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Analysis I
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Day 14 — Summary — Dimensionality of vector spaces

81. Definition: A collection of vectors is linearly dependent if there is a nontrivial linear combination that equals the zero vector.

82. Definition: The span of a collection of vectors is the set of all finite linear combinations of those vectors.

83. Definition: A finite collection of vectors in the space V is a basis if the collection is linearly independent and spans the whole space.

84. If a space has a basis of n elements, than any collection of more than n elements is linearly dependent.

85. If a space has a finite basis, then any collection of vectors that spans V contains a basis.

86. If a space has a basis of n elements, then any collection of n linearly independent elements is a basis.

87. Definition: The dimensionality of a space is the cardinality of any basis. If there is no (finite) basis, then the dimensionality is infinite.