Lecture 4 - Functions of several variables, partial derivatives - 7/9/2014 — Interphase 2014 Calc 3

15. Functions of several variables

a. There are three main mays to visualize functions of several variables:

- Level sets (aka level curves, level surfaces)
- 3d surfaces drawn in perspective
- Sequences of cross sections

b. The level curves of $f(x, y)$ are curves in the xy-plane along which f has a constant value. For functions of three variables, they are called level surfaces. Generally, they are called level sets.

The level set for the value c of $f(x, y)$ is the set of points (x, y) such that $f(x, y) = c$.

To visualize a function using level sets,

- Draw the level set for the value 0
- Draw the level set for each of several negative values
- Draw the level set for each of several positive values

c. When drawing 3d surface in perspective, consider drawing cross-sections in x and y. Consider drawing the level sets onto the surface.

d. If one variable is time, drawing a sequence of graphs may be a good way to visualize the function.

16. Partial derivatives

a. The partial derivative of $f(x, y)$ with respect to x is the normal (1d) derivative, where y is treated as a constant.

b. Notation: $\frac{\partial f}{\partial x}(x, y) = \partial_x f(x, y) = f_x(x, y)$ is the partial derivative of f with respect to x at (x, y).

c. Higher derivatives are written as $\frac{\partial^2 f}{\partial x^2} = \partial_{xx} f = f_{xx}$ or $\frac{\partial^2 f}{\partial x \partial y} = \partial_{xy} f = f_{yx}$

d. The mixed partials of a smooth function are equal: $\partial_{xy} f = \partial_{yx} f$.

e. Geometrically $\partial_x f(x, y)$ is the slope in the x direction at (x, y) of the surface given by $z = f(x, y)$.

f. For a differentiable function $f(x, y)$ and small values of Δx and Δy, we have the first-order approximation

$$f(x + \Delta x, y + \Delta y) \approx f(x, y) + \partial_x f(x, y)\Delta x + \partial_y f(x, y)\Delta y$$