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Analysis I
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Day 21 — Summary — Extension of Linear Operators

123. Definition: A linear operator (aka function or map) L from a normed vector space to another normed vector space is bounded if $\|L(x)\| \leq C\|x\|$ for all x. The constant C is an operator bound for L. The smallest such C is the operator norm of L.

124. A linear map from a normed vector space to another normed vector space is continuous if and only if it is bounded (as an operator).

125. Let F be a normed vector space, and let F_0 be a subspace. The closure of F_0 in F is a subspace of F.

126. Let F be a normed vector space, and let F_0 be a subspace. Let $L : F_0 \to E$ be a continuous linear map from F_0 into the complete normed vector space E. Then L has a unique extension to a continuous linear map $\bar{L} : \bar{F_0} \to E$ with the same operator bound.