CS3000: Algorithms & Data
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Lecture 11:

- Midterm review

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Course Contents

- Stable Matching Problem
  - Choosing definitions of Stability
  - Found counter examples of stability
  - Proved that some outcome is or is not stable
  - Gale Shapley Algorithm
    - Proved correctness via Induction
    - Computed the run time
    - Asymptotics

- Asymptotics
  - Way that we evaluate algorithms
  - Only care about behavior for large n (limit as n -> infinity)
  - Theta, Omega, Big-Oh, little omega, little oh
  - All exponential grow faster than all polynomials
  - All polynomials grow faster than logarithms to any power

- Induction
  - Prove mathematical facts
  - Used this to prove correctness of algorithms

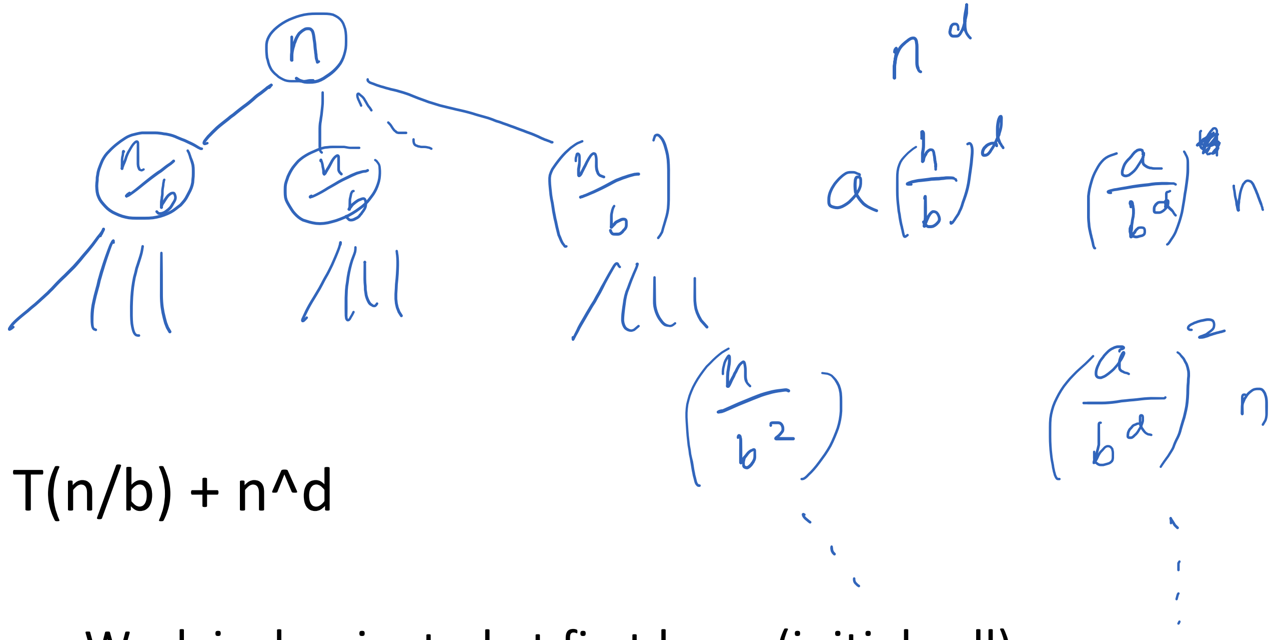
- Divide and Conquer Algorithms
    - Merge Sort
    - Binary Search in a sorted list
    - Max Subarray Sum
    - Karatsuba

- Sorting
  - Insertion Sort Theta(n^2)
  - Can sort in n log(n) time. Via MergeSort
  - Sorting makes searching easier (via binary search)

Karatsuba

$$T(n) = 3 \cdot T(n/2) + cn$$

$$\left(\frac{3}{2^1}\right) = \frac{3}{2} > 1 \quad O(n^{\log_2 3})$$



- Master Theorem
  - Solution to the recurrence T(n) = a T(n/b) + n^d
  - Three cases:
    - (a/b^d) <1. T(n) = Theta(n^d) Work is dominated at first layer (initial call)
    - (a/b^d) = 1 T(n) = Theta(Log(n) * n^d) Work at each layer is equal
    - (a/b^d) > 1 T(n) = Theta(a^(log_b n) = n^log_b(a)). Work is dominated at last layer (base cases)
    - Where In the recursion tree was the work done?
    - Number of layers log_b (n)

- Create Algorithms
- Proved Correctness
- Analyzed Runtimes
- Found examples that cause algorithms to fail
- Write Pseudocode

Which is asymptotically smaller

2^n or 3^n

Can we say $2^n = o(3^n)$? YES

$$\lim_{n \to \infty} \frac{2^n}{3^n} = \lim_{n \to \infty} \left(\frac{2}{3}\right)^n = 0 \quad ✓$$

Is $2^{n/2} = o(2^n)$?

$$\lim_{n \to \infty} \frac{2^{n/2}}{2^n} = \lim 2^{-n/2} = 0$$

2^n vs $3^{n/2}$

$a^b = e^{b \ln a}$

$e^{n \lg 2}$ $e^{\frac{n}{2} \lg 3}$