CS3000: Algorithms & Data — Fall '18 — Jonathan Ullman

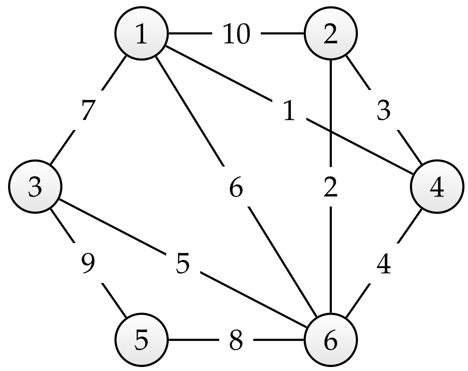
Homework 7
Due Friday November 9 at 11:59pm via Gradescope

Name:
Collaborators:

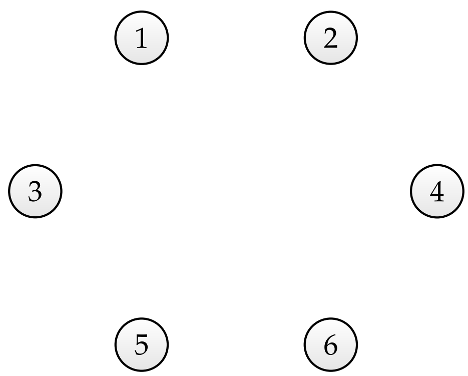
- Make sure to put your name on the first page. If you are using the LaTeX template we provided, then you can make sure it appears by filling in the `yourname` command.
- This assignment is due Friday November 9 at 11:59pm via Gradescope. No late assignments will be accepted. Make sure to submit something before the deadline.
- Solutions must be typeset in LaTeX. If you need to draw any diagrams, you may draw them by hand as long as they are embedded in the PDF. I recommend using the source file for this assignment to get started.
- I encourage you to work with your classmates on the homework problems. *However, you must write all solutions by yourself, in your own words.* Do not share any written or typed solutions. Do not submit anything you cannot explain. Please list all your collaborators in your solution for each problem by filling in the `yourcollaborators` command.
- Finding solutions to homework problems on the web, or by asking students not enrolled in the class is strictly forbidden.

Problem 1. *MST Practice*

Compute an MST in the following graph. You do not need to justify your answer. **Hint:** Draw your MST by filling in the edges on the skeleton provided.

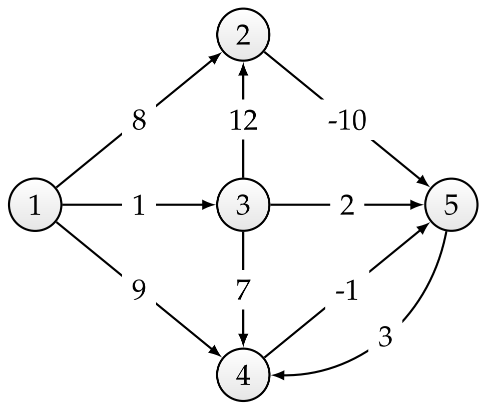


Solution:



Problem 2. *Shortest Paths Practice*

Solve the single-source shortest path problem on the following graph, using node 1 as the source. Write the distance from s to each node and write the parent of each node in the shortest path tree. **Hint:** Write your solution using the skeleton table provided.



Solution:

Node	1	2	3	4	5
Distance	???	???	???	???	???
Parent	???	???	???	???	???

Problem 3. *Cycle killer: qu'est-ce que c'est*

Suppose we are given an undirected, connected, weighted graph $G = (V, E, \{w(e)\})$, with non-negative edge weights $w(e) \geq 0$. In this problem we will design an algorithm to remove a set of edges $F \subseteq E$ from the graph such that the resulting graph $G' = (V, E \setminus F)$ is acyclic and the total weight of edges in F is as small as possible. Such a set is called a *feedback edge set* and this problem is sometime called the *minimum weight feedback edge set problem.*

(a) Design an algorithm for finding a *maximum spanning tree* of G. **Hint:** Transform the graph from G to G' so that the minimum spanning tree of G' is a maximum spanning tree of G.

  (i) Describe your algorithm in pseudocode.

  Solution:

  (ii) Justify that your algorithm is correct.

  Solution:

  (iii) Analyze your algorithm's running time.

  Solution:

(b) Design an algorithm to compute a minimum weight feedback edge set of G. **Hint:** Show that if F^* is a minimum weight feedback edge set, then $E \setminus F^*$ is a spanning tree.

  (i) Describe your algorithm in pseudocode.

  Solution:

  (ii) Justify that your algorithm is correct.

  Solution:

  (iii) Analyze your algorithm's running time.

  Solution:

Problem 4. *Anti-Kruskal*

In this problem we will see a new algorithm for finding an MST—the*anti-Kruskal* algorithm. Recall that *Kruskal's algorithm* starts wth $T = \emptyset$, considers edges e in *ascending* order of weight, and adds e to T as long as doing so would not create a cycle. The *anti-Kruskal algorithm* starts with $T = E$, considers edges e in *descending* order of weight, and removes e from T as long as doing so would not make T disconnected.

Explain why the anti-Kruskal algorithm outputs an MST. You may assume that all edge-weights are distinct and you may use the cut and cycle properties of MSTs without proof.

Solution: