9 February 2012
Computational Science and Engineering I
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Problem Set 1 [Revised]

Due: **23 February 2012** in class

1. (10 points) Show that

$$L = \begin{pmatrix} 1 & 0 & 0 \\ \ell_{21} & 1 & 0 \\ \ell_{31} & 0 & 1 \end{pmatrix} \text{ is the inverse of } S = \begin{pmatrix} 1 & 0 & 0 \\ -\ell_{21} & 1 & 0 \\ -\ell_{31} & 0 & 1 \end{pmatrix}$$

2. (10 points) By trial and error, find examples of 2 by 2 matrices such that
   (a) $LU \neq UL$
   (b) $A^2 = -I$, with only real entries in A
   (c) $B^2 = 0$, with no zeros in B

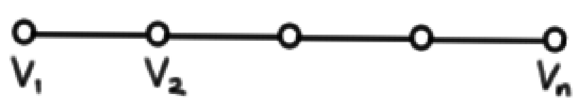
3. (10 points) Factor the matrix $A = LU$, where L is lower triangular, U is upper triangular, and

$$A = \begin{pmatrix} 2 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 2 \end{pmatrix}$$

4. (10 points) Use back substitution twice to solve $LUx = f$, where

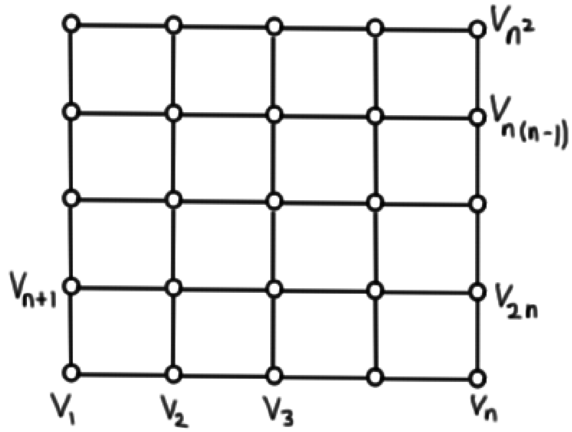
$$L = \begin{pmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 0 & 2 & 1 \end{pmatrix}, U = \begin{pmatrix} 2 & 8 & 0 \\ 0 & 3 & 5 \\ 0 & 0 & 7 \end{pmatrix}, \text{ and } f = \begin{pmatrix} 0 \\ 3 \\ 6 \end{pmatrix}$$

5. (20 points) Consider a line of n nodes, each connected to its neighbors by a resistor of resistance R. At the first node, potential is set to 1. At the nth node, potential is set to 0.



   (a) Write down n equations relating $v_1, v_2, \cdots, v_n$. For $n = 5$, write out by hand the equations in the form $Ax = b$.
   (b) Write a Matlab program that, for arbitrary n, forms A and b and solves for x. At approximately what value of n does it take 1 second to solve the linear system. Make sure you use the sparse data structure for A

6. (20 points) Consider the 2d lattice of points from $(1, 1)$ to (n, n). Each is connected to its neighbors by a resistor of resistance R. At the first node $v_1 = 1$. At the last node, $v_{n^2} = 0$.



(a) In the $n = 3$ case, write out by hand the 9 linear equations in the form $Ax = b$.

(b) Write a Matlab program that, for arbitrary n, forms A and b and solves for x. At approximately what value of n does it take 1 second to solve the linear system. Make sure you use the sparse data structure for A.

(c) Based on the results of 5b and 6b: is a one-dimensional problem involving a million nodes more, less, or equally expensive as a two-dimensional problem involving a million nodes?