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Analysis I
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HW 8

Due: Oct 28 in class. Justify all of your work.

1. VII.1.5
2. VII.3.3
3. VII.3.7
4. Draw and justify the Venn diagram for the following sets of sequences of functions:
 - $\{\{f_n\}_{n=1}^{\infty} \mid f_n \rightarrow 0 \text{ pointwise}, f_n : \mathbb{R} \rightarrow \mathbb{R}\}$
 - $\{\{f_n\}_{n=1}^{\infty} \mid f_n \rightarrow 0 \text{ uniformly}, f_n : \mathbb{R} \rightarrow \mathbb{R}\}$
 - $\{\{f_n\}_{n=1}^{\infty} \mid f_n \rightarrow 0 \text{ in } L_1, f_n : \mathbb{R} \rightarrow \mathbb{R}\}$
 - $\{\{f_n\}_{n=1}^{\infty} \mid f_n \rightarrow 0 \text{ in } L_2, f_n : \mathbb{R} \rightarrow \mathbb{R}\}$

Provide proofs of containment and counterexamples for noncontainment. For your counterexamples, restrict yourself to f_n that are bounded, piecewise-continuous functions with bounded support.

5. Same as (4) but with $f_n : [0, 1] \rightarrow \mathbb{R}$.
6. Same as (4) but with $f_n : \mathbb{N} \rightarrow \mathbb{R}$. This problem is the same as comparing convergence of a sequence of sequences in the following senses: pointwise, uniform, ℓ_1 , and ℓ_2 .