27 August 2015
Analysis I
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Day 2— Summary — Cauchy sequences, Bolzano-Weierstrass, limsup and liminf

9. The sequence $\{x_n\}$ is Cauchy if $\forall\, \varepsilon > 0$, there exists N such that $m, n \geq N \Rightarrow |x_m - x_n| < \varepsilon$.
10. $\mathbb{R}$ is complete: If $\{x_n\}$ is a Cauchy sequence of $\mathbb{R}$, then $\{x_n\}$ converges to an element of $\mathbb{R}$.
11. Let $x = \{x_n\}$ be a sequence. A subsequence of x is obtained by keeping (in order) an infinite number of the items x_n and discarding the rest. Two ways to denote a subsequence are $x_{(n)}$ and x_{n_k}.
12. Let $\{x_n\}$ be a sequence. The number x is an accumulation point (or point of accumulation) of the sequence if $\forall \varepsilon$ there are infinitely many n such that $|x_n - x| < \varepsilon$.
13. Bolzano-Weierstrass Theorem: Every bounded sequence of real numbers has a convergent subsequence.
14. (a) $\limsup\{x_n\}$ is defined as supremum of the accumulation points of $\{x_n\}$. An alternative way to think about it is through $\limsup\{x_n\} = \lim_{n->\infty} \sup_{m \geq n} x_m$.
    (b) $\liminf\{x_n\}$ is defined analogously.