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Analysis I
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HW 1

Due: 31 January 2017 in class

1. If $X \sim \mathcal{N}(\mu, \sigma^2)$, compute $\mathbb{E}\Big[|X-\mu|\Big]$.

2. Let $X_i \sim \mathcal{N}(\mu, \sigma^2)$ for $i = 1 \dots n$. Suppose you estimate μ and σ by

$$\hat{\mu} = \frac{1}{n}\sum_{i=1}^{n} X_i, \quad \hat{\sigma}^2 = \frac{1}{n}\sum_{i=1}^{n}(X_i - \hat{\mu})^2. \tag{1}$$

   Compute $\mathbb{E}[\hat{\sigma}^2]$ and show that it does not equal σ^2. Modify the formula for $\hat{\sigma}$ so that its expected value is σ^2.

3. Let $X \sim$ Exponential(λ). Compute the mean μ and standard deviation σ of X. Find $\mathbb{P}(|X-\mu| \geq k\sigma)$. How does this compare to the bound you get from Chebyshev's Inequality.

4. Let $X_i \sim$ Bernoulli(p) for $i = 1 \dots n$. Find a reasonable bound for $\mathbb{P}(\frac{1}{n}\sum_{i=1}^{n} X_i < p/2)$ for large n and small p.

5. Let $X_i \sim$ Exponential(λ) for $i = 1 \dots n$. Roughly how big will $\max_{i\in[n]} X_i$ be (for arbitrary λ and large n)? Create and prove a probability bound that shows that it is improbable for the maximum of these random variables to be much larger than your answer.

6. Let $X_i \sim$ Uniform$([0, L])$ for $i = 1 \dots n$. Roughly how big will $\min_{i\in[n]} X_i$ be (for arbitrary $L > 0$ and large n)? Create and prove a probability bound that shows that it is improbable for the minimum of these random variables to be much smaller than your answer.

7. Let $X, Y \sim \mathcal{N}(0, 1)$ be independent. Show that XY follows the same distribution as the difference of two independent χ_1^2 random variables. Hint: Expand out $(X + Y)^2 - (X - Y)^2$.