

Navigation in Object Graphs

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and
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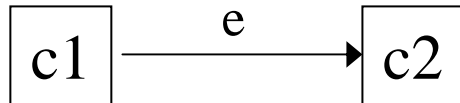
Searching for Reachable Objects

- Task: Given an object o_1 of class c_1 in an object graph, find all objects of type c_2 that are reachable from o_1 .
- Assumptions: we know the class structure that describes the object graph, but we know nothing else about the object graph except the class of the current object.

Search using meta information

- we could visit the entire object but that
 - would be wasteful or
 - might lead to wrong results

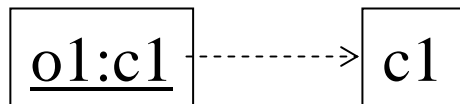
Classes and Objects: Basic Notations



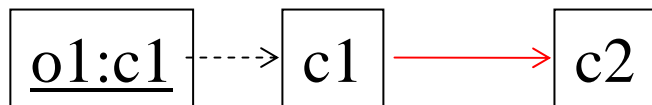
Class c1 has a part e of type c2



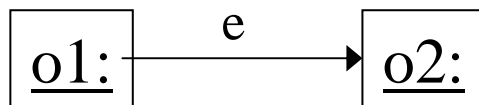
Class c1 inherits from class c2



Object o1 is of class c1

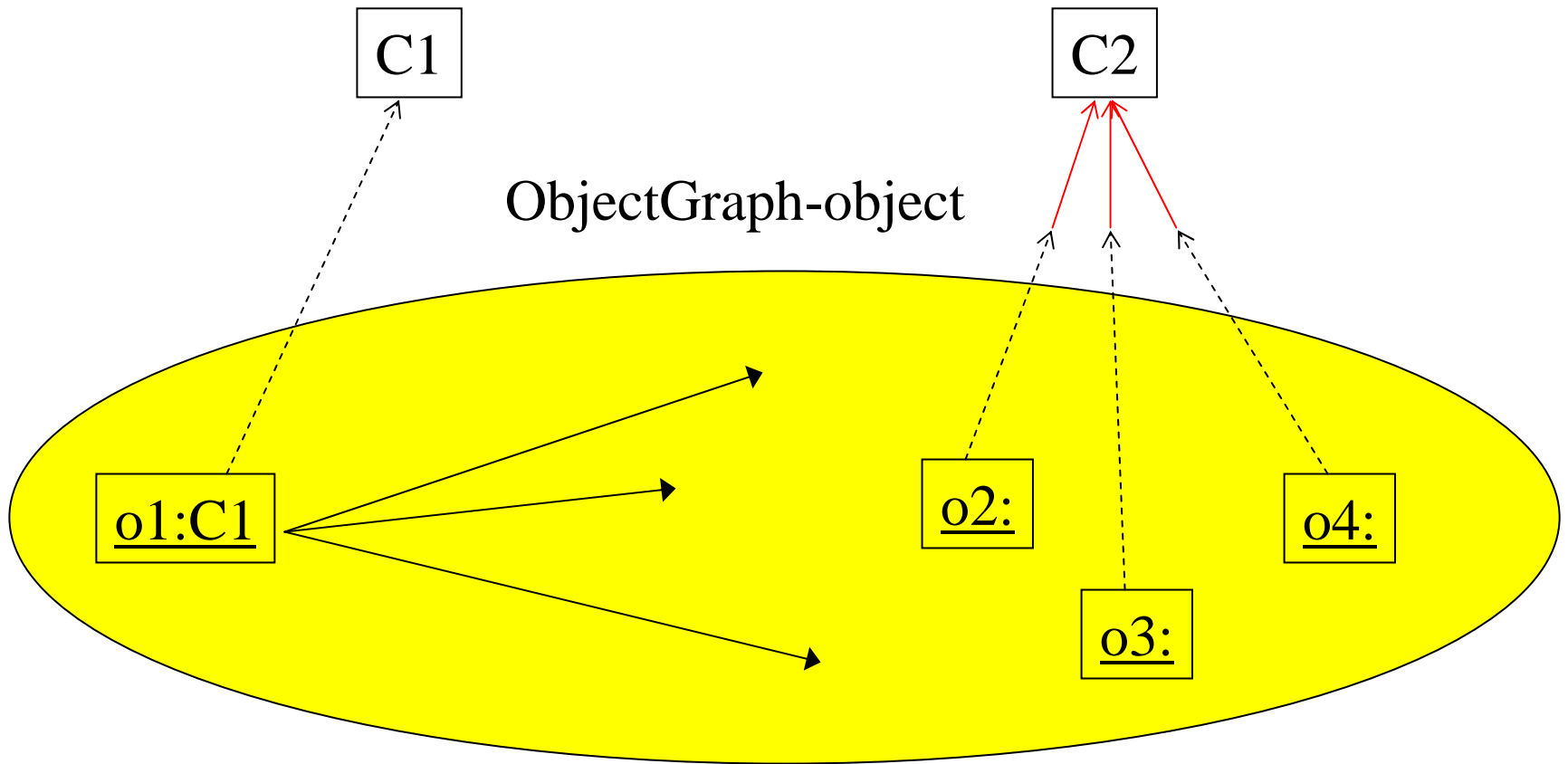


Object o1 is of type c2
(i.e., its class is a subclass of c2)



Object o1 has a part e which is object o2

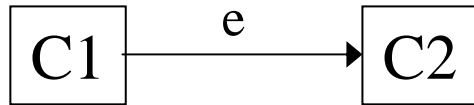
Finding the first step for the search



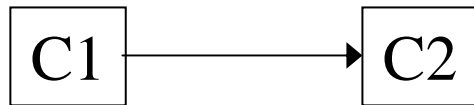
Which arrows might lead to an object of type C2?

Traversal Strategy:
(// C1 C2)

Relations between Classes



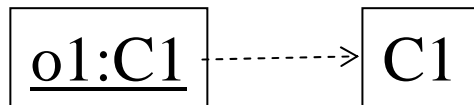
$e(C1, C2)$



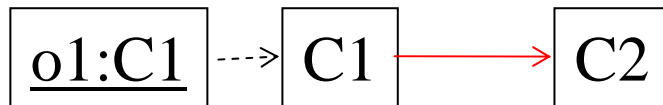
$C(C1, C2)$ (that is, $e(C1, C2)$ for some e)



$\leq(C1, C2), \Rightarrow(C2, C1)$

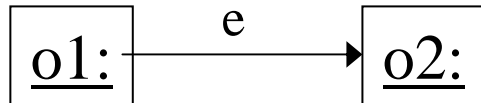


$\text{Class}(o1) = C1$

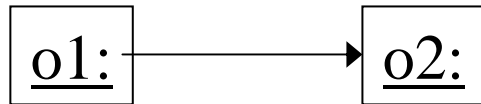


Object $o1$ is of type $C2$:
 $\text{Class}(o1) \leq C2$

Relations between Objects



$e(o1,o2)$



$O(o1,o2)$ (that is, $e(o1,o2)$ for some e)

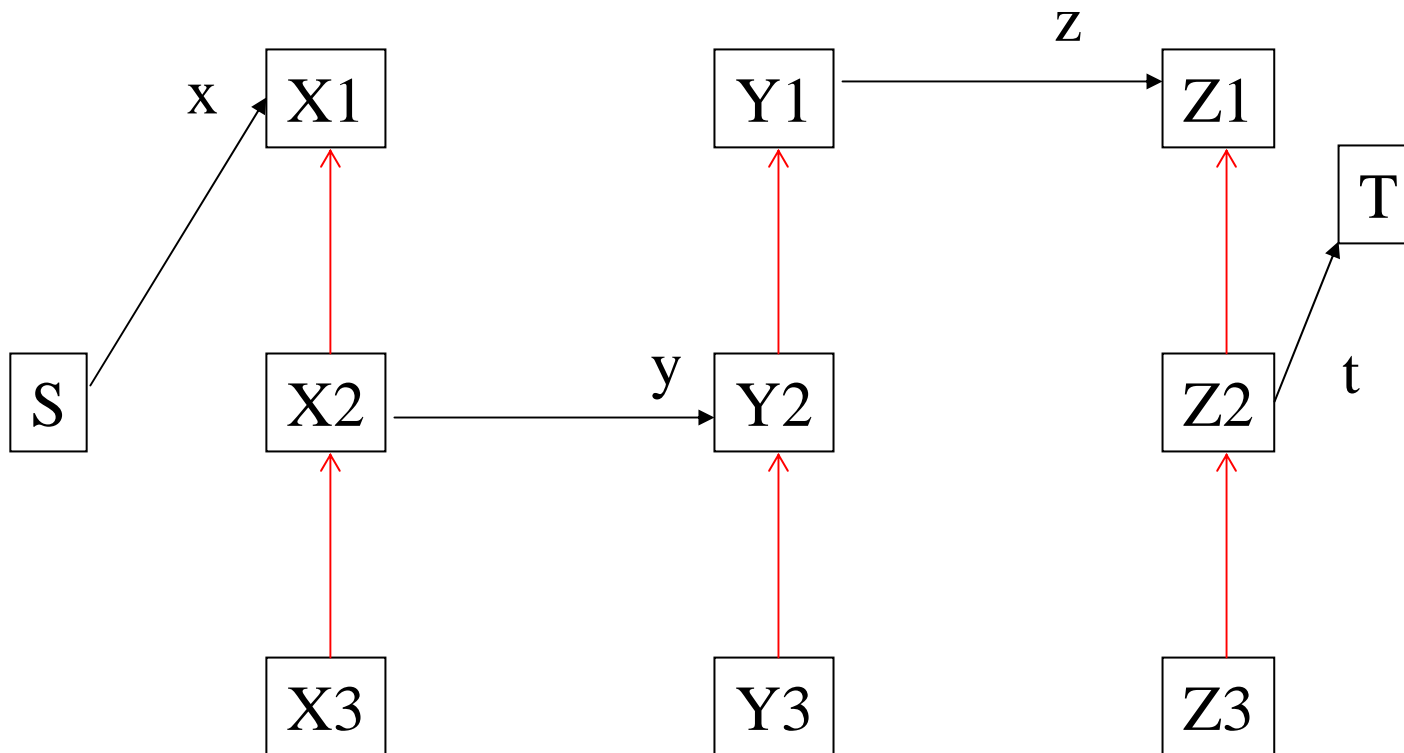
Operations on Relations

- $R.S = \{(x,z) \mid \text{exists } y \text{ s.t. } R(x,y) \text{ and } S(y,z)\}$
- $R^* = \text{reflexive, transitive closure of } R$

Write graph in terms of relations

Set: {S,T,X1,X2,X3,Y1,Y2,Y3,Z1,Z2,Z3}

Relations are sets of pairs, ordering is irrelevant.



Relations:

$x,y,z,t,<=,>$

$x(S,X1)$

$y(X2,Y2)$

$z(Y1,Z1)$

$t(Z2,T)$

$<=(Y2,Y1)$

$<=(X2,X1)$

$<=(X3,X2)$

...

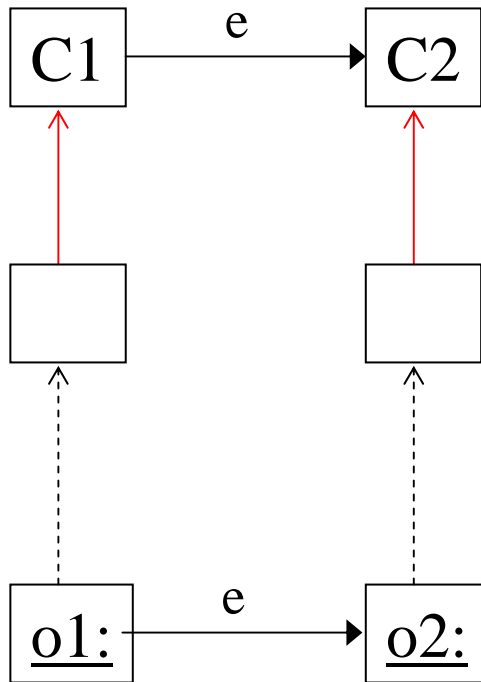
$=>(X2,X3)$

$=>(X1,X2)$

$=>(Z1,Z2)$

...

Possible edges in the object graph

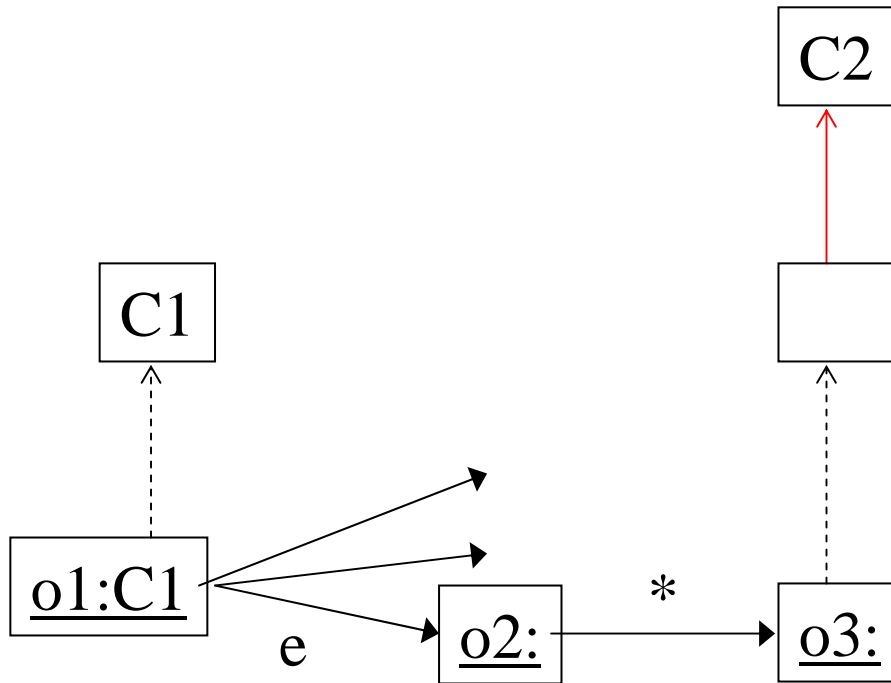


$e(o1, o2)$ implies
 $class(o1) (\leq .e .\Rightarrow) class(o2)$
in the class graph

“up, over, and down”

$O(o1, o2)$ implies
 $class(o1) (\leq .C .\Rightarrow) class(o2)$
in the class graph

Which edges to follow to C2?



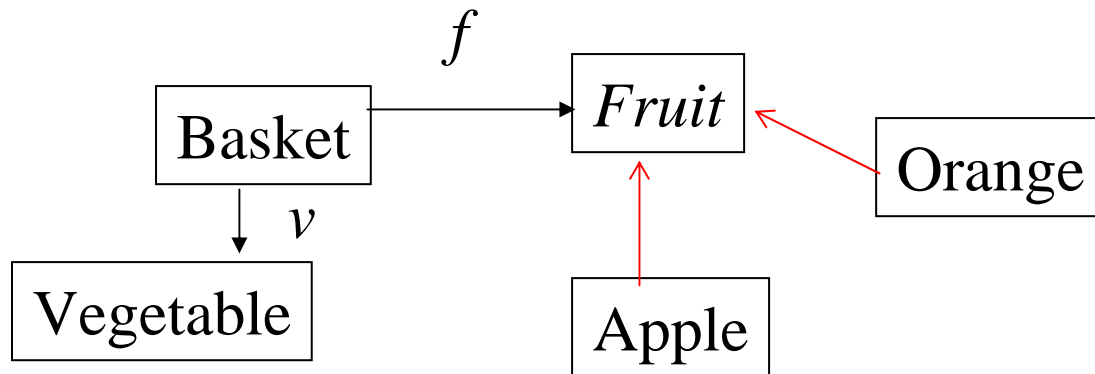
From $o1$ of class $C1$, follow edge e iff *there is some object graph O and some $o2, o3$ s.t.*

- (1) $e(o1, o2)$,
- (2) $O^*(o2, o3)$, and
- (3) $\text{class}(o3) \leq C2$

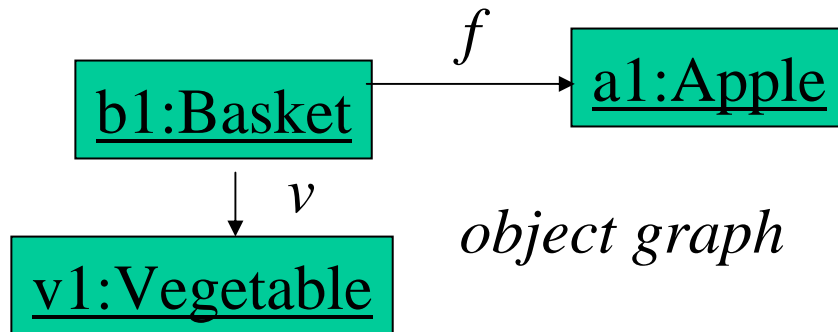
The existential quantifier “there is some object graph” represents our lack of knowledge about the rest of the object graph

from Basket to Orange = (from-to Basket Orange) =
(// Basket Orange)

Example

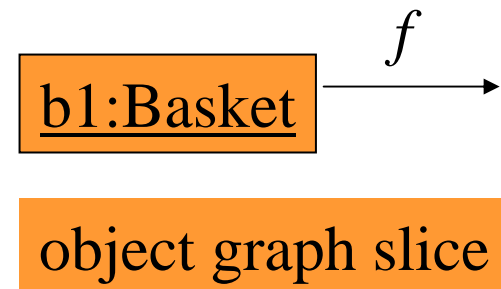


class graph



object graph

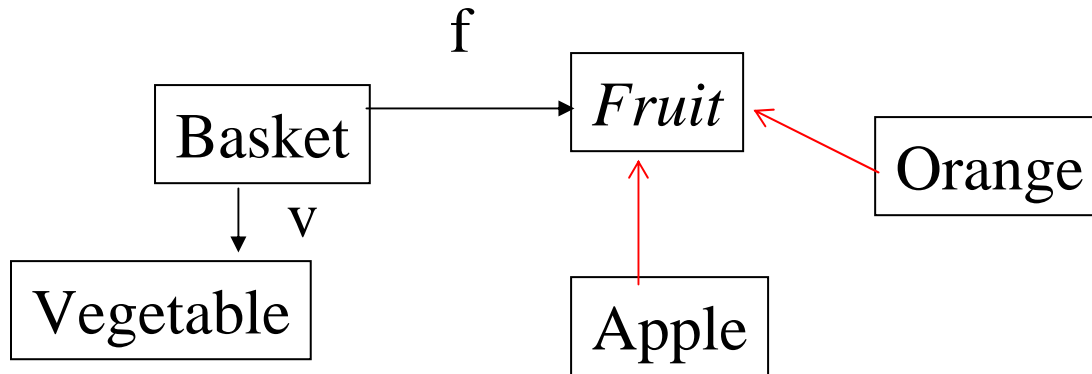
*premature
termination*



object graph slice

(// Basket Orange)

Example



mapping:

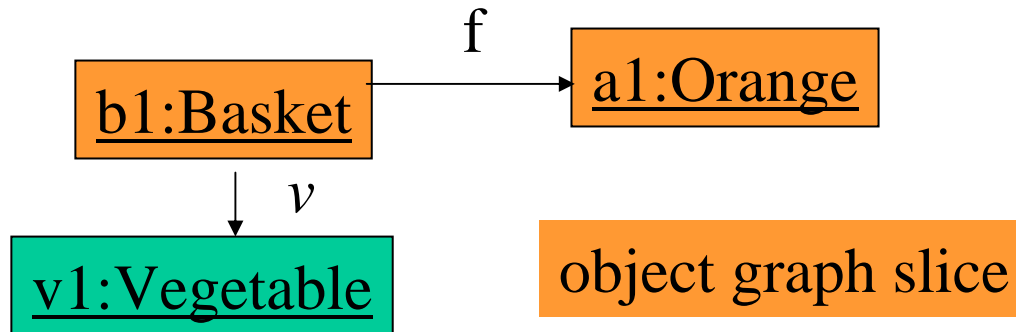
o1 b1

o2 a1

o3 a1

e f

class graph

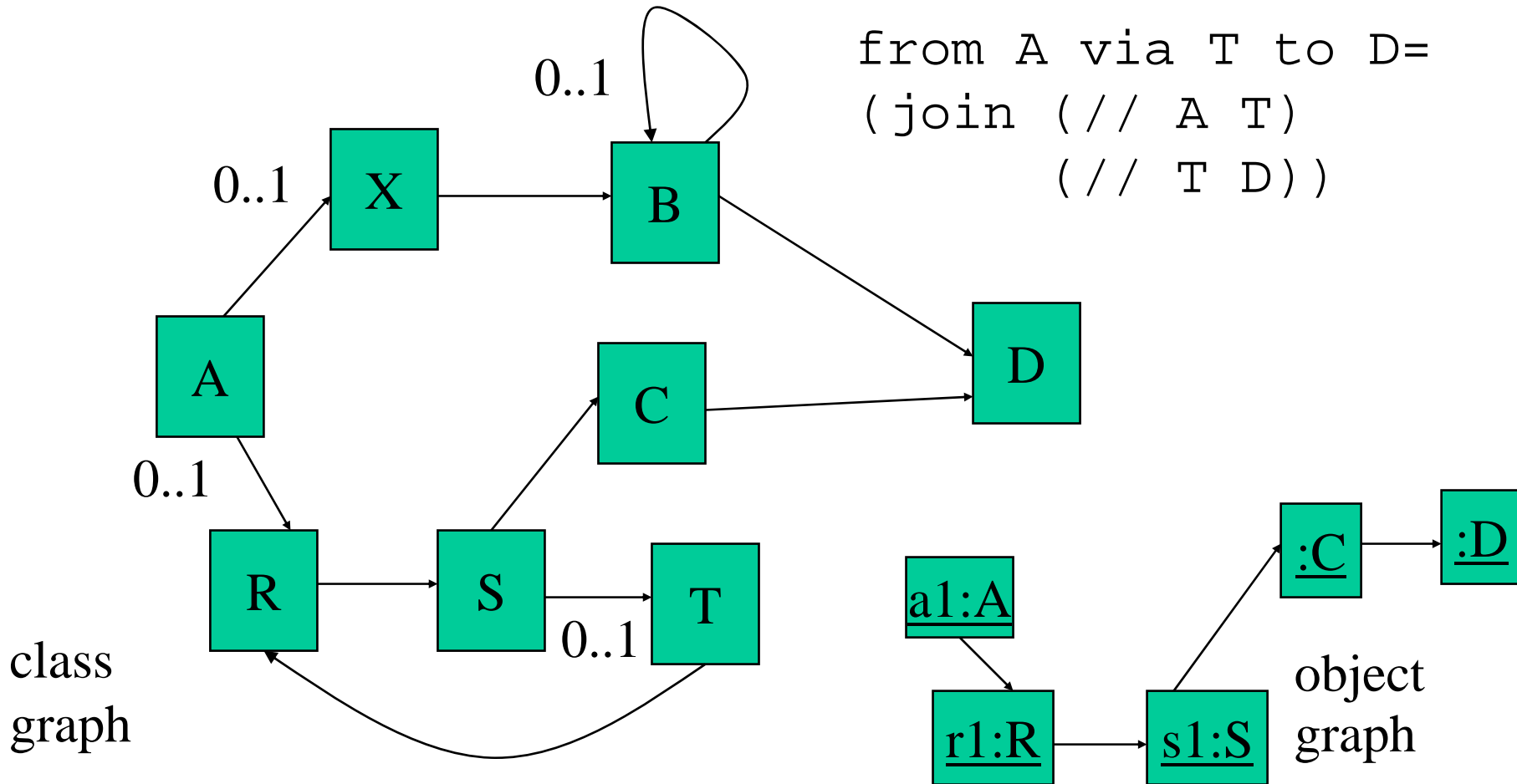


object graph

Example B1

strategy

```
from A via T to D =
(join (// A T)
      (// T D))
```



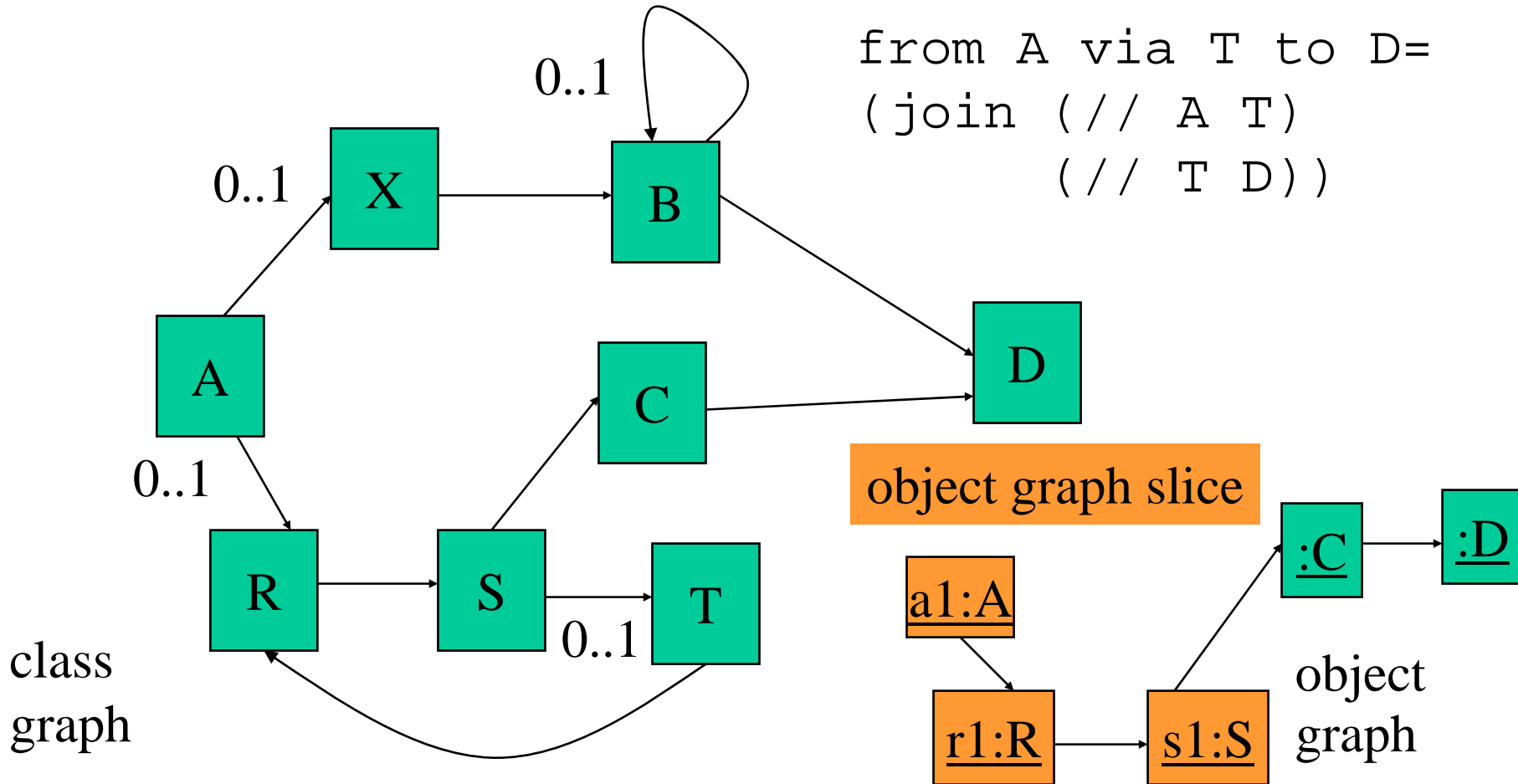
class
graph

object
graph

Example B1

strategy

```
from A via T to D=  
(join (// A T)  
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```



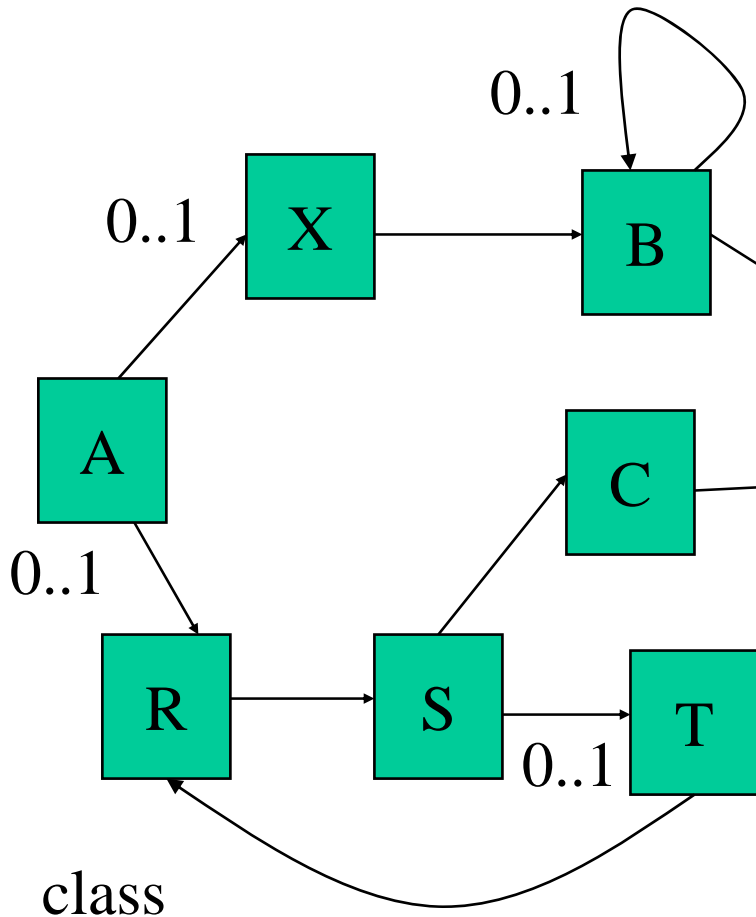
class
graph

object graph slice

object
graph

Example B2

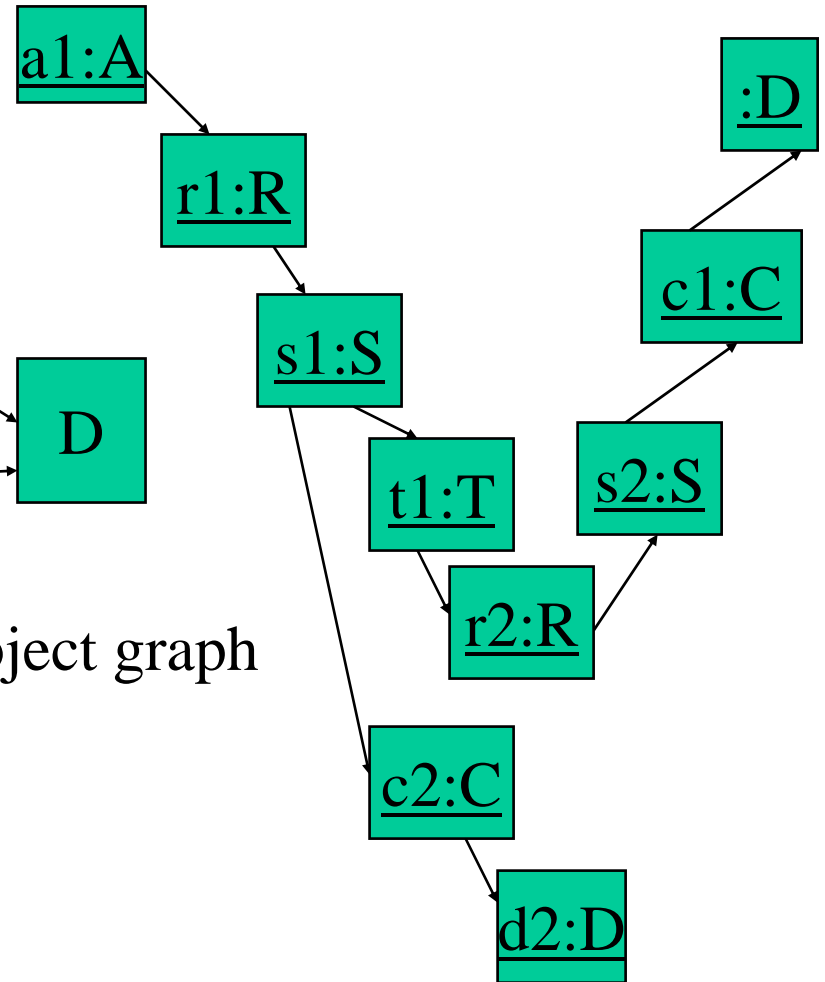
strategy from A via T to D=
(join (// A T)
 (// T D))



class graph

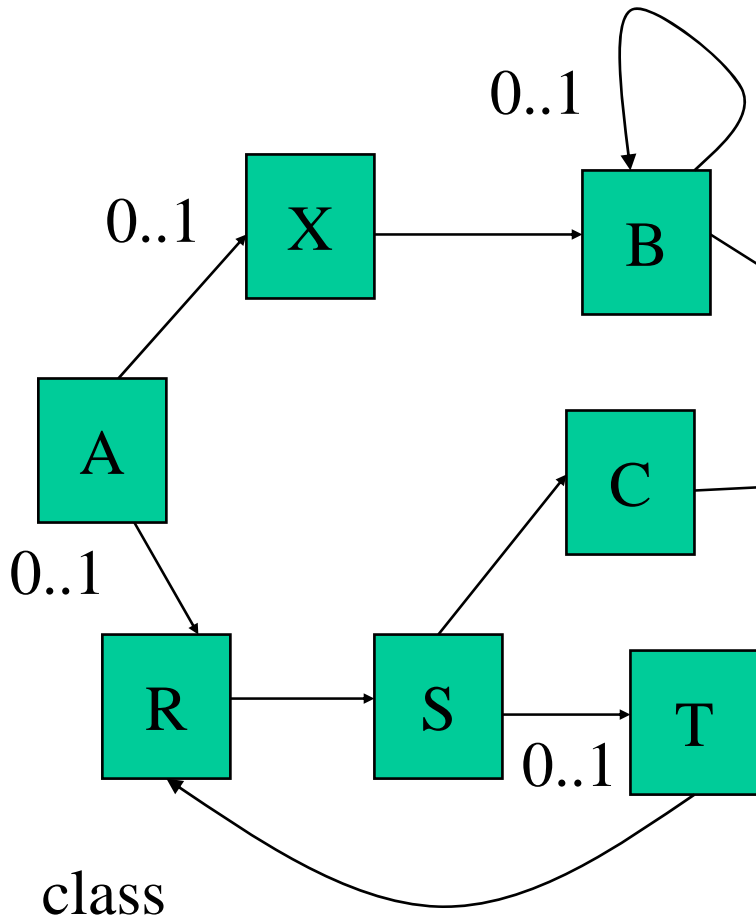
f1/1/2005

object graph



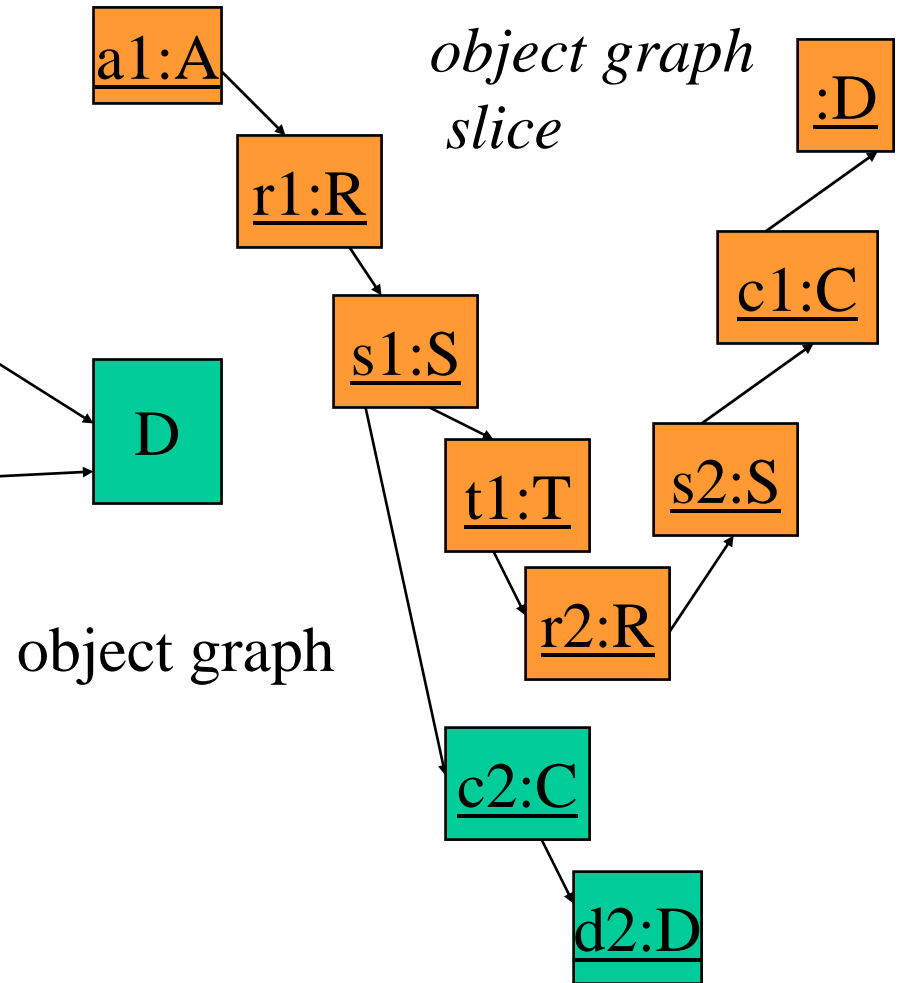
Example B2

strategy from A via T to D=
(join (// A T)
(// T D))



class
graph

11/1/2005



object graph
slice

object graph

Lack of Knowledge

- Objects of a given class may be very different.
- We want to go down edges without looking ahead!
- We don't want to go down edges that are guaranteed to be unsuccessful (never reaching a target object).

Object graph conforms to class graph

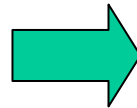
- The object graph O must follow the rules of the class graph: the object graph cannot contain more information than the class graph allows.

For all edges $e(o1, o2)$ in the object graph:
 $e(o1, o2)$ implies
 $class(o1) \text{ (} \leq \text{.e.} \text{)} class(o2)$
in the class graph

From dynamic to static characterization

From o_1 of class c_1 , follow edge e iff there is some object graph O and some o_2, o_3 s.t.

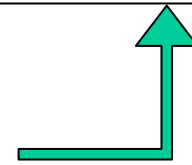
1. $e(o_1, o_2)$,
2. $O^*(o_2, o_3)$, and
3. $\text{class}(o_3) \leq c_2$



From o_1 of class c_1 , follow edge e iff there are classes c', c'' s.t.

1. $c_1 \leq_{\text{e}} c'$
2. $c' \leq_{\text{C}} c''$ and
3. $c'' \leq c_2$

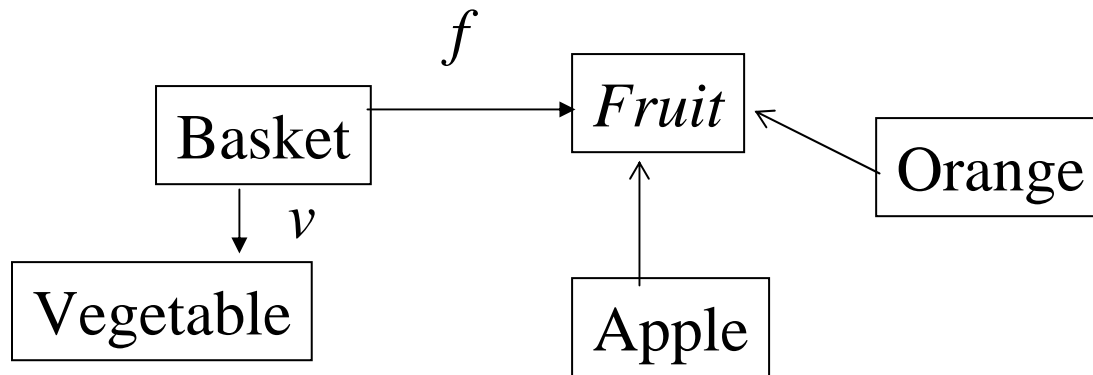
Let c' be $\text{class}(o_2)$, c'' be $\text{class}(o_3)$



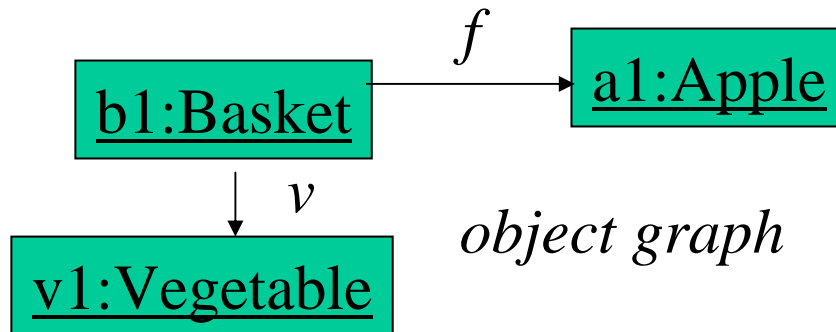
from Basket to Orange

static characterization

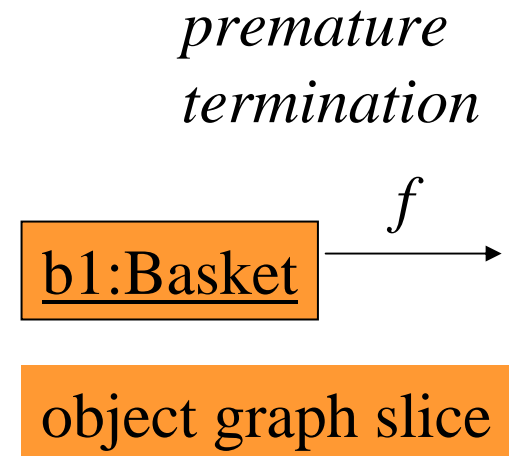
Example



class graph



object graph



*premature
termination*

from Basket to Orange

static characterization

Example

mapping:

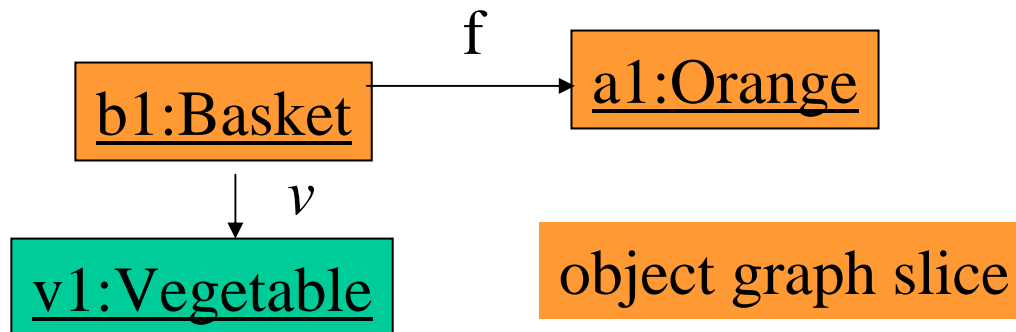
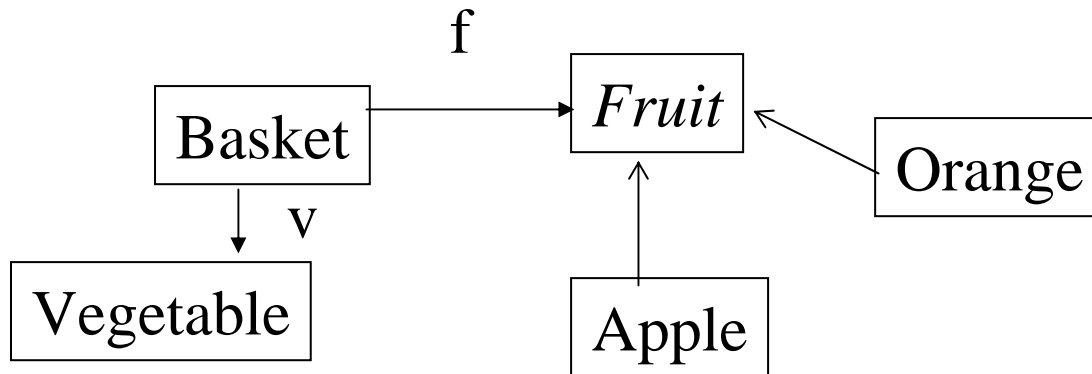
c1 Basket

c' Orange

c'' Orange

e f

class graph



object graph

from x1 to x2

Relational Formulation

From object o of class x1, to get to x2, follow edges in the set
 $\text{POSS}(x1,x2)=\{e \mid x1 \leq.e.\Rightarrow.(\leq.C.\Rightarrow)^*.\leq x2 \}$

Can easily compute these sets for every x1, x2
via transitive-closure algorithms.

POSS = abbreviation for: following these edges it
is still **possible** to reach a x2-object for some x1-
object rooted at o.

from x1 to x2

Relational Formulation

From object o of class x1, to get to x2, follow edges in the set
 $\text{POSS}(x1,x2)=\{e \mid x1 \leq.e.\Rightarrow.(\leq.C.\Rightarrow)^*.\leq x2 \}$

Simplification for class graphs obtained from hw class graphs:

$\text{POSS}(x1,x2)=\{e \mid x1 e.\Rightarrow.(C.\Rightarrow)^*.\leq x2 \}$ (flat)

up-over-and-down becomes over-and-down

\Rightarrow means only one edge

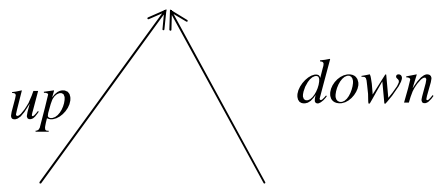
\leq means only one edge

Negative formulation

Positive Formulation:

From object o of class x_1 , to get to x_2 , follow edges e in the set $\text{POSS}(x_1, x_2) = \{ e \mid x_1 \leq e \Rightarrow (\leq C \Rightarrow)^* \leq x_2 \}$

Build paths anyway you like but don't follow \Rightarrow (down) immediately after \leq (up) and the first has-a edge must have label e .

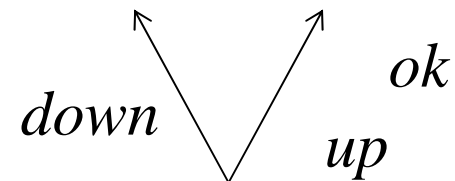


Forbidden

Edge kinds:

Is-a (down, up)

Has-a



ok

Followed-by relationships

Followed by	Is-a: down \Rightarrow	Is-a: up \Leftarrow	Has-a e, C
Is-a: down	yes	yes	yes
Is-a: up	NO	yes	yes
Has-a	yes	yes	yes

Linking the terminologies

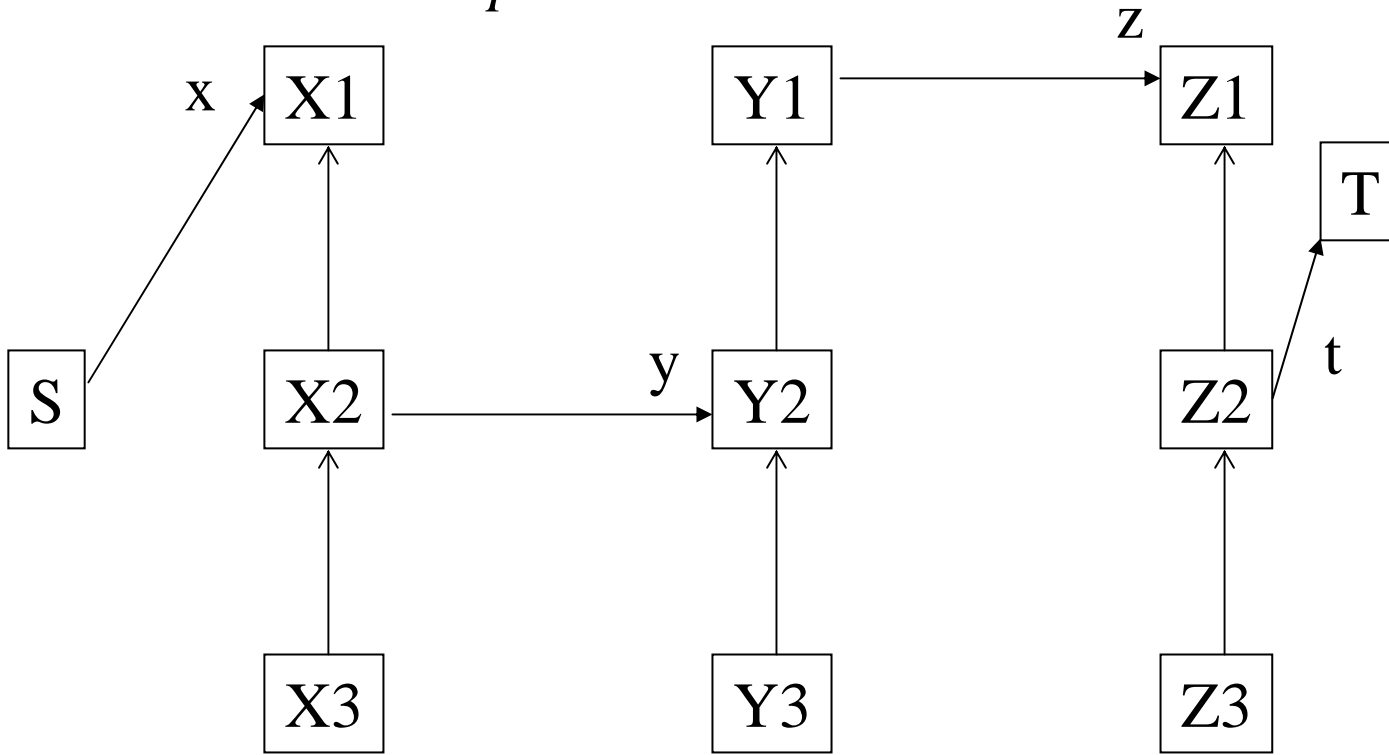
program			theory
$\rightarrow A, b, B$	Has-a	over	e, C
$\Rightarrow A, B$	Is-a in reverse	down	\Rightarrow
$:> A, B$	Is-a	up	\Leftarrow

Generalizations

- More complex strategies
- “from c1 through c2 to c3”
 - Use “waypoint navigation”; get to a c2 object, then search for a c3 object.
- More complex strategy graphs also doable in this framework

Examples

Example A



Strategy

$S \rightarrow T$

Relations:

$x(S, X1)$

$\Rightarrow (X1, X2)$

$y(X2, Y2)$

$\Leftarrow (Y2, Y1)$

$z(Y1, Z1)$

$\Rightarrow (Z1, Z2)$

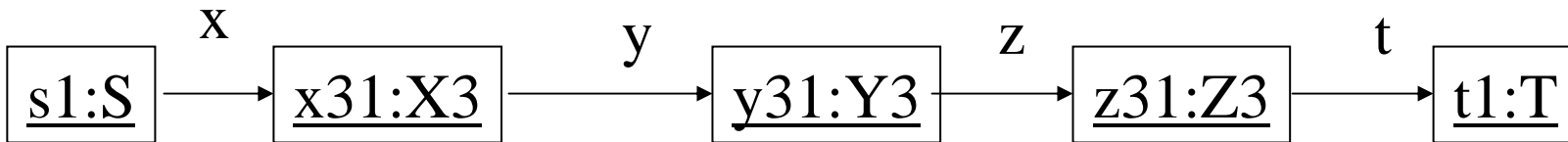
$t(Z2, T)$

$\Leftarrow (X2, X1)$

$\Leftarrow (X3, X2)$

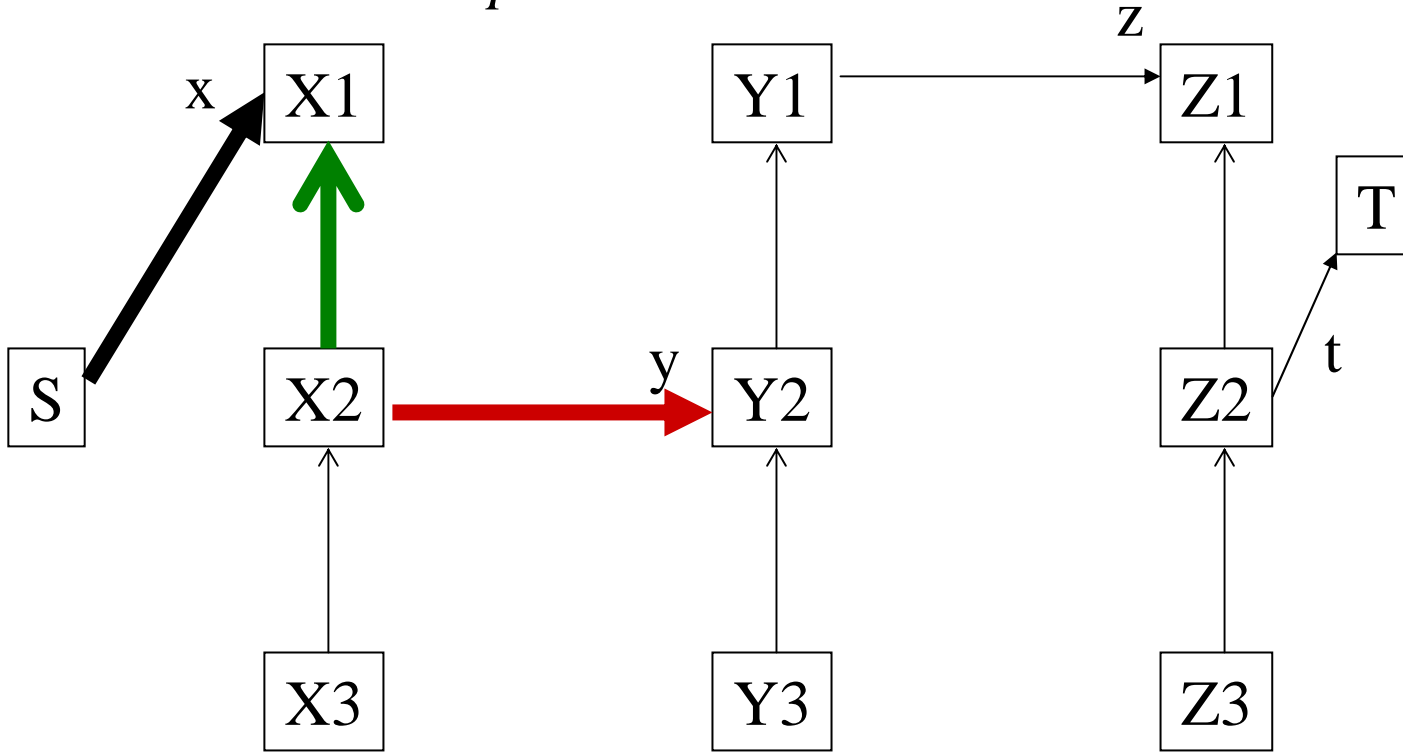
$\Rightarrow (X2, X3)$

...



go down x iff S and T are in relation: $\Leftarrow.x.\Rightarrow.(\Leftarrow.C.\Rightarrow)^*.\Leftarrow$

Example A

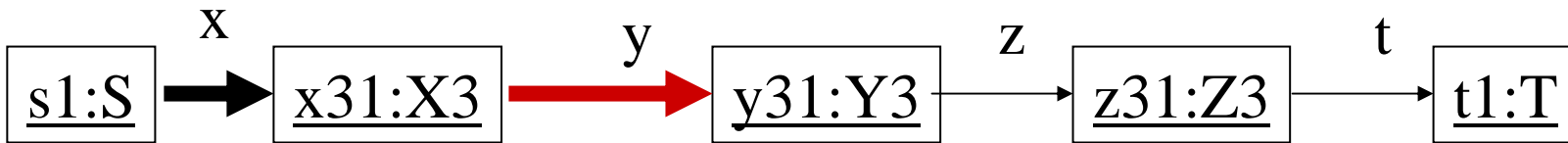


Strategy

S -> T

Relations:

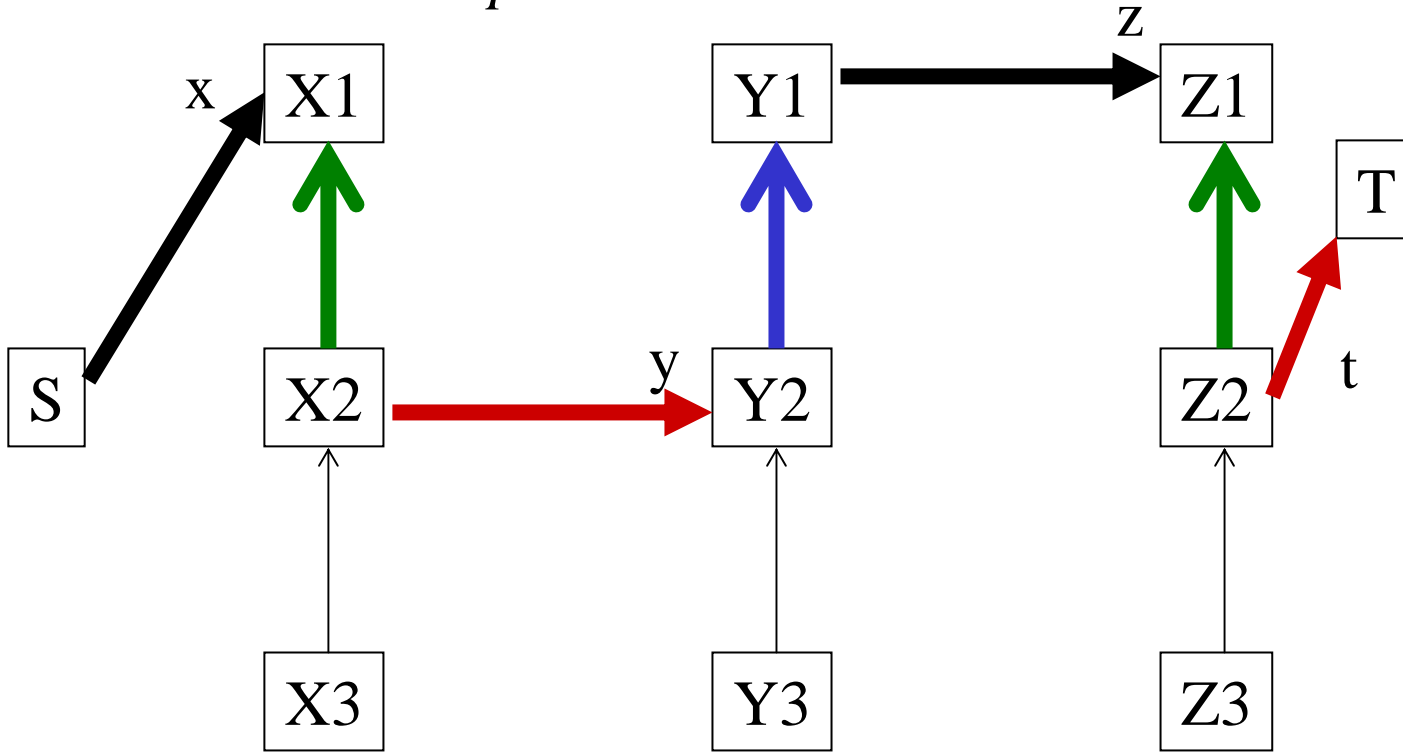
- x(S,X1)
- =>(X1,X2)
- y(X2,Y2)
- <=(Y2,Y1)
- z(Y1,Z1)
- =>(Z1,Z2)
- t(Z2,T)
- <=(X2,X1)
- ...



⟨=,=⟩
not
used

go down x iff S <=.x. =>.(<=.C.=>)*.<= T

Example A

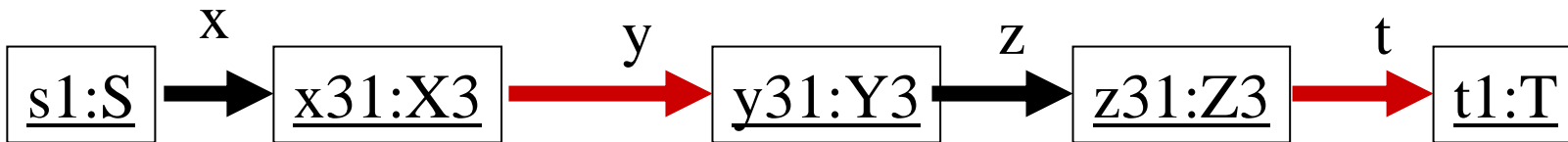


Strategy

S -> T

Relations:

- x(S,X1)
- =>(X1,X2)
- y(X2,Y2)
- <=(Y2,Y1)
- z(Y1,Z1)
- =>(Z1,Z2)
- t(Z2,T)
- <=(X2,X1)
- ...



go down x iff S <=.x.=>.(<=.C.=>.<=.C.=>.<=.C.=>).<= T

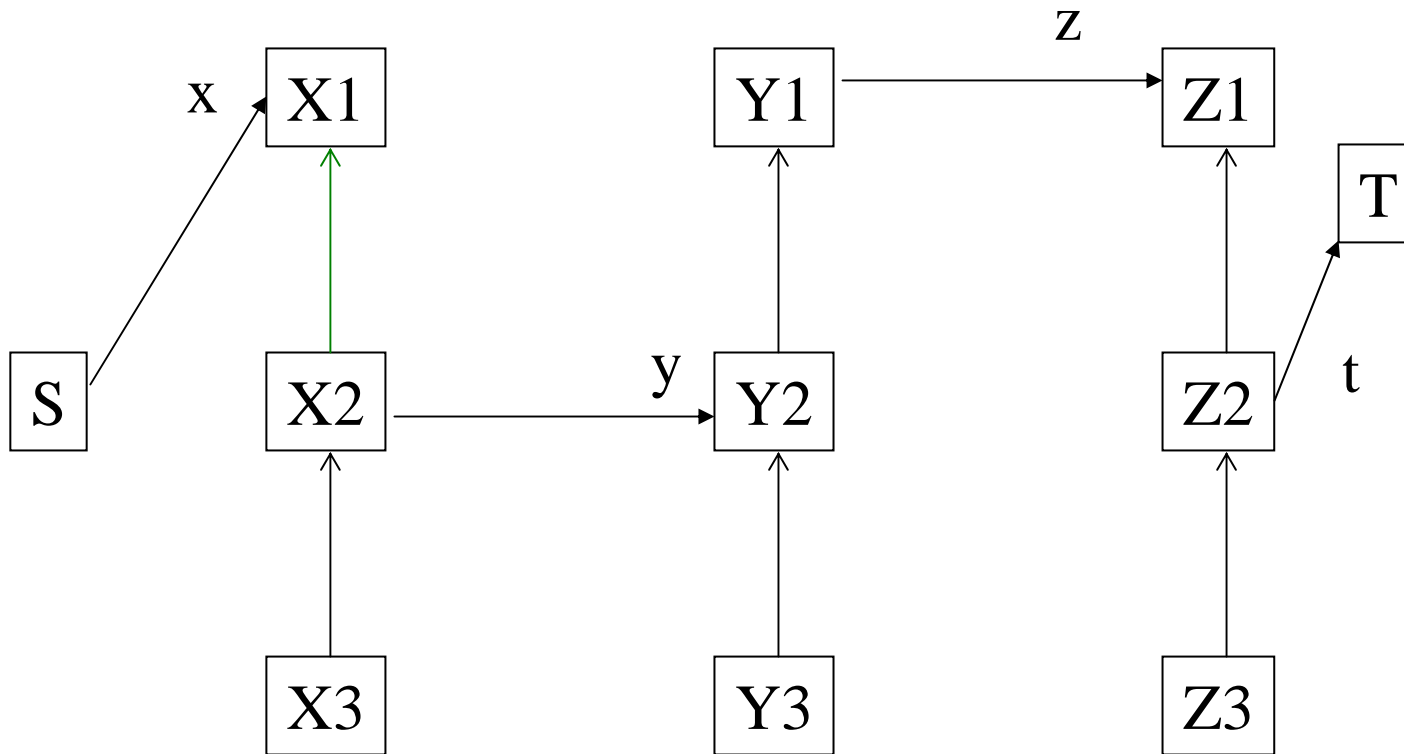
<=,=>

not

used

Just in terms of relations

Relations are sets of pairs, ordering is irrelevant.

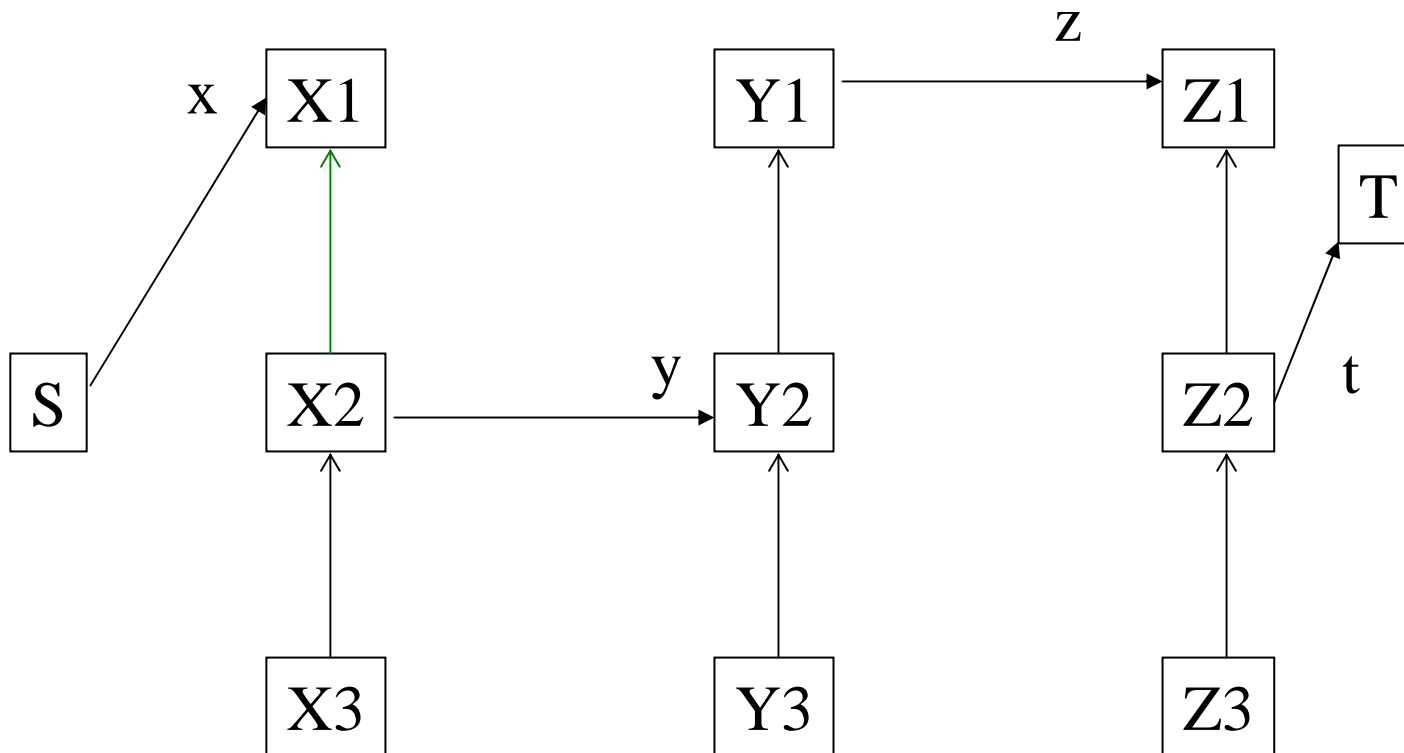


Set: {S,T,X1,X2,X3,Y1,Y2,Y3,Z1,Z2,Z3}

Write graph in terms of relations

Set: {S,T,X1,X2,X3,Y1,Y2,Y3,Z1,Z2,Z3}

Relations are sets of pairs, ordering is irrelevant.



Relations:

$x, y, z, t, \leq, \Rightarrow$

$x(S, X1)$

$y(X2, Y2)$

$z(Y1, Z1)$

$t(Z2, T)$

$\leq(Y2, Y1)$

$\leq(X2, X1)$

$\leq(X3, X2)$

...

$\Rightarrow(X2, X3)$

$\Rightarrow(X1, X2)$

$\Rightarrow(Z1, Z2)$

...

Set: {S,T,X1,X2,X3,Y1,Y2,Y3,Z1,Z2,Z3}

Compose relations

Relations are sets of pairs, ordering is irrelevant.

Relations: x,y,z,t, \leftarrow , \Rightarrow *← inheritance in reverse*

x(S,X1)

y(X2,Y2)

z(Y1,Z1)

t(Z2,T)

\leftarrow =(Y2,Y1)

\leftarrow =(X2,X1)

\leftarrow =(X3,X2)

...

\Rightarrow =(X2,X3)

\Rightarrow =(X1,X2)

\Rightarrow =(Z1,Z2)

...

inheritance

Are S and T in the relation: \leftarrow . **x** . \Rightarrow . (\leftarrow . **C** . \Rightarrow \leftarrow . **C** . \Rightarrow . \leftarrow . **C** . \Rightarrow) . \leftarrow

Set: {S,T,X1,X2,X3,Y1,Y2,Y3,Z1,Z2,Z3}

Just in terms of relations

Relations are sets of pairs, ordering is irrelevant.

Relations: x,y,z,t,<=,=>

x(S,X1)

y(X2,Y2)

z(Y1,Z1)

t(Z2,T)

<=(Y2,Y1)

<=(X2,X1)

<=(X3,X2)

...

=>(X2,X3)

=>(X1,X2)

=>(Z1,Z2)

...

The order in which we consume the pairs

Relations:

x(S,X1)

=>(X1,X2)

y(X2,Y2)

<=(Y2,Y1)

z(Y1,Z1)

=>(Z1,Z2)

t(Z2,T)

go down x iff $S \leftarrow \mathbf{x} \rightarrow \cdot (\leftarrow \mathbf{C} \rightarrow \leftarrow \mathbf{C} \rightarrow \leftarrow \mathbf{C} \rightarrow) \leftarrow \mathbf{T}$

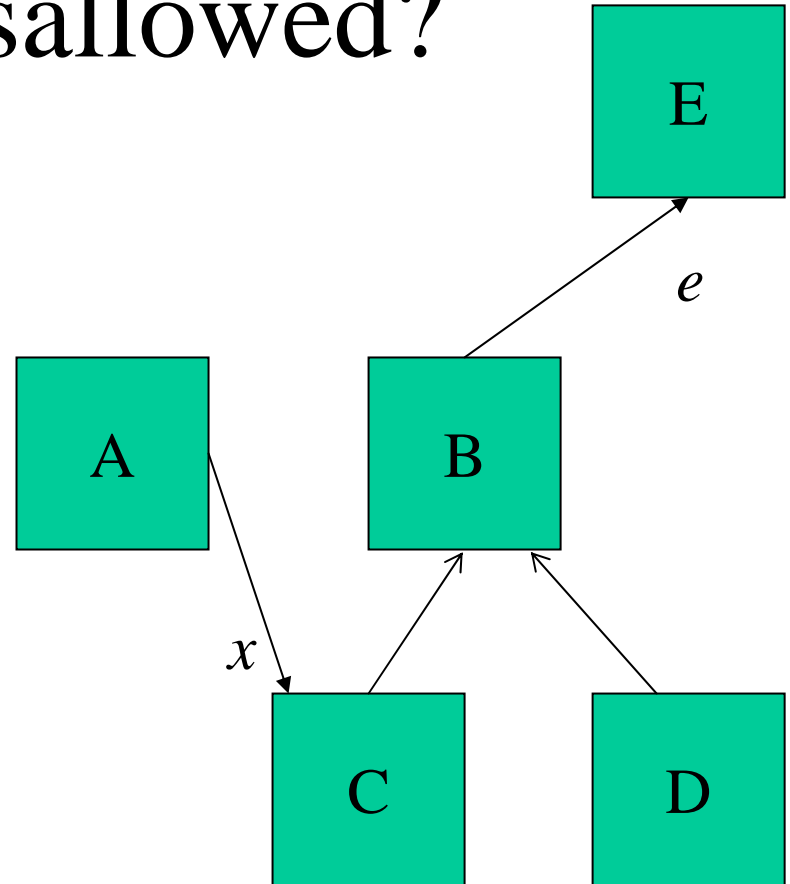
How big is the relation?

How many pairs does the relation contain in this example?

$\langle \cdot, x \cdot \rangle, \langle \cdot, C \cdot \rangle, \langle \cdot, C \cdot \rangle, \langle \cdot, C \cdot \rangle, \langle \cdot, C \cdot \rangle$

What is disallowed?

Where can you go from A?



go down x iff S and T are in relation: $\langle =.x. = \rangle . (\langle =.C.= \rangle)^* . \langle =$

class dictionary

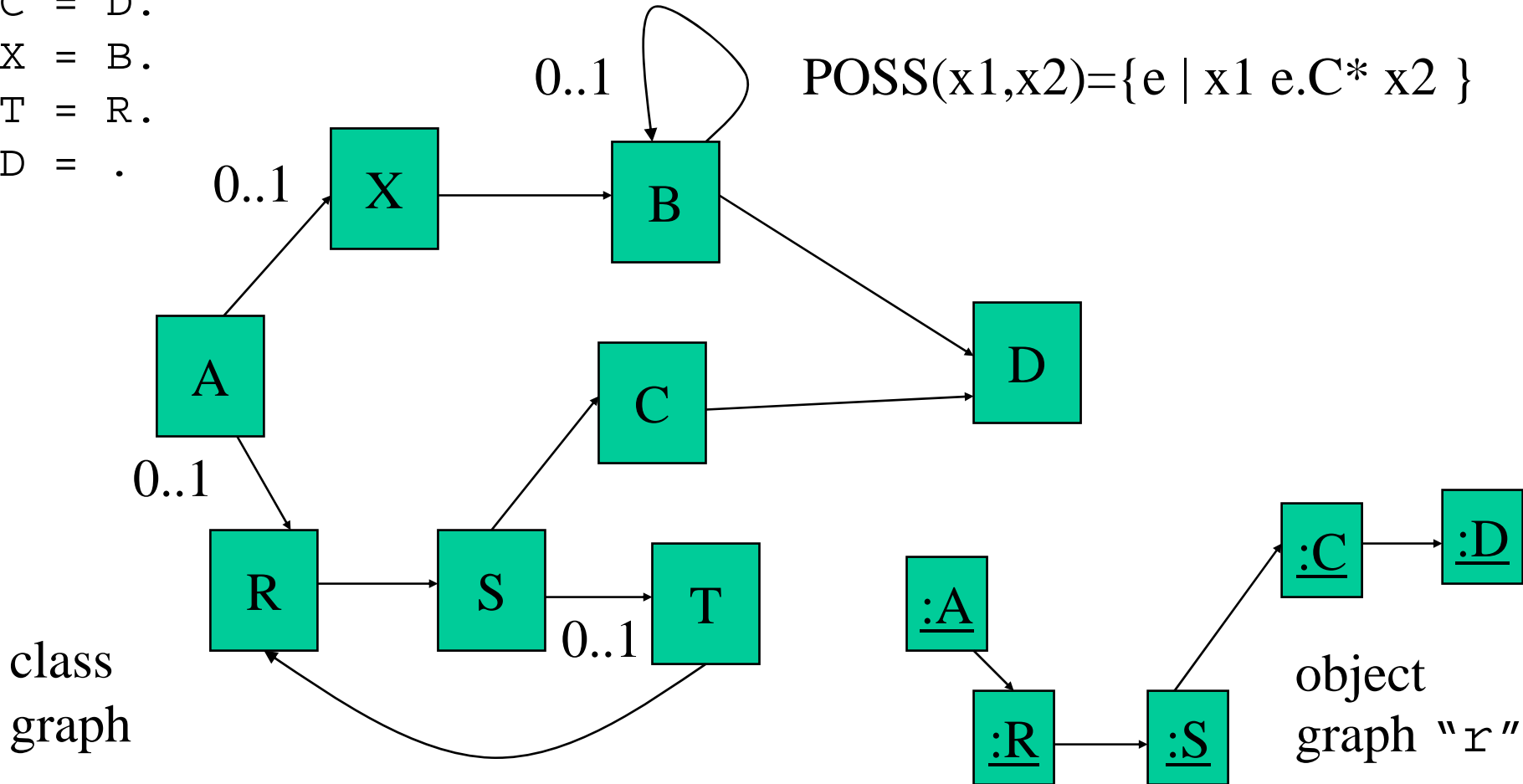
A = ["x" X] ["r" R].
B = ["b" B] D.
R = S.
S = ["t" T] C
C = D.
X = B.
T = R.
D = .

strategy

A -> T
T -> D

Example B

$POSS(x1,x2)=\{e \mid x1 e.C^* x2 \}$



class dictionary

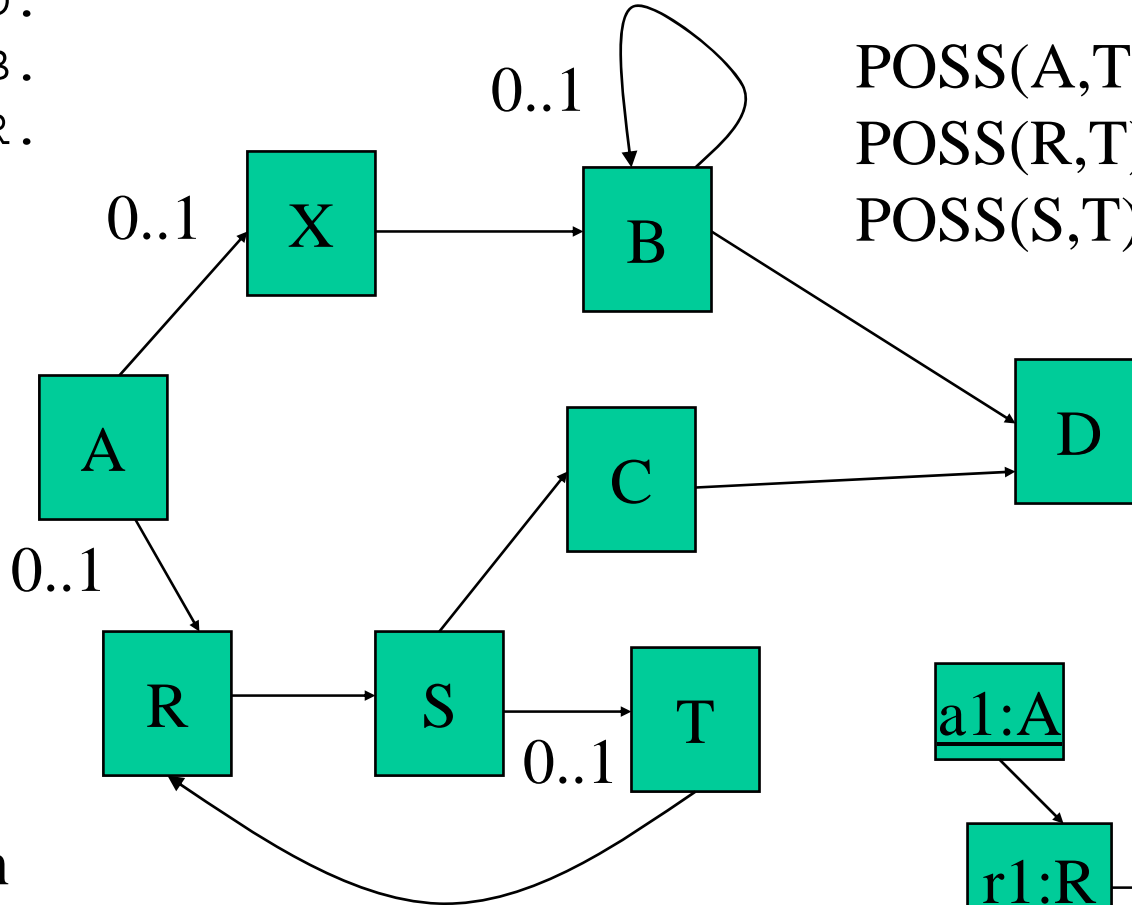
A = ["x" X] ["r" R].
 B = ["b" B] D.
 R = S.
 S = ["t" T] C
 C = D.
 X = B.
 T = R.
 D = .

strategy

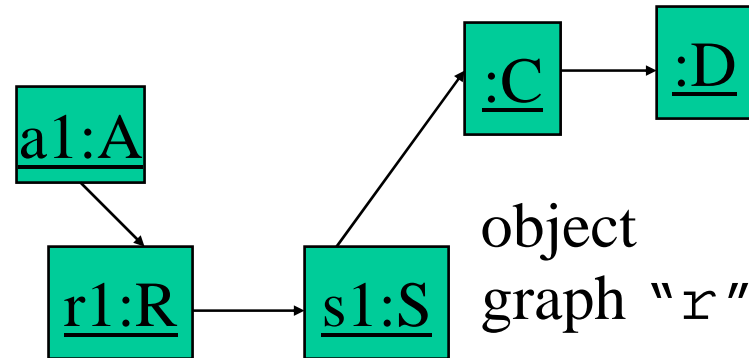
A -> T
 T -> D

Example B1

POSS(A,T) = 1 edge
 POSS(R,T) = 1 edge
 POSS(S,T) = 0 edges



class graph



object graph "r"

$$POSS(c1, c2) = \{ e \mid c1 \xrightarrow{e} C^* c2 \}$$

class dictionary

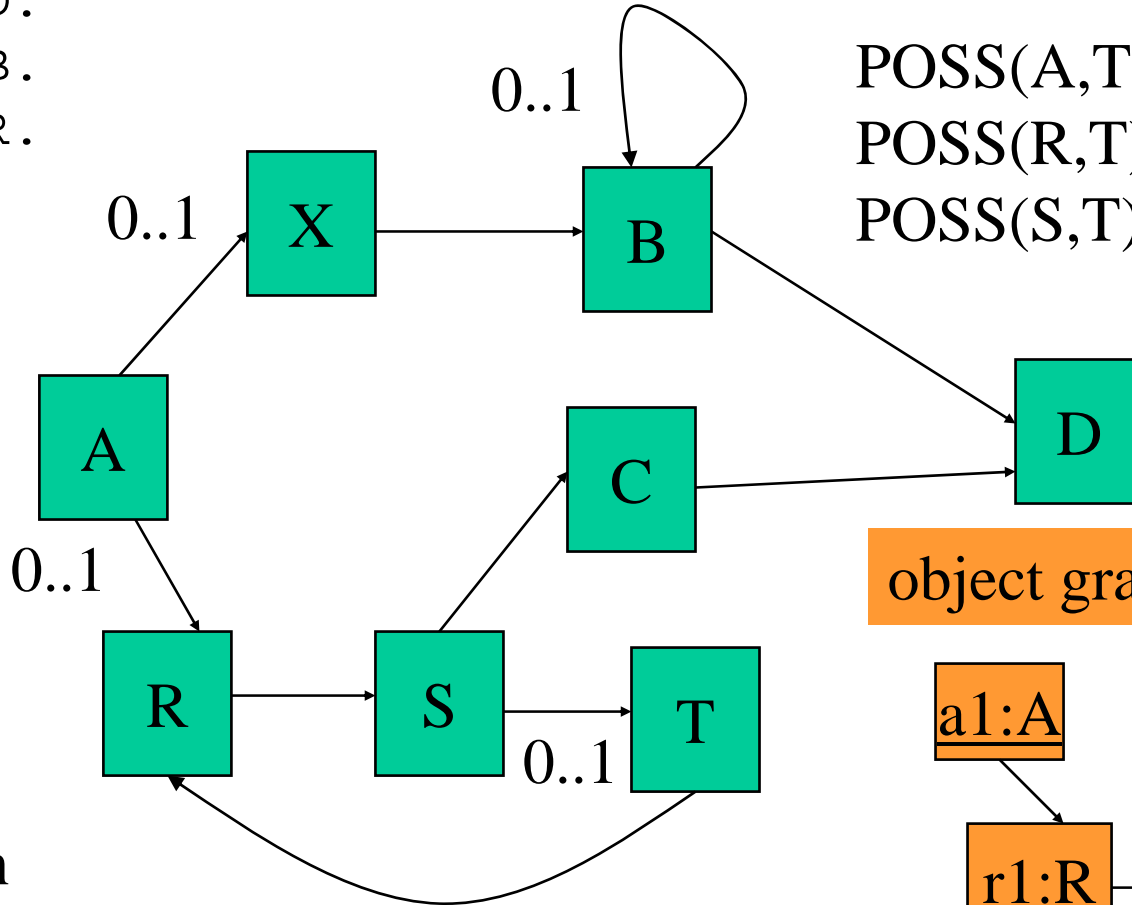
A = ["x" X] ["r" R].
 B = ["b" B] D.
 R = S.
 S = ["t" T] C
 C = D.
 X = B.
 T = R.
 D = .

strategy

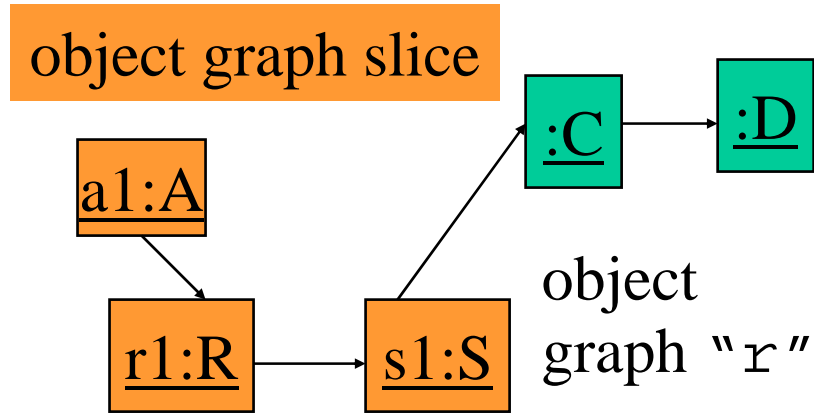
A -> T
 T -> D

Example B1

POSS(A,T) = 1 edge
 POSS(R,T) = 1 edge
 POSS(S,T) = 0 edges



class graph



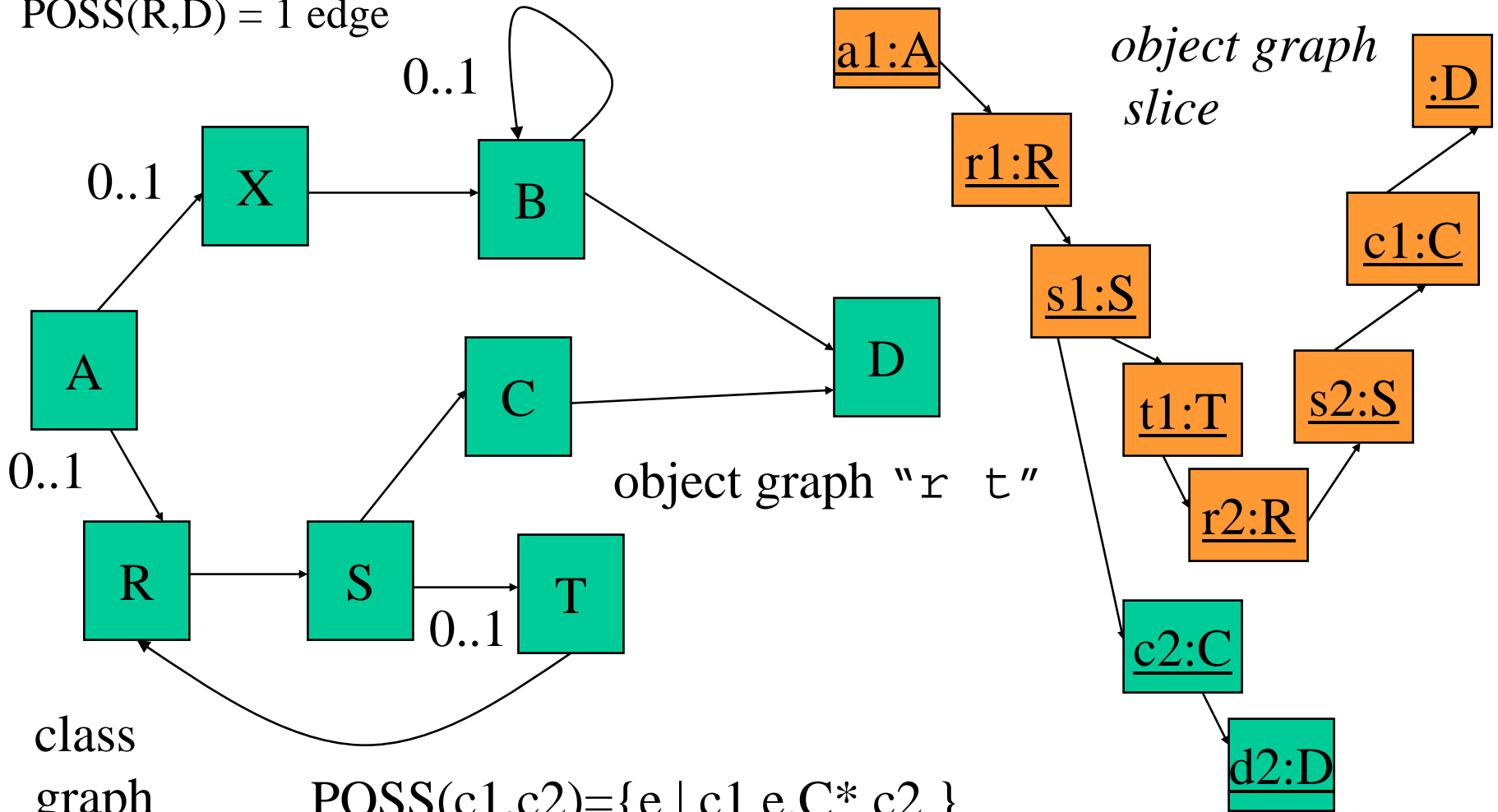
$$POSS(c1,c2) = \{e \mid c1 \xrightarrow{e} C^* c2\}$$

Example B2

POSS(A,T) = 1 edge
 POSS(R,T) = 1 edge
 POSS(S,T) = 1 edge
 POSS(T,D) = 1 edge
 POSS(R,D) = 1 edge

strategy

A -> T
 T -> D



class graph

$$POSS(c1,c2) = \{ e \mid c1 \xrightarrow{e} C^* c2 \}$$

POSS(A,T) = 1 edge
 POSS(R,T) = 1 edge
 POSS(S,T) = 1 edge
 POSS(T,D) = 1 edge
 POSS(R,D) = 1 edge

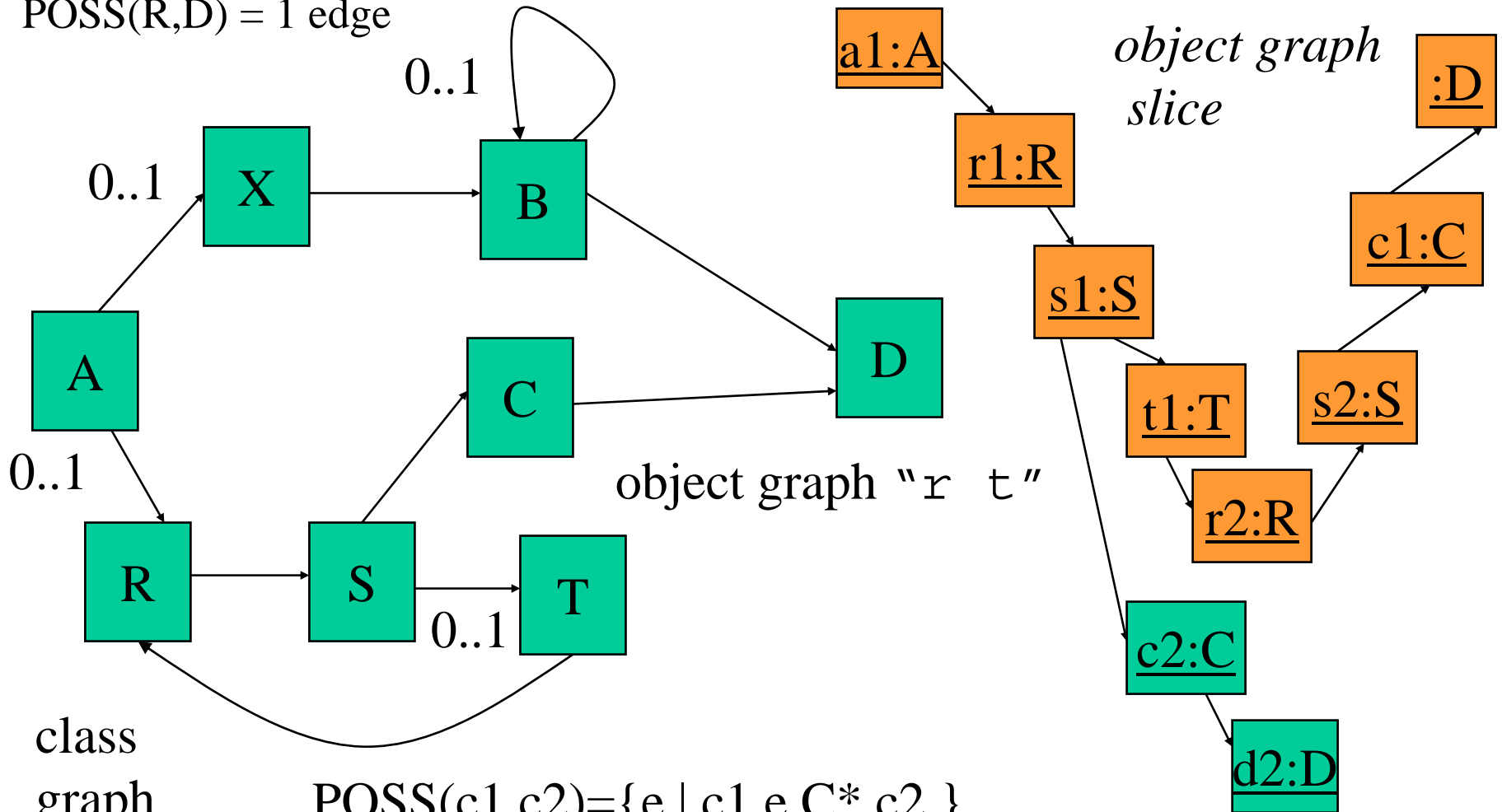
Example B2

strategy

A -> T

T -> D

object graph slice



class graph

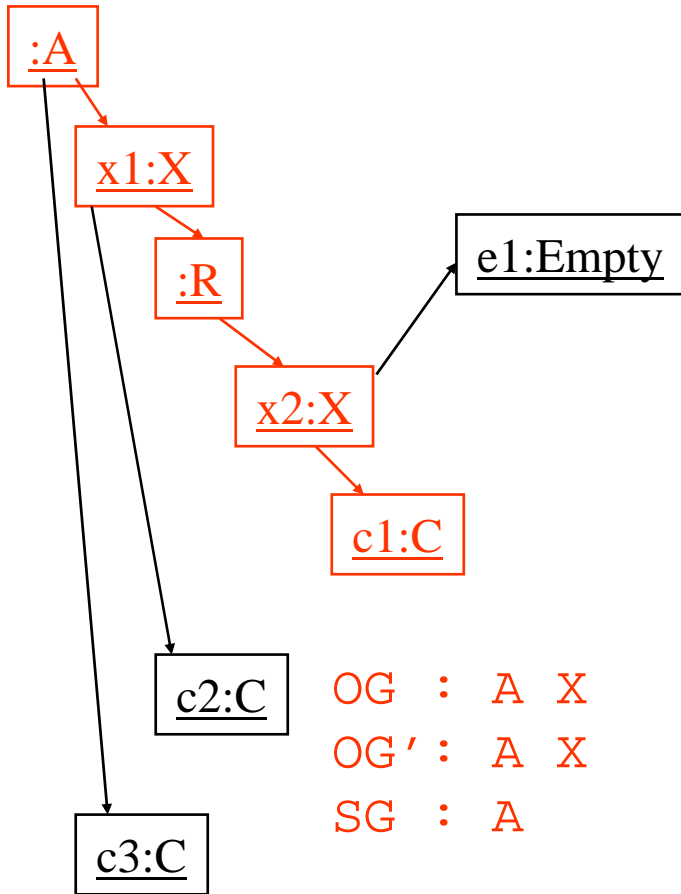
$$POSS(c1,c2) = \{e \mid c1 \xrightarrow{e} C^* c2\}$$

Only node paths shown for space reasons

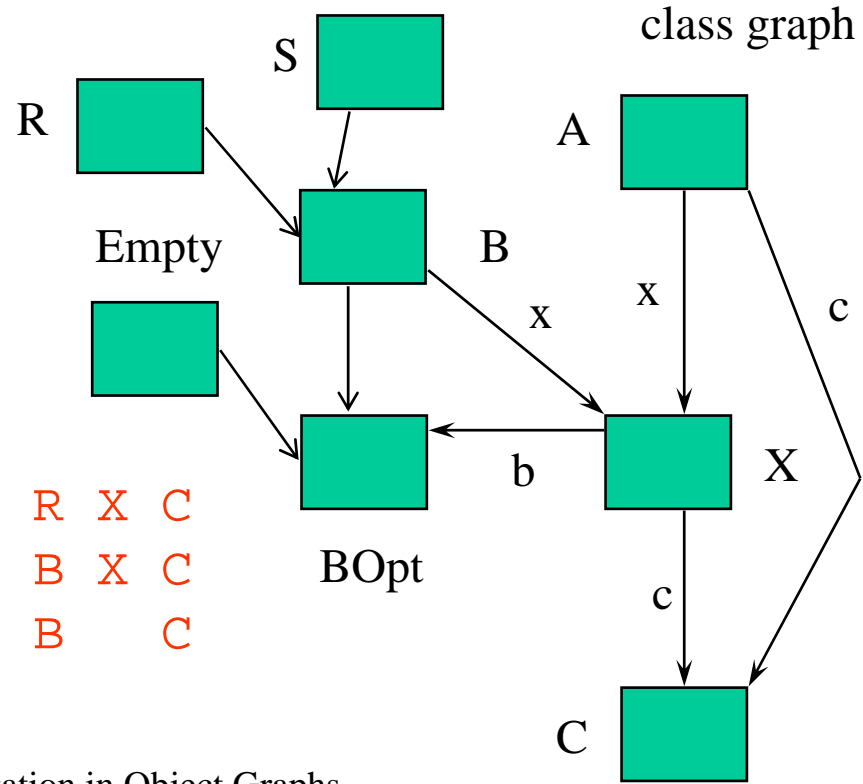
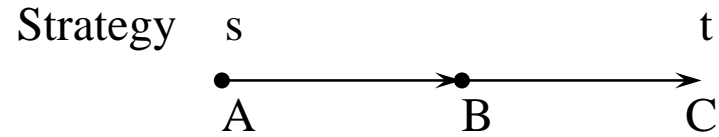
Example C

strategy SG:
 { A -> B
 B -> C }

Object graph



OG : A X
 OG' : A X
 SG : A



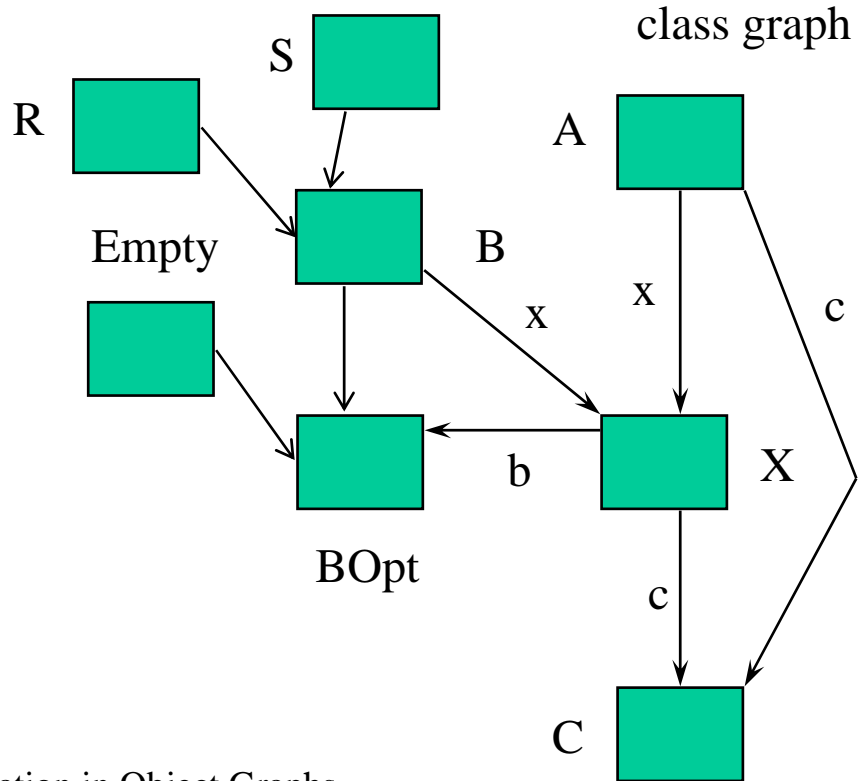
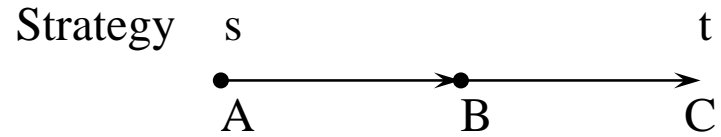
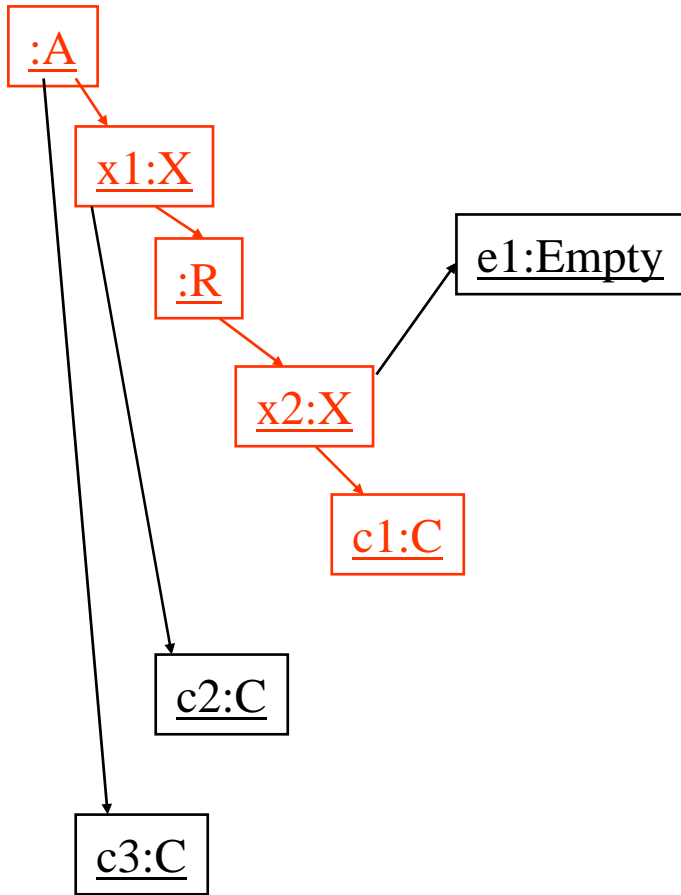
R X C
 B X C
 B C

Write Java code that does the traversal

Example C

strategy:
{ A -> B
 B -> C }

Object graph

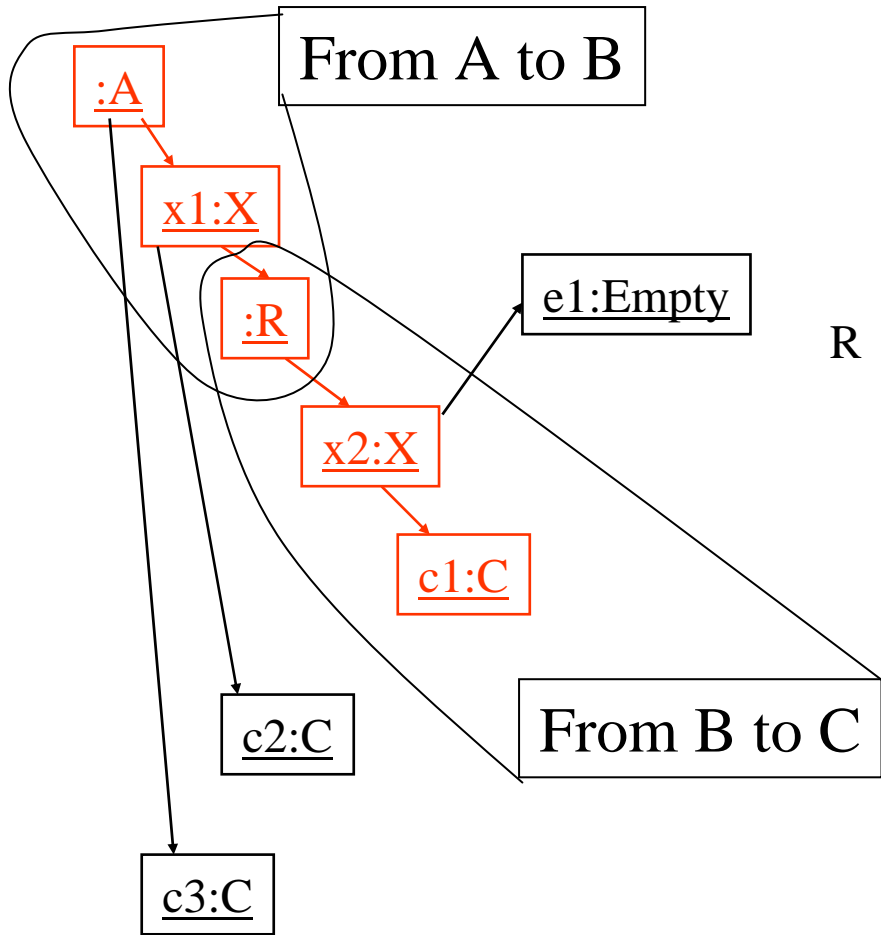


Not the complete story

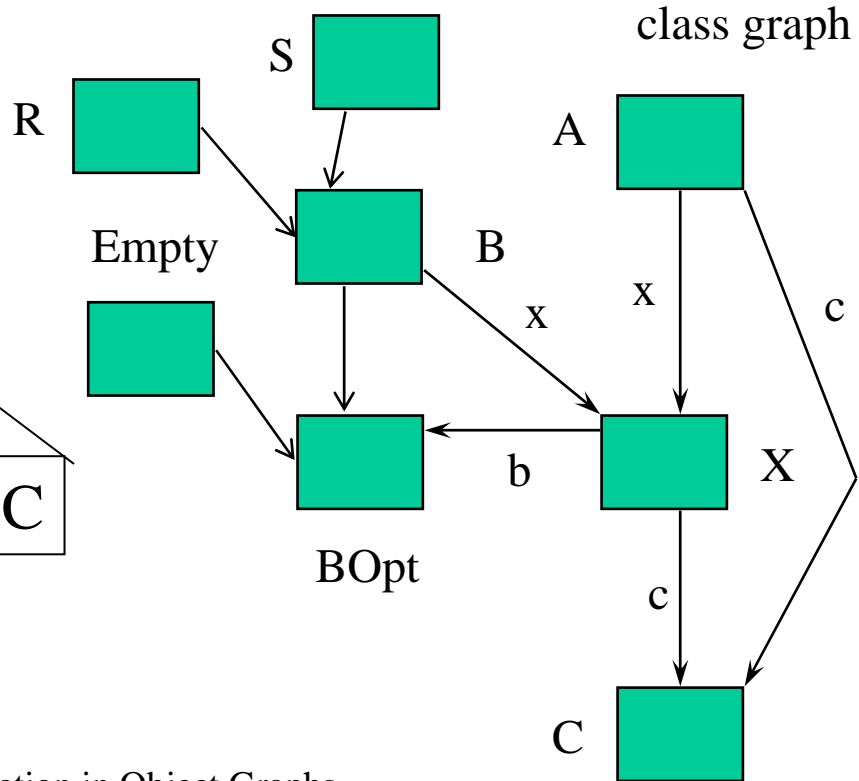
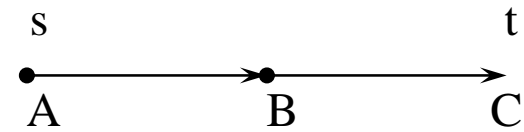
Example C

strategy:
{ A -> B
 B -> C }

Object graph



Strategy

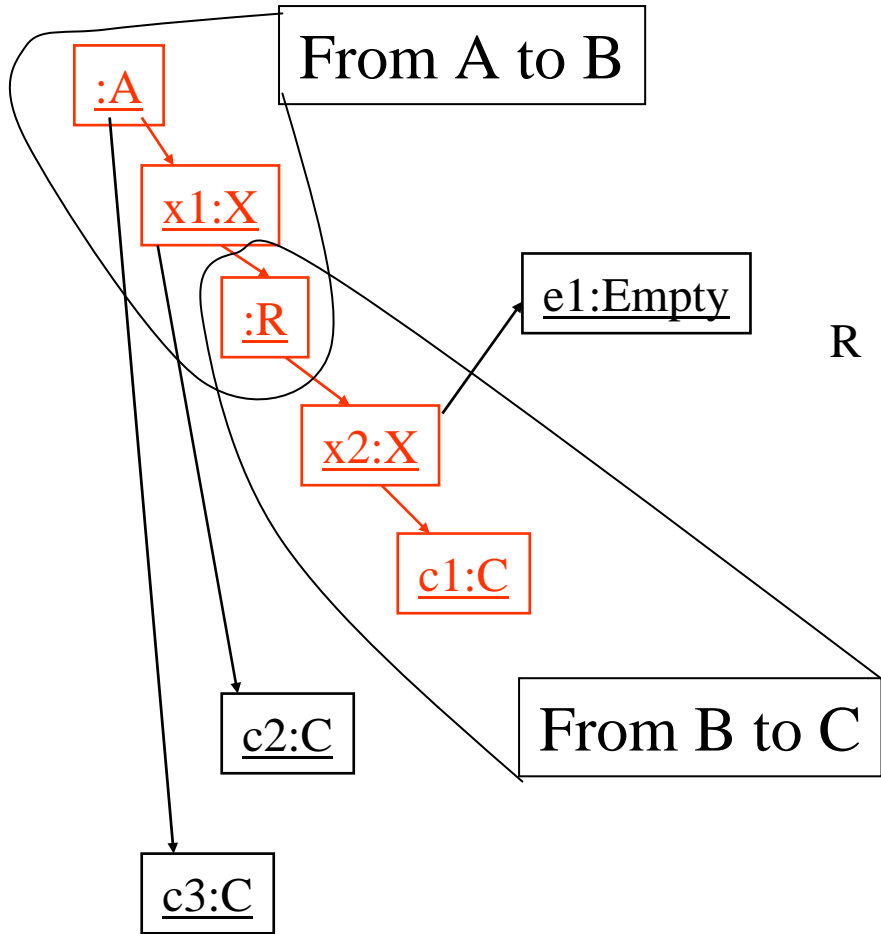


Not the complete story: traversal must look for further B

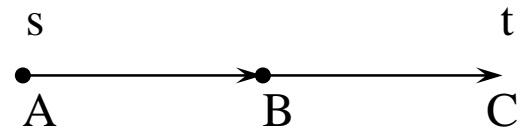
Example C

strategy:
 { A -> B
 B -> C }

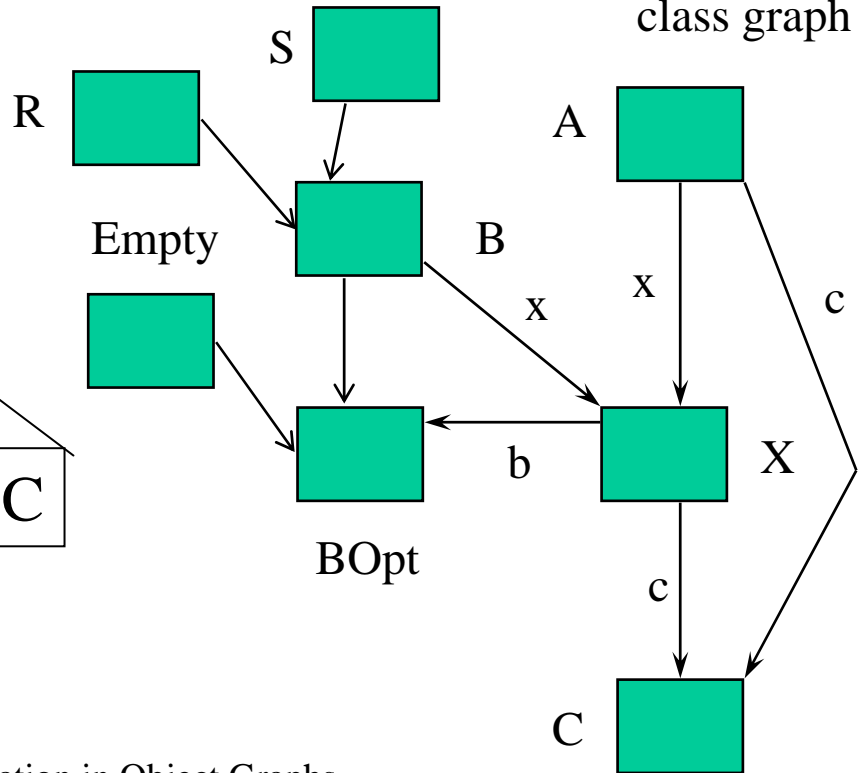
Object graph



Strategy



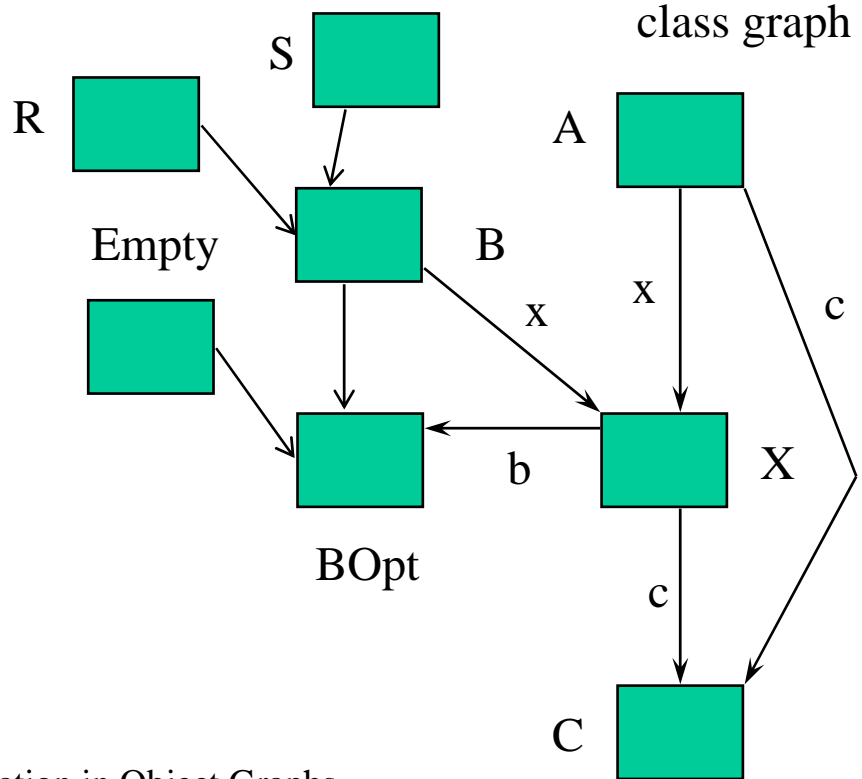
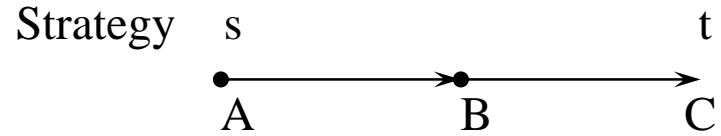
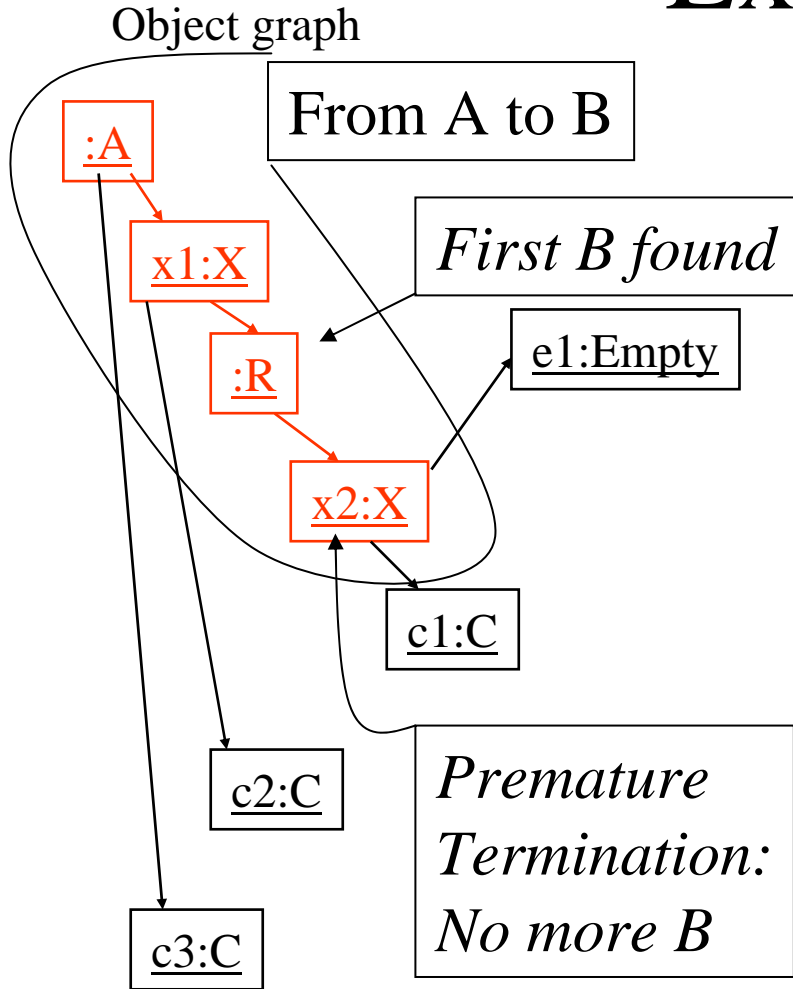
class graph



Not the complete story: traversal must look for further B

Example C

strategy:
 { A -> B
 B -> C }

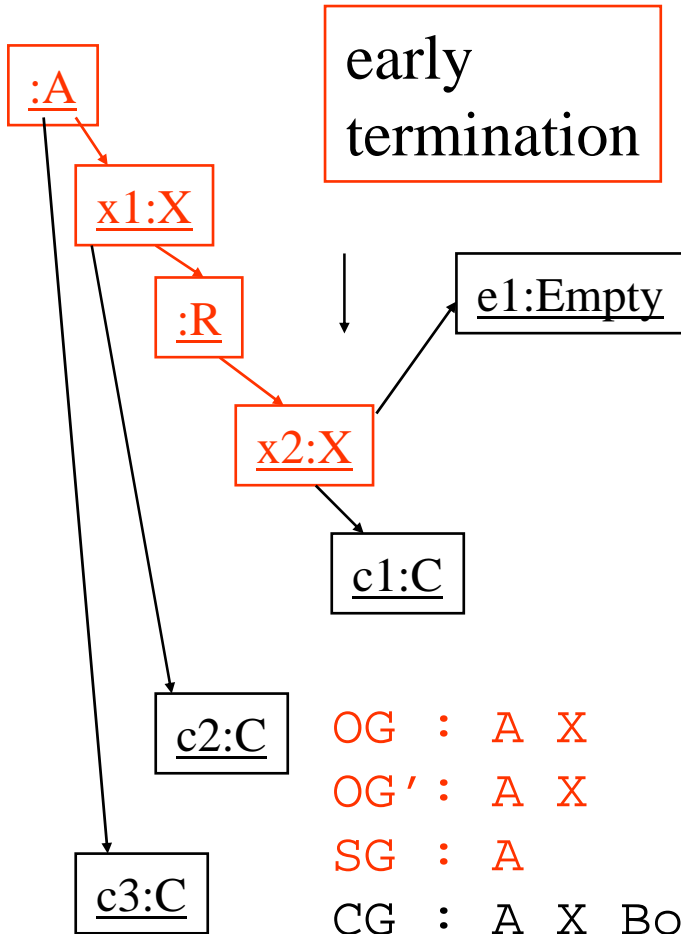


Only node paths shown for space reasons

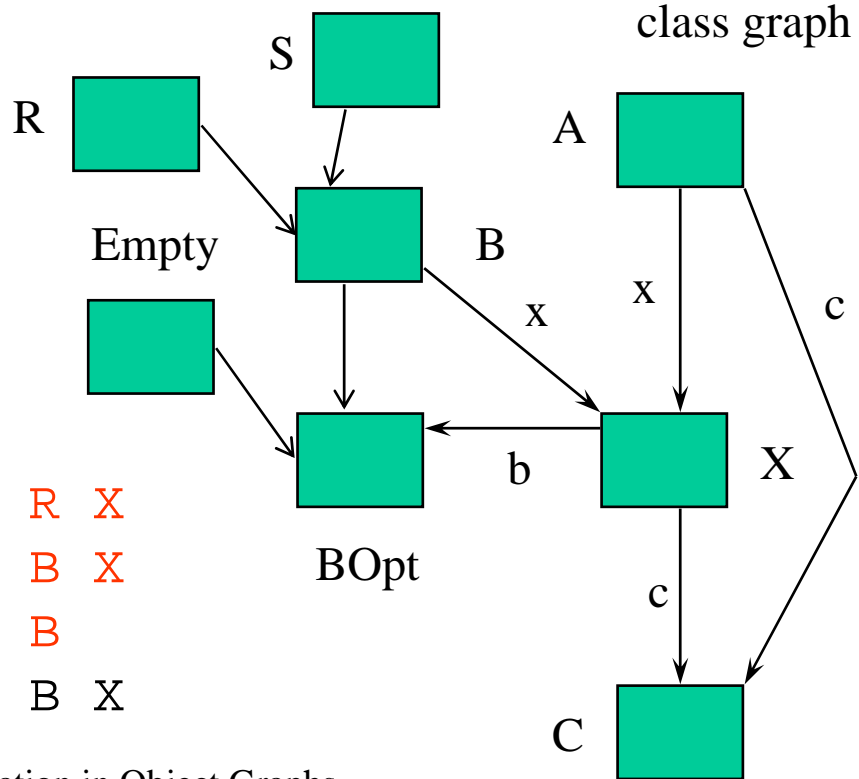
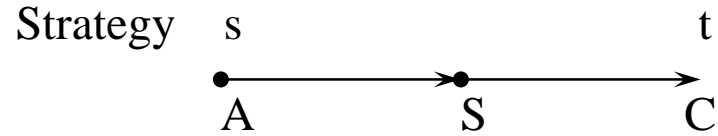
Example C1

strategy SG:
 $\{A \rightarrow s$
 $s \rightarrow C\}$

Object graph

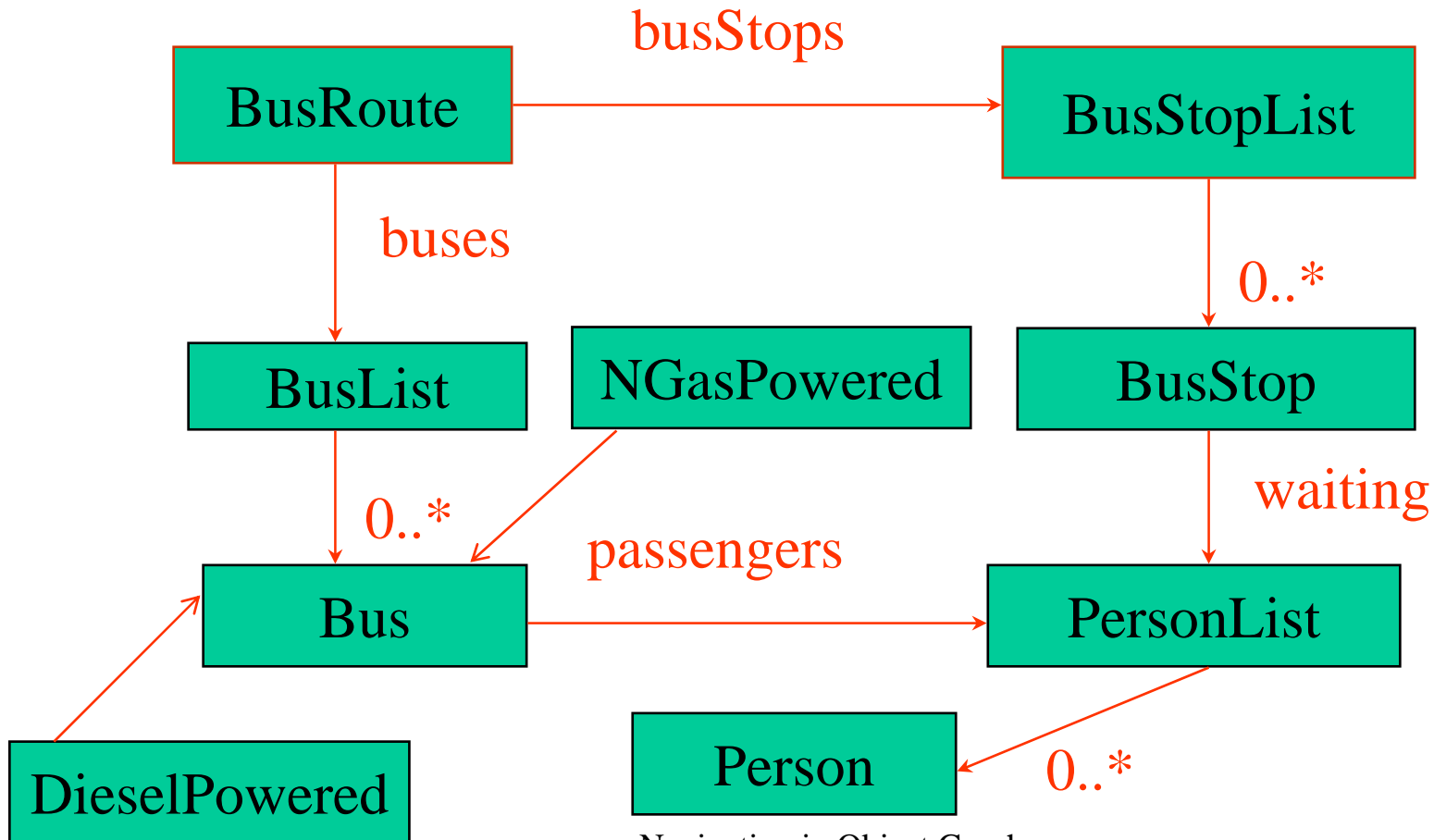


OG	:	A	X	R	X	
OG'	:	A	X	B	X	
SG	:	A		B		
CG	:	A	X	Bopt	B	X



S = from BusRoute through Bus to Person

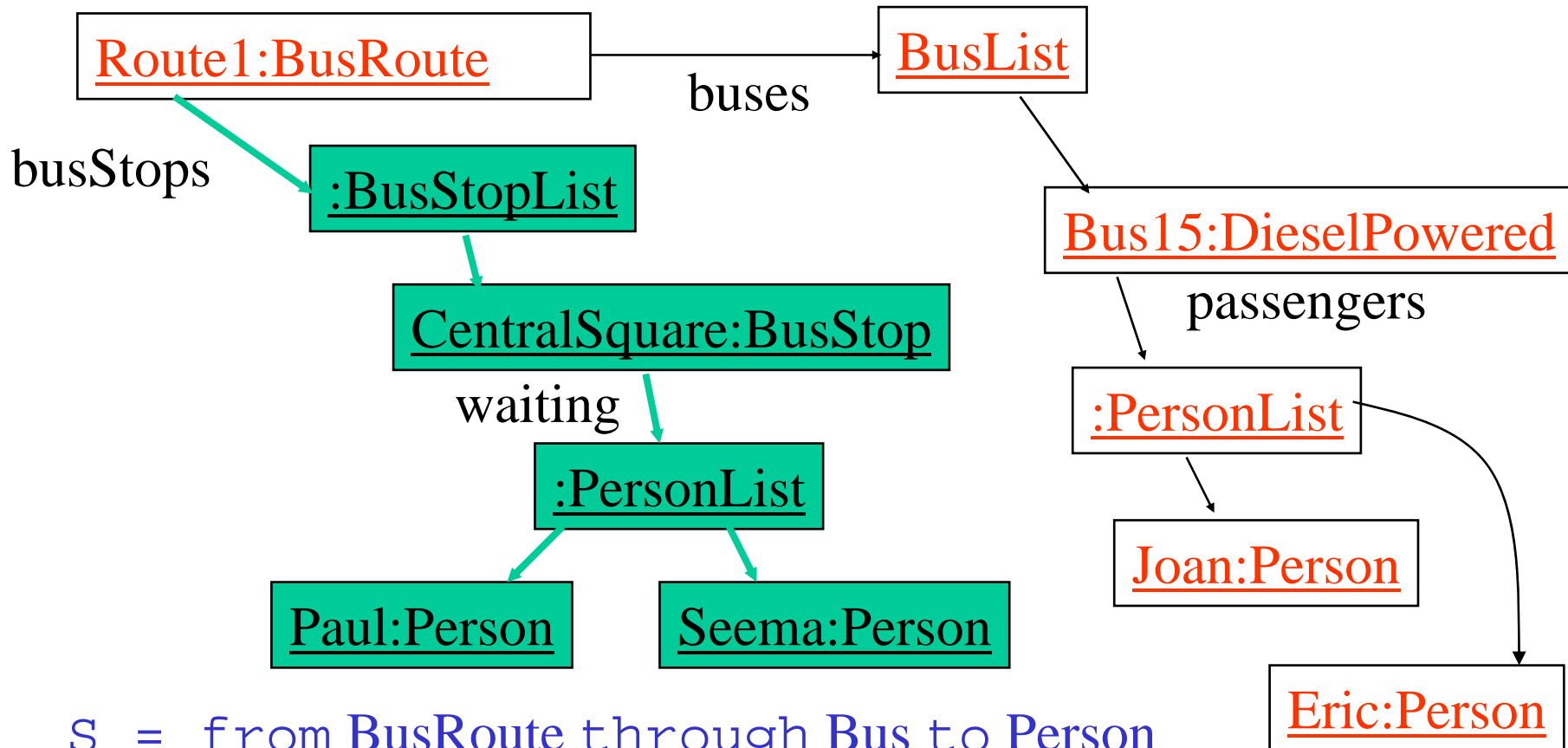
Example D



OG : BR BL DP PL P
 OG' : BR BL B PL P
 S : BR B P

Example D1

Only node paths shown for space reasons



S = from BusRoute through Bus to Person

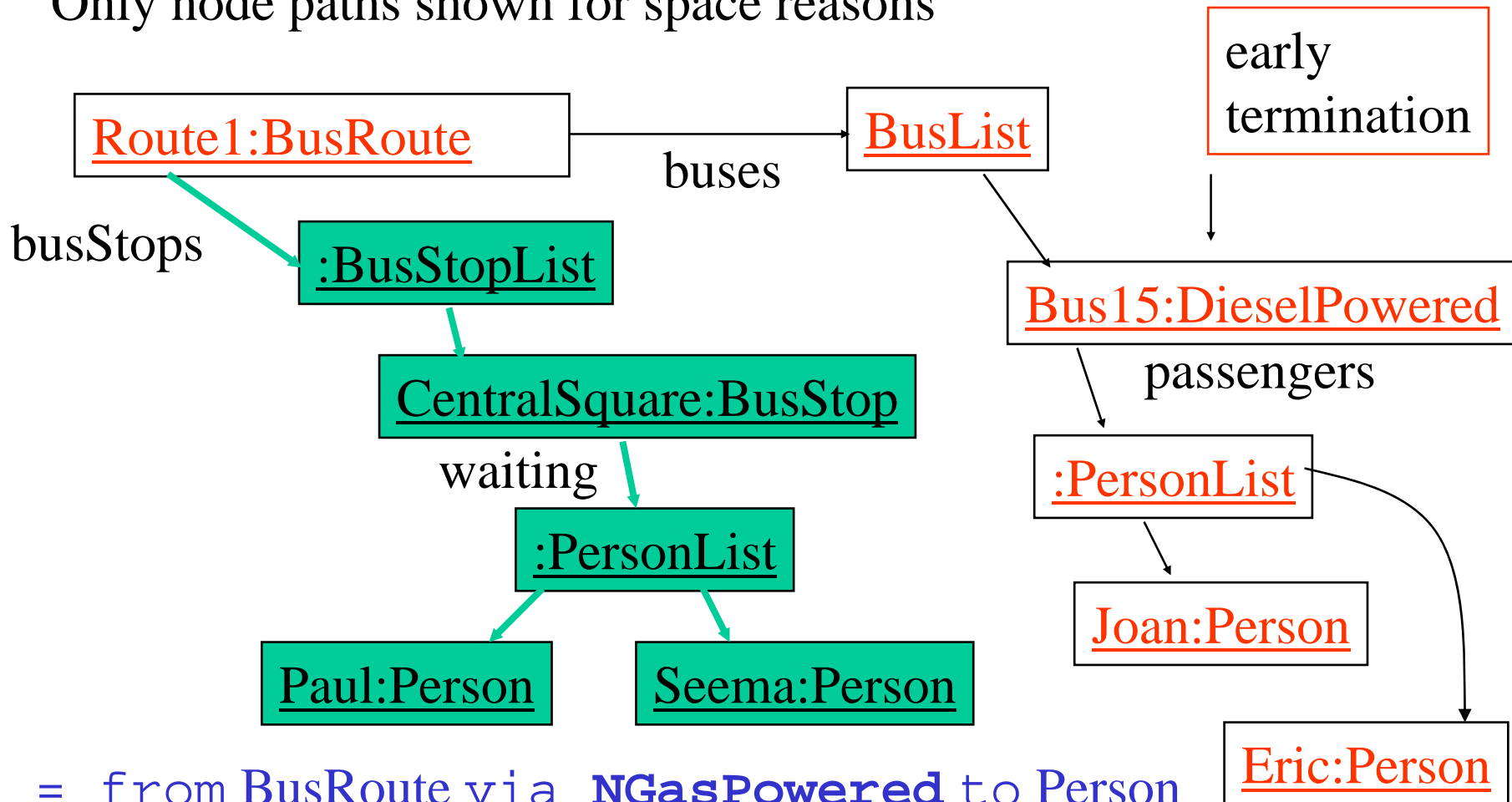
OG : BR BL

OG' : BR BL

S : BR

Example D2

Only node paths shown for space reasons



S = from BusRoute via **NGasPowered** to Person

from c1 **bypassing** {x1,x2, ... ,xn} to c2

Relational Formulation

From object o of class c1, to get to c2, follow edges in the set

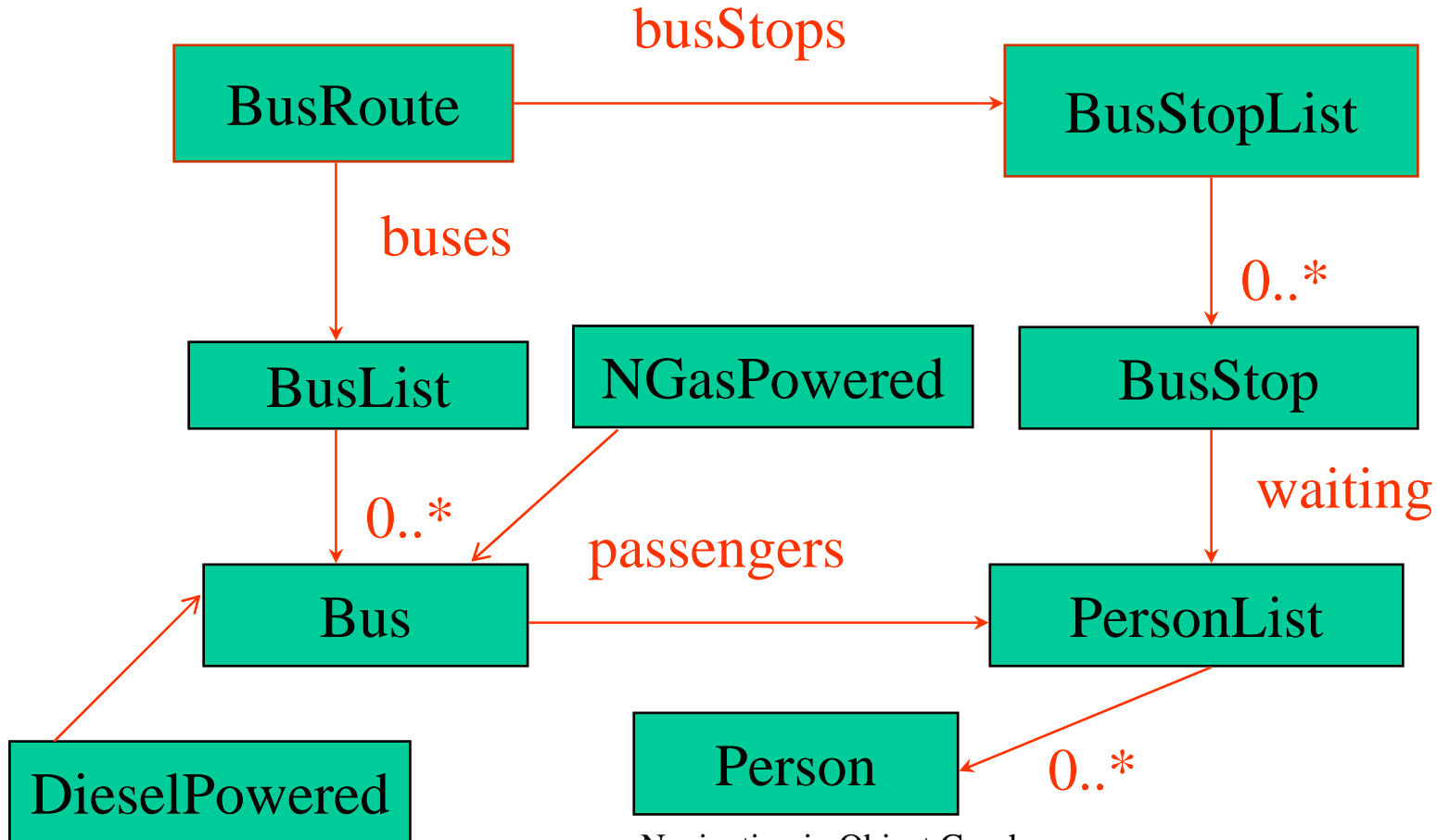
$$\text{POSS}(c1,c2)=\{e \mid c1 \leq.e.\Rightarrow (\leq.C.\Rightarrow)^* \leq c2 \}$$

POSS = abbreviation for: following these edges it is still possible to reach a c2-object for some c1-object rooted at o.

Delete x1,x2, ... ,xn and all edges incident with these nodes from the class graph (assumed to be different from c1, c2).

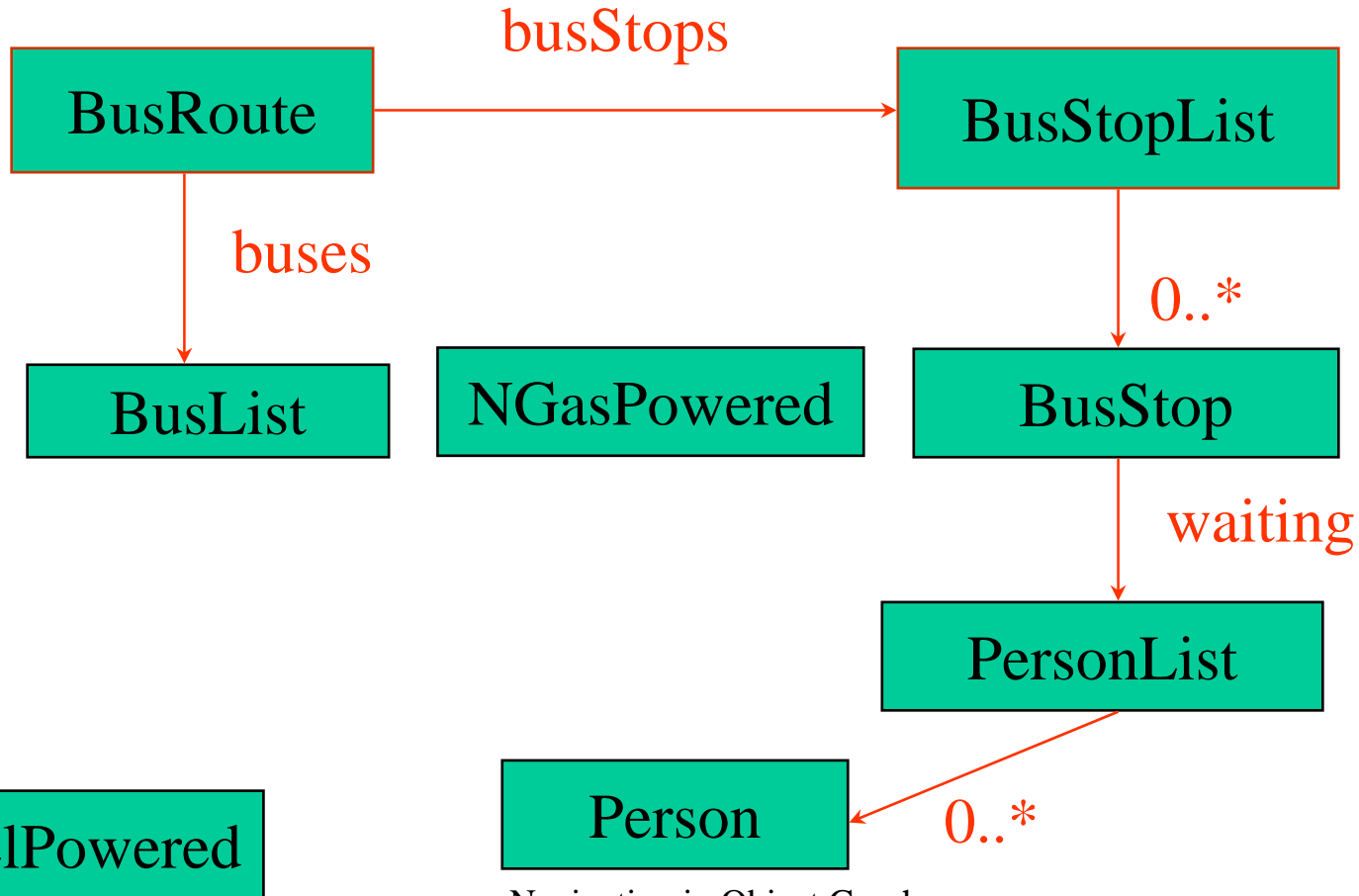
S = from BusRoute bybassing Bus to Person

Example D



S = from BusRoute bybassing Bus to Person

Example D



Conclusions

- Programming language elements are mathematical objects having precise mathematical definitions.
- Exercise in applying an abstract algorithm to concrete inputs. Mapping abstract situations to concrete situations.
- Separation of concerns is also useful for defining programming language elements
 - separate subgraph selected from
 - how the subgraph is traversed (depth-first etc.)
- In earlier works: meaning of a traversal strategy for an object graph
 - was a traversal history
 - now it is a subgraph of the object graph. A traversal history can be defined ...