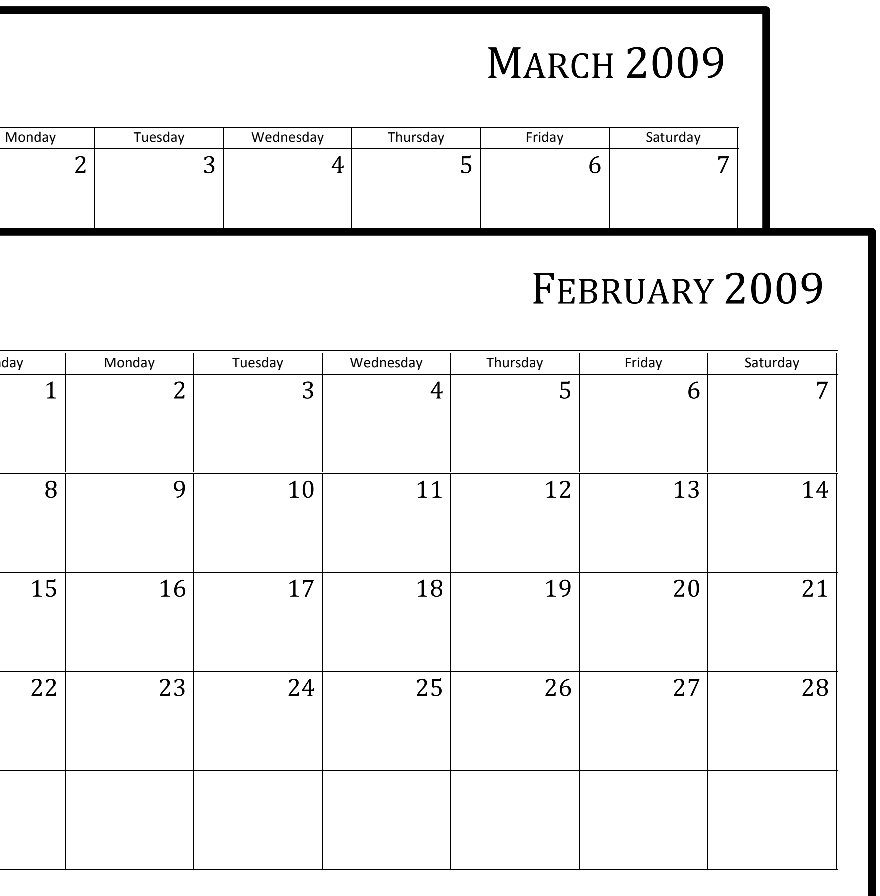
- A single additional player who is trying to solve STFL.
- Each commodity is its own player.

- How does the fact that our graph was created as a result of a game help or hurt the algorithm? The equilibria in the game?

Results Mentioned are from...

- Nikolaos Laoutaris, Laura J. Poplawski, Rajmohan Rajaraman, Ravi Sundaram, Shang-Hua Teng. *Bounded Budget Connection (BBC) Games or How to Make Friends and Influence People, on a Budget*.
  In PODC ’08, pages 165–174, 2008.
- Nikolaos Laoutaris, Laura J. Poplawski, Rajmohan Rajaraman, Ravi Sundaram, Shang-Hua Teng. *Bounded Budget Connection (BBC) Games or How to make friends and influence people, on a budget*.
  arXiv:0806.1727v1 [cs.GT]
- Laura J. Poplawski, Rajmohan Rajaraman, Ravi Sundaram, Shang-Hua Teng. *Preference Games and Personalized Equilibria, with Applications to Fractional BGP*.
  arXiv:0812.0598v2 [cs.GT]

Thesis Plan
February - March

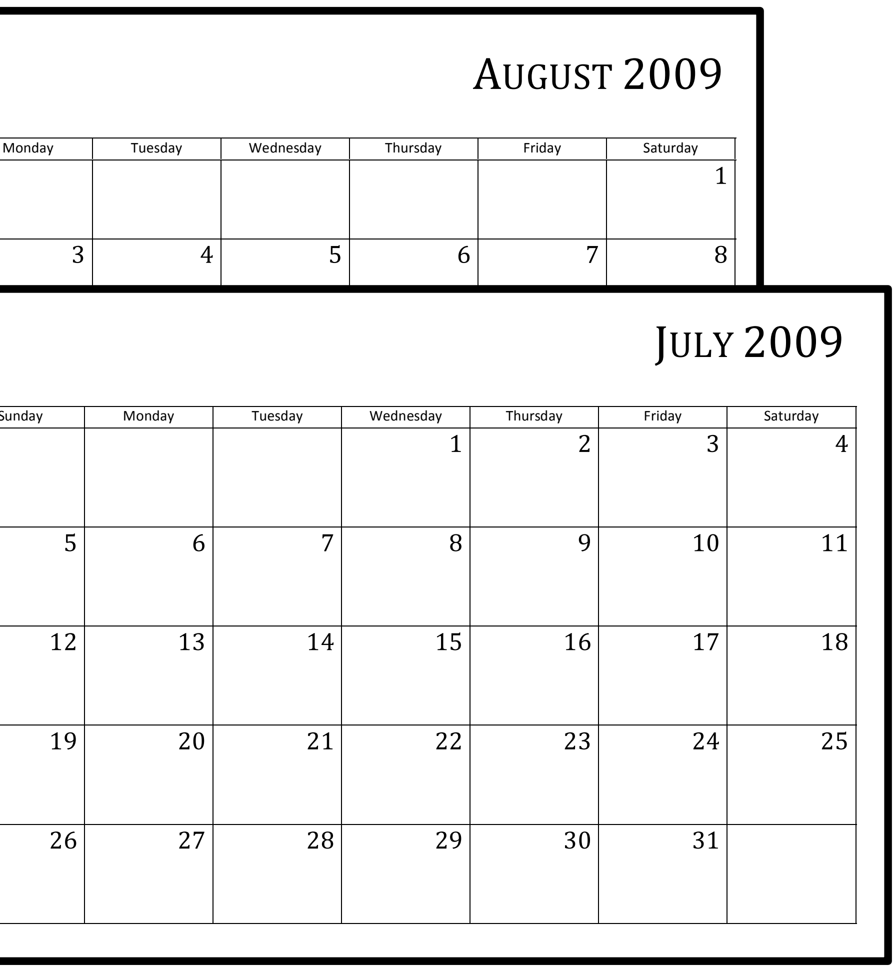


- Uniform Fractional BBC Game, k=1
- More on the Preference Game
- Integral BBC Game, all uniform except budget, all uniform except length.

Thesis Plan
April - June

- STFL: general problem approximation algorithm
- Commodities are controlled by BBC players

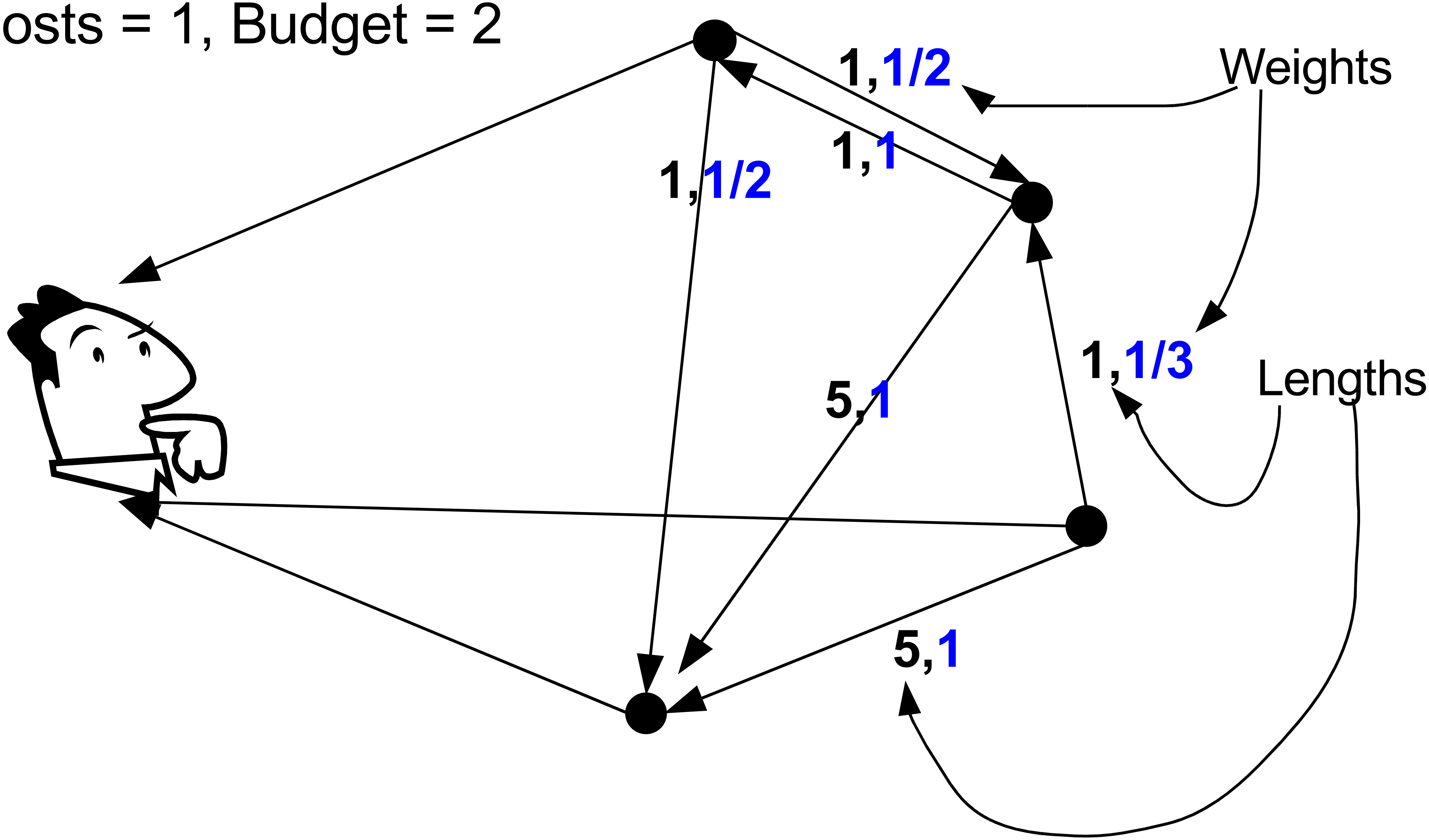
Thesis Plan
July - August



- Follow loose ends
- Write thesis
- Defense

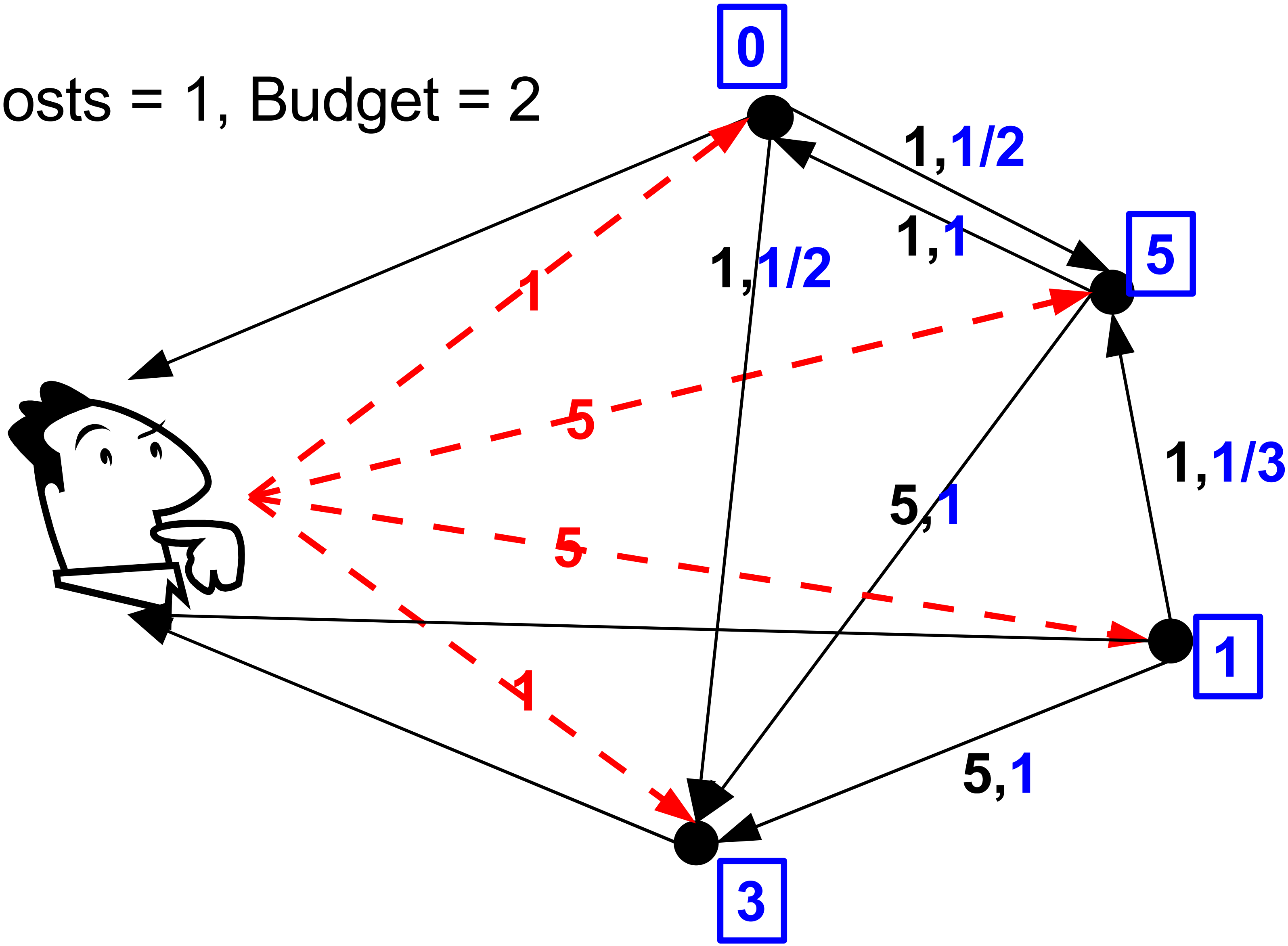
Example

Costs = 1, Budget = 2



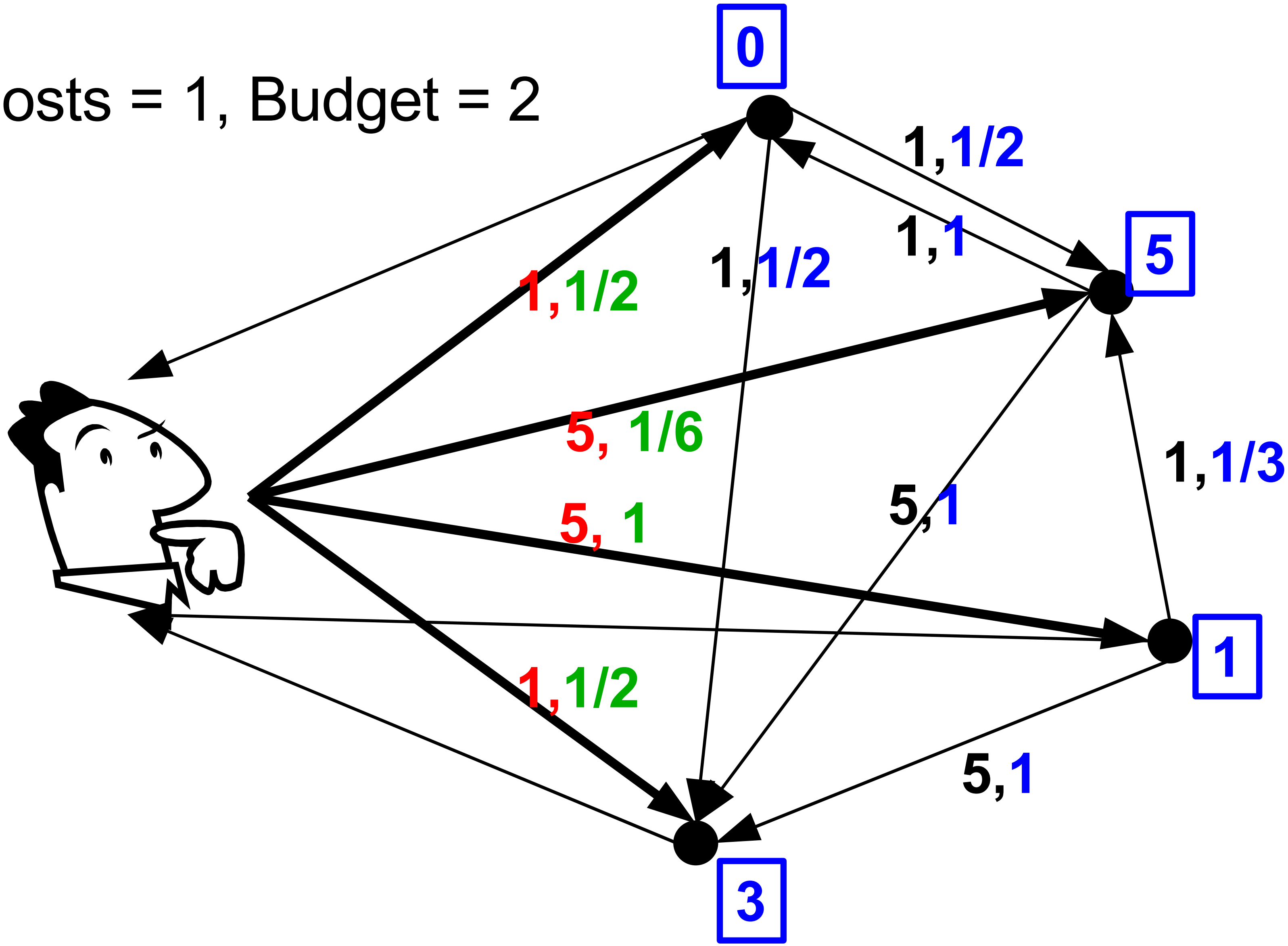
Example

Costs = 1, Budget = 2



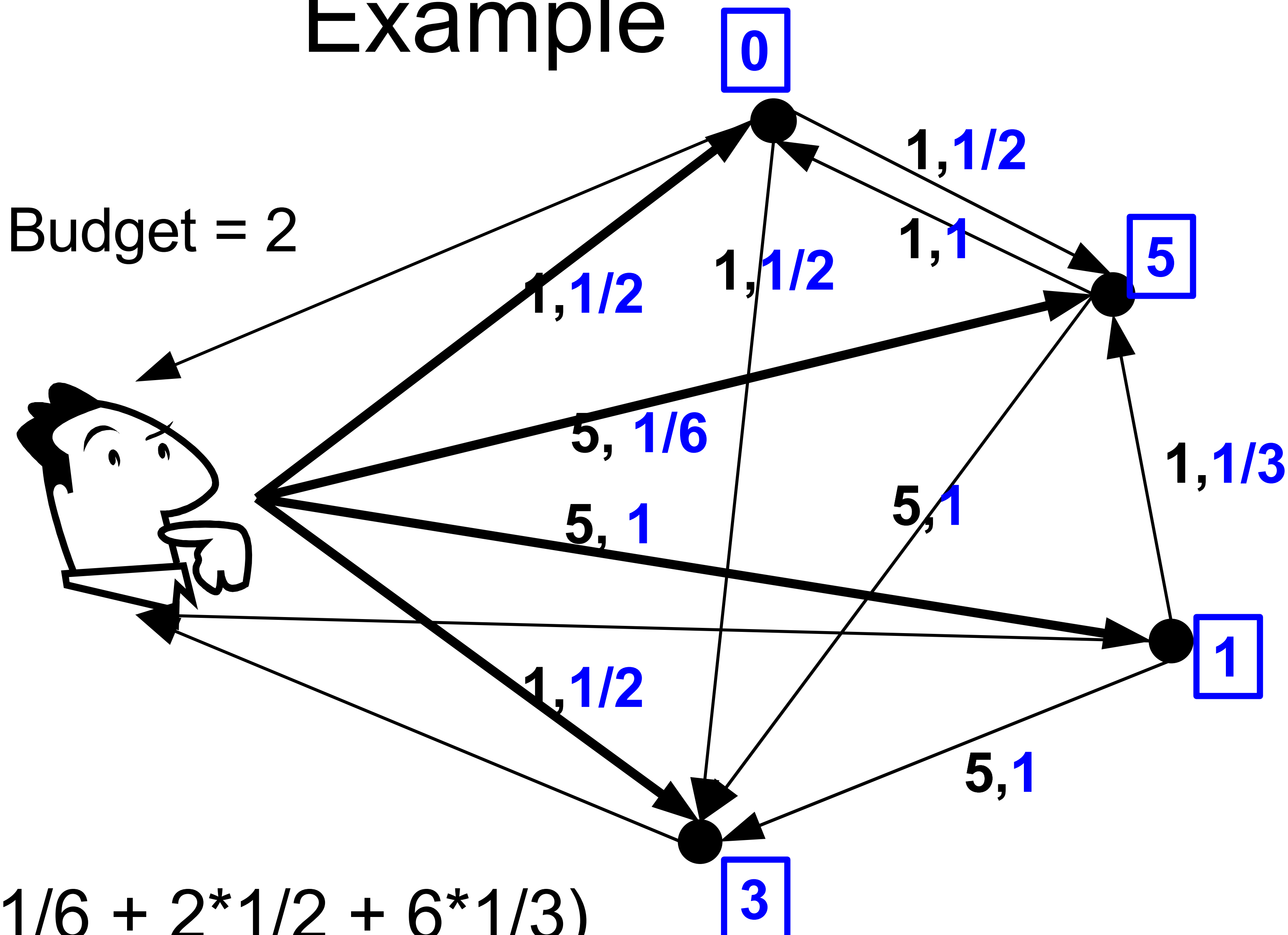
Example

Costs = 1, Budget = 2



Example

Costs = 1, Budget = 2



Cost = 5(5*1/6 + 2*1/2 + 6*1/3)
+ 1(5) + 3(2*1/2 + 1*1/2)
= 28 2/3