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Analysis I
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Week 8 — Summary — Compactness

84. The function spaces L^p can be defined as the completion of continuous functions under the L^p norm (for $1 \leq p < \infty$).

85. Definition: A subset S of a normed vector space is (sequentially) compact if every sequence within the set has a subsequence that converges to an element of S.

86. A compact subset of a normed vector space is closed and bounded.

87. A closed subset of a compact set is compact.

88. A subset of $\mathbb{R}^k$ is compact if and only if it is closed and bounded

89. Let S be a compact subset of a normed vector space V, and let f be a continuous function from S to the normed vector space F . Then the image of S under f is compact.

90. A continuous function over a compact set in a normed vector space achieves its maximum and minimum.

91. A continuous function from a compact subset of a normed vector space to a normed vector space is uniformly continuous.

92. Consider a subset S of a normed vector space. S is sequentially compact if and only if any open cover of S has a finite subcover. This provides an alternative definition of compactness.