

19 October 2016  
Analysis I  
Paul E. Hand  
hand@rice.edu

## HW 7

Due: 25 Oct 2016

The problems are written in the format ‘chapter.section.problem-number’ from Lang’s book. Practice problems are not to be handed in. The HW problems will be graded thoroughly and may be revised once, by the Tuesday after they were returned. Please submit each problem on a detached sheet of paper with your name on it.

Practice problems:

1. VI.3.4
2. VI.3.5
3. VI.3.6. Make sure your norms are in fact norms on the space. In particular, make sure they are finite for any element in the vector space.
4. VI.4.2
5. VI.4.4
6. VI.5.2
7. VII.3.3
8. VII.3.7
9. VIII.1.2

Homework problems:

- P16. VI.4.1
- P17. Let  $V = \ell_\infty$ . Let  $S = \{x \mid \|x\|_1 \leq 1\}$ . Is  $S$  open with respect to the  $\ell_\infty$  norm? Is it closed with respect to the  $\ell_\infty$  norm? Prove it.
- P18. Is  $C^1([-1, 1])$  complete with respect to the sup norm? Prove your answer.
- P19. VIII.2.5
- P20. Find a sequence  $\{a_n\}$  in an incomplete normed vector space such that  $\sum_{n=1}^{\infty} \|a_n\|$  converges, yet  $\sum_{n=1}^{\infty} a_n$  does not converge (to an element of the space).