22 September 2015
Analysis I
Paul E. Hand
hand@rice.edu

Coverpage to Pledged HW 5

Time limit: 3 hours. You may not use your books, your homeworks, your notes, or any electronics during the exam. Please write the start and finish times on your paper. Each subproblem is worth 10 points. To receive full credit, you must name all major theorems and state definitions used in your arguments. All counter examples must be accompanied by a proof. You may cite results from class and well-known theorems.

This homework is pledged. On the first page, please write your signature and the Rice University pledge: “On my honor, I have neither given nor received any unauthorized aid on this homework.”

Due: Tuesday, 29 September 2015 at the beginning of class.

[The exam is on the next page]

23 September 2015
Analysis I
Paul E. Hand
hand@rice.edu

Pledged HW 5

Time limit: 3 hours. You may not use your books, your homeworks, your notes, or any electronics during this timed homework. Please write the start and finish times on your paper. Each subproblem is worth 10 points. To receive full credit, you must name all major theorems and state definitions used in your arguments. All counter examples must be accompanied by a proof. You may cite results from class and well-known theorems.

This homework is pledged. On the first page, please write your signature and the Rice University pledge: "On my honor, I have neither given nor received any unauthorized aid on this homework."

Due: Tuesday, 29 September 2015 at the beginning of class.

1. Let $\{x_n\}$ and $\{y_n\}$ be nonnegative sequences of real numbers indexed by $n \in \mathbb{N}$.
   (a) Prove that $\limsup_{n\to\infty}(x_n y_n) \leq \limsup_{n\to\infty} x_n \limsup_{n\to\infty} y_n$
   (b) Provide an example for which the inequality is strict.
2. Let $f(x) = \begin{cases} x\sin(\log x) & \text{if } x > 0 \\ 0 & \text{if } x = 0 \end{cases}$
   (a) Plot f on $[0, \infty)$.
   (b) Is f uniformly continuous on $[0, \infty)$? Prove your answer.
3. Prove that there does not exist a $f : \mathbb{R} \to \mathbb{R}$ that is differentiable everywhere and such that $f'(x) < 0$ for $x < 0$ and $f'(x) > 0$ for $x \geq 0$.
4. Prove that if $f : [0, 1] \to \mathbb{R}$ is monotonic increasing, then it is Riemann integrable.