28 August 2014
Analysis I
Paul E. Hand
hand@rice.edu

Day 2— Summary — Cauchy sequences, Bolzano-Weierstrass, limsup and liminf

1. For real numbers, $|x+y| \leq |x| + |y|$ and $|x-y| \geq |x| - |y|$.
2. (a) The supremum of a set is the least upper bound of the set. It is denoted by $\sup(S)$. If S is unbounded from above, then $\sup(S) = \infty$.
   (b) The infimum of a set is its greatest lower bound. It is denoted by $\inf(S)$. If S is unbounded from below, then $\inf(S) = -\infty$.
3. The sequence $\{x_n\}$ is Cauchy if $\forall\, \varepsilon > 0$, there exists N such that $m, n \geq N \Rightarrow |x_m - x_n| < \varepsilon$.
4. If $\{x_n\}$ is a Cauchy sequence of $\mathbb{R}$, then $\{x_n\}$ converges.
5. Let $x = \{x_n\}$ be a sequence. A subsequence of x is obtained by keeping (in order) an infinite number of the items x_n and discarding the rest. Two ways to denote a subsequence are $x_{(n)}$ and x_{n_k}.
6. Let $\{x_n\}$ be a sequence. The number x is an accumulation point (or point of accumulation) of the sequence if $\forall \varepsilon$ there are infinitely many n such that $|x_n - x| < \varepsilon$.
7. (Bolzano-Weierstrass Theorem) Every bounded sequence of real numbers has a convergent subsequence.
8. (a) $\limsup\{x_n\}$ is defined as supremum of the accumulation points of $\{x_n\}$. A better way to think about it is through $\limsup\{x_n\} = \lim_{n->\infty} \sup_{m \geq n} x_n$.
   (b) $\liminf\{x_n\}$ is defined analogously.