

25 September 2014  
Analysis I  
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## Pledged HW 6

Time limit: 3 hours. You may not use your books, your homeworks, your notes, or any electronics during this timed homework. Please write the start and finish times on your paper. Each subproblem is worth 10 points. To receive full credit, you must name all major theorems and state definitions used in your arguments. All counter examples must be accompanied by a proof. You may cite results from class and well-known theorems.

This homework is pledged. On the first page, please write your signature and the Rice University pledge: "On my honor, I have neither given nor received any unauthorized aid on this homework."

Due: Tuesday, 7 October 2014 at the beginning of class.

- Let  $x$  be a sequence of real numbers.
  - If every subsequence of  $x$  has a subsequence that converges, does  $x$  converge? Prove it or find a counterexample.
  - If every subsequence of  $x$  has a subsequence that converges to the number  $L$ , does  $x$  converge? Prove it or find a counterexample.
- Assume that  $f : \mathbb{R} \rightarrow \mathbb{R}$  is such that  $|f(x)| \leq x^2$ . Does  $f'(0)$  exist? If so find it and prove it. If not, find a counter example.
  - Assume that  $f : \mathbb{R} \rightarrow \mathbb{R}$  is such that  $|f(x)| \leq |x|^3$ . Does  $f''(0)$  exist? If so find it and prove it. If not, find a counter example.
- At a horse race of length  $L$ , two horses start at time  $t = 0$ . Both horses finish in an exact tie at time  $t = T$ .
  - Is it necessary that there is an intermediate time  $t \in (0, T)$  at which the horses have equal position? Prove it or find a counter example. Assume that the position of each horse is a  $C^0$  function of time.
  - Is it necessary that there is an intermediate time  $t \in (0, T)$  at which the horses have equal velocity? Prove it or find a counter example. Assume that the position of each horse is a  $C^1$  function of time. Recall that velocity is the first derivative of position with respect to time.
  - Is it necessary that there is an intermediate time  $t \in (0, T)$  at which the horses have equal acceleration? Prove it or find a counter example. Assume that the position of each horse is a  $C^2$  function of time. Recall that acceleration is the second derivative of position with respect to time.
- If  $f(x)$  is Riemann integrable, is  $|f(x)|$  Riemann integrable? Prove it or find a counterexample.
  - If  $|f(x)|$  is Riemann integrable, is  $f(x)$  Riemann integrable? Prove it or find a counterexample.