CS 5800: Algorithms — K.Lieberherr, V. Pavlu
Due: April. 7, 2014

Homework Module 10

1 Submission Rules

`http://www.ccs.neu.edu/home/lieber/courses/algorithms/cs5800/sp14/homeworks/submission-rules.pdf`

We have removed the competition component from the debates. It is still important that you do the debates but whether you win or lose and which side you choose does not matter. However, it matters that you debate according to the rules.

2 Problems]

1. (30 points) Submission of debates and programs for the leafcovering problem.
2. (15 pts) Exercise 22.1-5. Square of a graph.
3. (10 pts) Exercise 22.3-7. Replace recursion by stack.
4. (15 pts) Exercise 22.3-12. DFS for connected components.
5. (20 pts) Exercise 22.4-4. Topological ordering with cycles.
6. (20 pts) Exercise 22.4-5. Topological ordering without DFS.
7. (15 pts) Two special vertices s and t in the undirected graph G=(V,E) have the following property: any path from s to t has at least $1 + |V|/2$ edges. Assume V is even. Show that all paths from s to t must have a common vertex v (not equal to either s or t) and give an algorithm with running time O(V+E) to find such a node v.
8. (30 points) We generalize the previous question. This claim is about networks and how easily networks with "long" paths can be disrupted by destroying a few nodes. We want to find the minimum number of nodes that need to be destroyed in a specific context.

   Claim Vulnerable

   $$\forall c \in Nat \text{ where c} > 1$$

   $$\exists d \in Nat$$

   $$\forall \text{ undirected graphs G} = (\text{V}, \text{E})$$

   where $|V|$ is divisible by c without remainder and such that $\exists s, t \in V : dist(s,t) > |V|/c$

   $$(\exists \text{ set w of nodes} \in \text{V}$$

where w does not contain s or t and set w contains d nodes such that $G' = (V - w, E -$ edges incident with a node in w) has no s-t path

and

there exists no smaller set w with less than d nodes which has above property.)

end Claim Vulnerable This ends the definition of claim Vulnerable.

Nat is the set of natural numbers 0,1,2, ...

The distance $dist(s, t)$ between two nodes s and t is the length of a shortest path from s to t.

Example claims: Consider the following subclaim of Vulnerable $c = 2, d = 1$ which is true; see previous question.

For representing graphs, use this notation:

http://www.ccs.neu.edu/home/lieber/courses/algorithms/cs5800/sp14/JSON/graphs-in-JSON

For representing sets, use this notation: [0,1,2,3,4,5]. You need this for representing the sets w.

What is the minimum value of d as a function of c? What is your defense strategy? KEEP YOUR DEFENSE STRATEGY SECRET but turn it in as part of your answer for this question. Turn in your defense strategy with proof and the debates within your team.

Here is what to turn in

(a) Express d as a function of c.

(b) Proof for previous item, including your clever algorithm and its analysis.

(c) Debates, including debate tree.

9. (25 points) Problem 22-3. Euler tour.

10. (25 pts) Exercise 23.1-3. Light edge crossing.

11. (25 pts) Exercise 23.2-4. Kruskal Analysis.