8 October 2015
Analysis I
Paul E. Hand
hand@rice.edu

Day 12 — Summary — Complete Normed Vector Spaces

63. Definition: A sequence x_n in a normed vector space is Cauchy if

$$\forall \varepsilon \; \exists N \text{ such that } n, m \geq N \Rightarrow \|x_n - x_m\| < \varepsilon.$$

64. In a normed vector space, we say that x_n converges to x if $\forall \varepsilon \; \exists N$ such that $n \geq N \Rightarrow \|x_n - x\| < \varepsilon$. We write this as $\lim_{n\to\infty} x_n = x$

65. Definition: A vector space is complete if any Cauchy sequence converges to an element in the set.

66. Definition: A Banach space is a complete normed vector space.

67. Definition: $\mathbb{R}^n$ is a Banach space under the ℓ_∞ norm. By equivalence of norms on finite dimensional spaces, it is a Banach space under any norm.