28 November 2015
Analysis I
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Coverpage to Pledged HW 11

Time limit: 3 hours. You may not use your books, your homeworks, your notes, or any electronics during the exam. Please write the start and finish times on your paper. Each subproblem is worth 10 points. To receive full credit, you must name all major theorems and state definitions used in your arguments. All problems must be accompanied by a proof. You may cite results from class and well-known theorems.

This homework is pledged. On the first page, please write your signature and the Rice University pledge: "On my honor, I have neither given nor received any unauthorized aid on this homework."

Due: Tuesday, 1 December 2015 at the beginning of class.

[The exam is on the next page]

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1. Show that $C^2([0,1])$ is not complete with respect to the C^1 norm.
   Recall that $\|f\|_{C^1} = \|f\|_\infty + \|f'\|_\infty$
2. Consider a bounded function $f : [0,1] \to \mathbb{R}$.
   (a) Can f be written as the sum of a nondecreasing and a nonincreasing function?
   (b) What if $f \in C^1([0,1])$?
3. Let $x_n = \left(\sum_{k=1}^n \frac{1}{k}\right) - \log n$. Prove that x_n converges as $n \to \infty$.
4. Let E be a complete normed vector space and let A be a non-empty set in E. Define $f_A : E \to \mathbb{R}$ by

   $$f_A(x) = \inf\{\|y - x\|, y \in A\}.$$

   This function is called the distance from x to A.
   (a) Prove that f_A is uniformly continuous on E.
   (b) Prove that the closure of A is given by $\overline{A} = \{x \in E, f_A(x) = 0\}$.