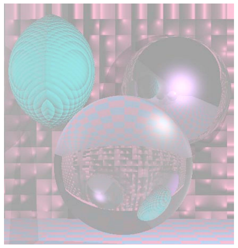
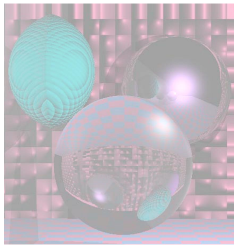


CS5310
Graduate Computer Graphics

Prof. Harriet Fell
Spring 2011
Lecture 8 – March 16, 2011

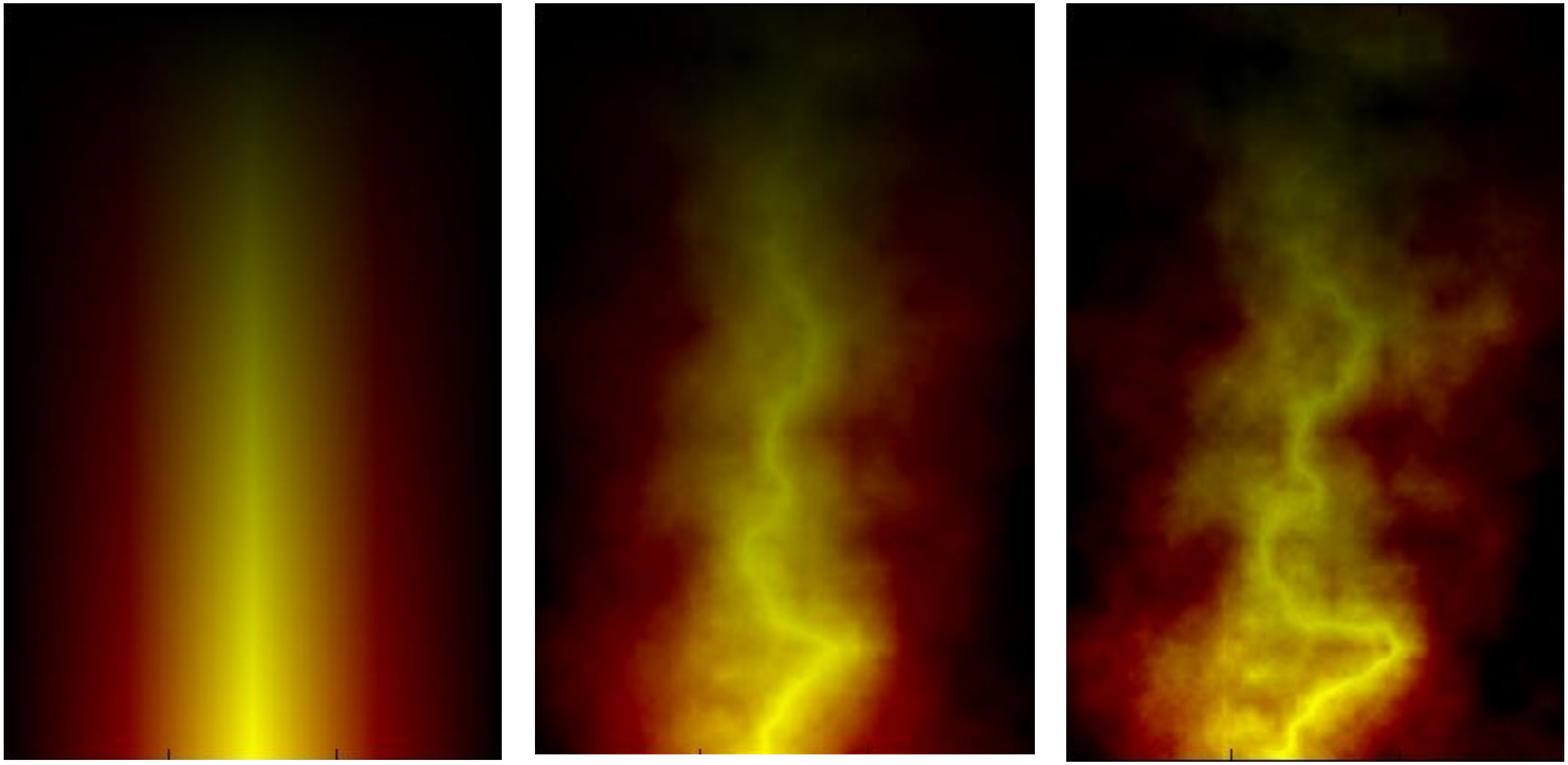


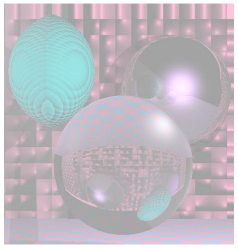
Today’s Topics

- A little more Noise
- -----------------------------
- Gouraud and Phong Shading

- Color Perception mostly ala Shirley *et al.*
    - Light – Radiometry
    - Color Theory
    - Visual Perception

Fire

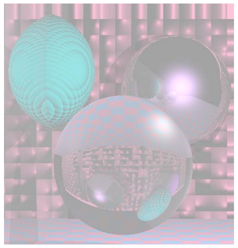




Plane Flame Code

(MATLAB)

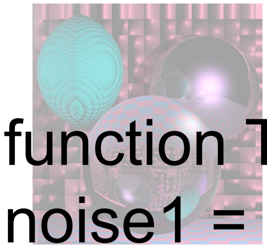
```
w = 300;     h = w + w/2;     x=1:w;     y=1:h;

flameColor = zeros(w,3); % Set a color for each x
flameColor(x,:)=…
            [1-2*abs(w/2-x)/w; max(0,1-4*abs(w/2-x)/w); zeros(1,w)]';

flame=zeros(h,w,3); % Set colors for whole flame
% 1 <= x=j <= 300=h, 1 <= y=451-i <= 450=h+h/2
for i = 1:h
    for j = 1:w
        flame(i,j,:)=(1-(h-i)/h)*flameColor(j,:);
    end
end
```



Turbulent Flame Code

(MATLAB)

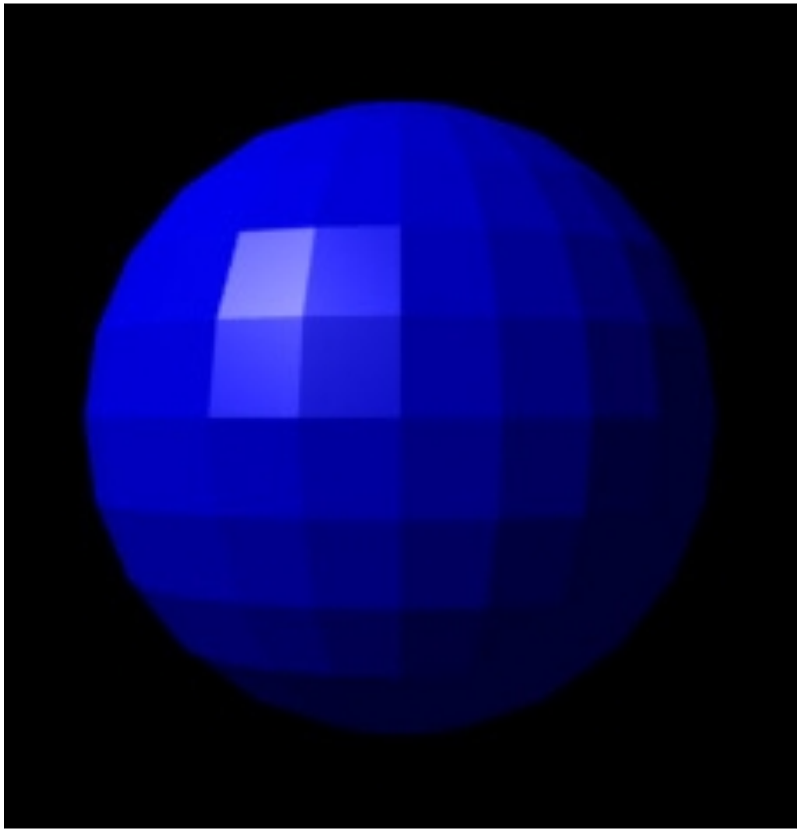
```
for u = 1:450
    for v = 1:300
        x = round(u+80*Tarray(u,v,1)); x = max(x,2); x = min(x,449);
        y = round(v+80*Tarray(u,v,2)); y = max(y,2); y = min(y,299);
        flame2(u,v,:) = flame(x,y,:);
    end
end
```



```
function Tarray = turbulenceArray(m,n)
noise1 = rand(39,39);
noise2 = rand(39,39);
noise3 = rand(39,39);
divisor = 64;
Tarray = zeros(m,n);

for i = 1:m
   for j = 1:n
      Tarray(i,j,1) = LinearTurbulence2(i/divisor, j/divisor, noise1, divisor);
      Tarray(i,j,2) = LinearTurbulence2(i/divisor, j/divisor, noise2, divisor);
      Tarray(i,j,3) = LinearTurbulence2(i/divisor, j/divisor, noise3, divisor);
   end
end
```

Flat Shading

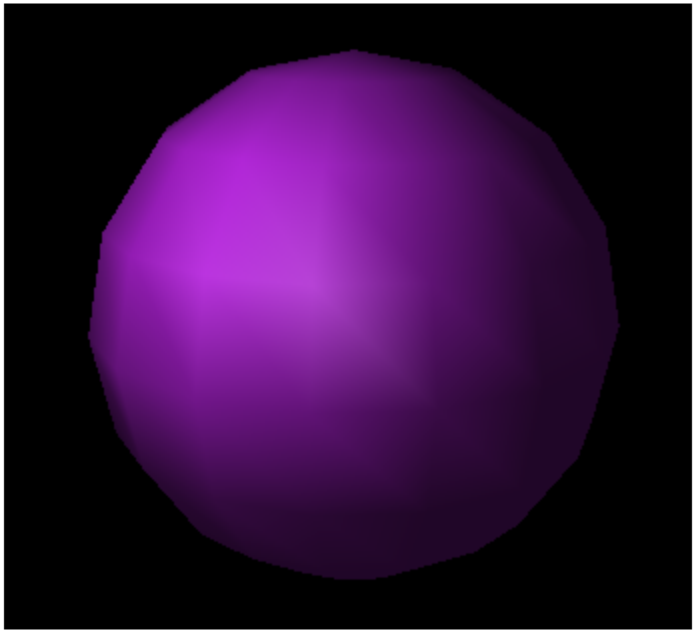
- A single normal vector is used for each polygon.
- The object appears to have facets.

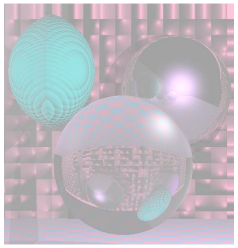


http://en.wikipedia.org/wiki/Phong_shading

Gouraud Shading

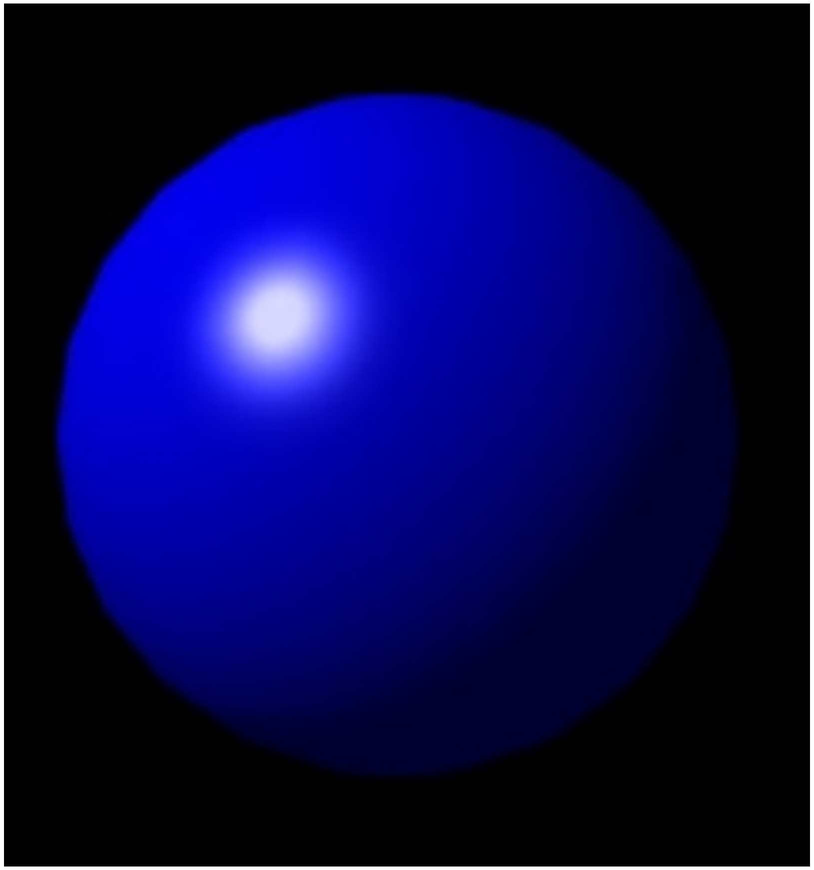
- Average the normals for all the polygons that meet a vertex to calculate its surface normal.
- Compute the color intensities at vertices base on the Lambertian diffuse lighting model.
- Average the color intensities across the faces.



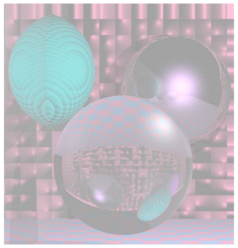


Phong Shading

- Gouraud shading lacks specular highlights except near the vertices.
- Phong shading eliminates these problems.
- Compute vertex normals as in Gouraud shading.
- Interpolate vertex normals to compute normals at each point to be rendered.
- Use these normals to compute the Lambertian diffuse lighting.

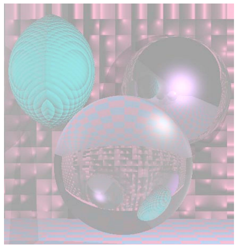


http://en.wikipedia.org/wiki/Phong_shading



Color Systems

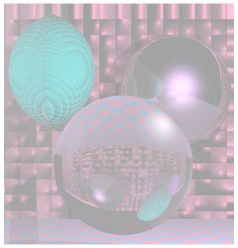
- RGB
- CMYK
- HVS
- YIQ
- CIE = XYZ for standardized color



Light – Radiometry

Things You Can Measure

- Think of light as made up of a large number of photons.
- A *photon* has position, direction, and wavelength λ.
    - λ is usually measured in nanometers
    - 1 nm = 10^{-9} m = 10 angstroms
- A photon has a speed c that depends only on the refractive index of the medium.
- The *frequency* $f = c/\lambda$.
    - The *frequency* does not change with medium.



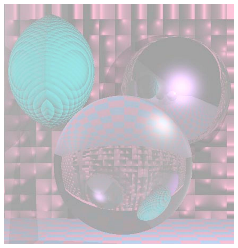
Spectral Energy

- The *energy* q of a photon is given by

$$q = hf = \frac{hc}{\lambda}$$

- h = 6.63*10^{-34} Js, is Planck's Constant.

$$Q_\lambda[\Delta\lambda] = \frac{\displaystyle\sum_{\substack{\text{all photons with}\\ \text{wavelength within}\\ \Delta\lambda/2 \text{ of } \lambda}} q(\text{photon})}{\Delta\lambda}$$

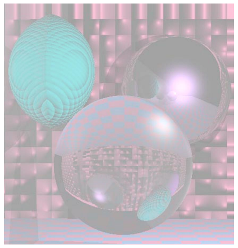


Spectral Energy

$$Q_\lambda = \lim_{\Delta\lambda \to 0} Q_\lambda\left[\Delta\lambda\right]$$

We just use small $\Delta\lambda$ for computation, but not so small that the quantum nature of light interferes.

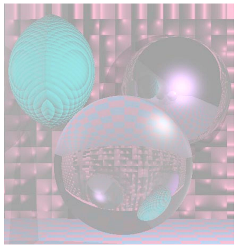
For theory we let $\Delta\lambda \to 0$.



Radiance

- *Radiance* and *spectral radiance* describe the amount of light that passes through or is emitted from a particular area, and falls within a given solid angle in a specified direction.

- Radiance characterizes total emission or reflection, while spectral radiance characterizes the light at a single wavelength or frequency.
  - http://en.wikipedia.org/wiki/Radiance



Radiance Definition

Radiance is defined by

$$L = \frac{d^2\Phi}{dA\,d\Omega\cos\theta} \simeq \frac{\Phi}{\Omega A\cos\theta}$$

where

the approximation holds for small A and Ω,

L is the radiance ($\mathrm{W \cdot m^{-2} \cdot sr^{-1}}$),

Φ is the radiant flux or power (W),

θ is the angle between the surface normal and the specified direction,

A is the area of the source ($\mathrm{m^2}$), and

Ω is the solid angle (sr).

The spectral radiance (radiance per unit wavelength) is written L_λ.

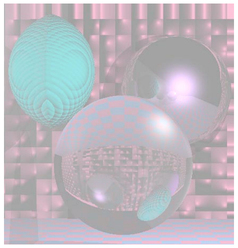
SI radiometry units

edit

Quantity	Symbol	SI unit	Abbr.	Notes
Radiant energy	Q	joule	J	energy
Radiant flux	Φ	watt	W	radiant energy per unit time, also called *radiant power*
Radiant intensity	I	watt per steradian	$W \cdot sr^{-1}$	power per unit solid angle
Radiance	L	watt per steradian per square metre	$W \cdot sr^{-1} \cdot m^{-2}$	power per unit solid angle per unit *projected* source area. Sometimes confusingly called "intensity".

Irradiance	E	watt per square metre	$W \cdot m^{-2}$	power incident on a surface. Sometimes confusingly called "intensity".
Radiant emittance / Radiant exitance	M	watt per square metre	$W \cdot m^{-2}$	power emitted from a surface. Sometimes confusingly called "intensity".
Spectral radiance	L_λ or L_ν	watt per steradian per metre^3 *or* watt per steradian per square metre per Hertz	$W \cdot sr^{-1} \cdot m^{-3}$ *or* $W \cdot sr^{-1} \cdot m^{-2} \cdot Hz^{-1}$	commonly measured in $W \cdot sr^{-1} \cdot m^{-2} \cdot nm^{-1}$
Spectral irradiance	E_λ or E_ν	watt per metre^3 *or* watt per square metre per hertz	$W \cdot m^{-3}$ *or* $W \cdot m^{-2} \cdot Hz^{-1}$	commonly measured in $W \cdot m^{-2} \cdot nm^{-1}$

http://en.wikipedia.org/wiki/Radiance



SI Units

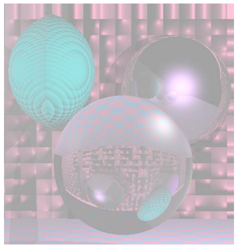
Spectral radiance has SI units

$\mathrm{W \bullet sr^{-1} \bullet m^{-3}}$

when measured per unit wavelength, and

$\mathrm{W \bullet sr^{-1} \bullet m^{-2} Hz^{-1}}$

when measured per unit frequency interval.



Photometry

Usefulness to the Human Observer

Given a spectral radiometric quantity $f_r(\lambda)$ there is a related photometric quantity

$$f_p = 683 \frac{lm}{W} \int_{\lambda=380\text{ nm}}^{800\text{ nm}} \bar{y}(\lambda) f_r(\lambda) d\lambda$$

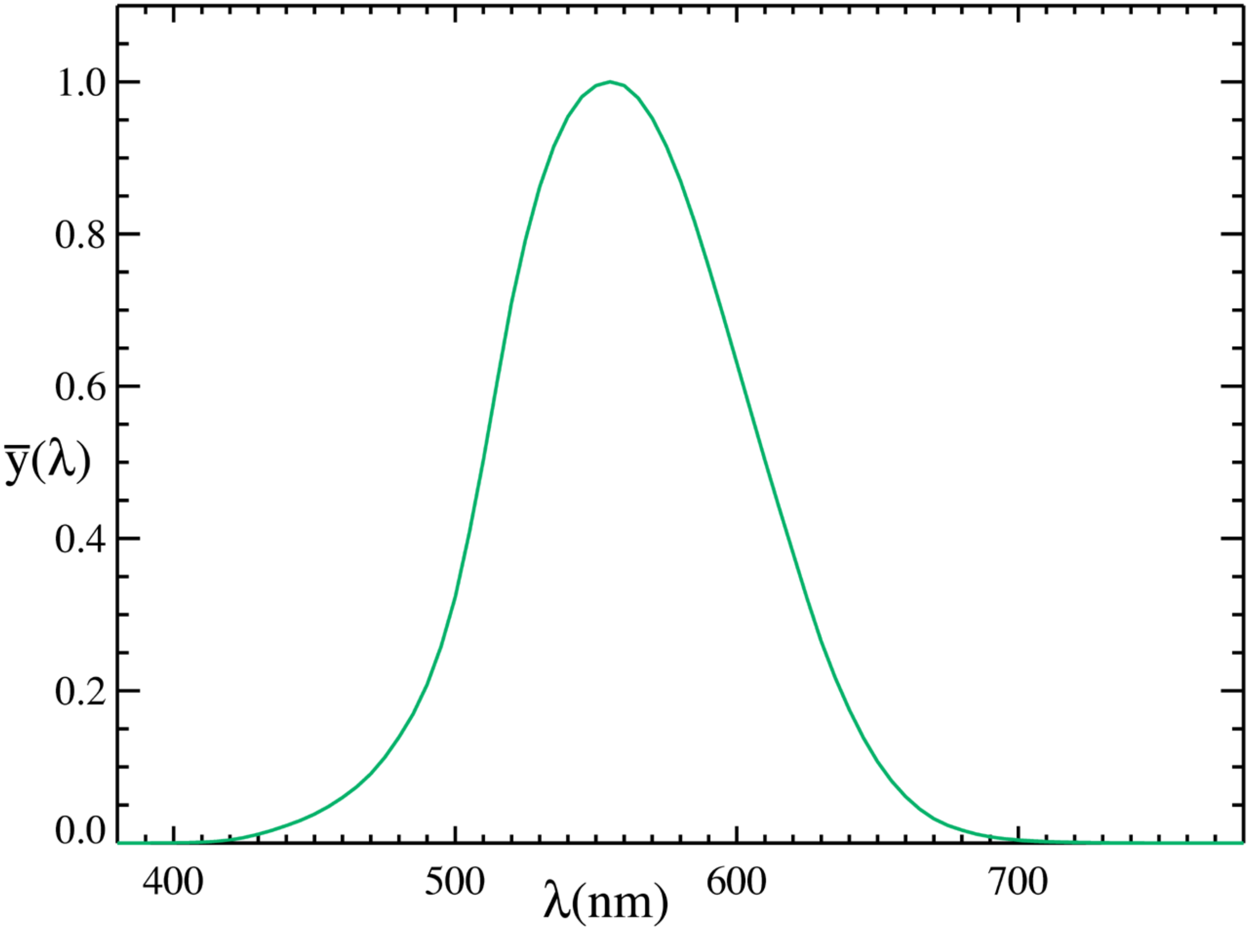
where $\bar{y}$ is the *luminous efficiency function* of the human visual system.

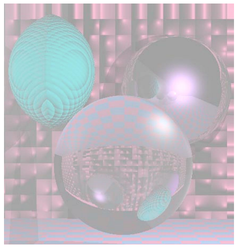
$\bar{y}$ is 0 for λ < 380 nm, the *ultraviolet range.*

$\bar{y}$ then increases as λ increases until λ = 555 nm, pure green.

$\bar{y}$ then decreases, reaching 0 when λ = 800 nm, the boundary with the *infrared region*.

1931 CIE Luminous Efficiency Function





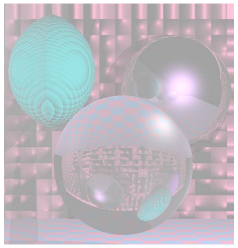
Luminance

$$Y = 683\frac{lm}{W}\int_{\lambda=380\text{ nm}}^{800\text{ nm}} \bar{y}(\lambda)L(\lambda)d\lambda$$

Y is *luminance* when L is spectral radiance.

lm is for *lumens* and W is for *watts*.

Luminance describes the amount of light that passes through or is emitted from a particular area, and falls within a given solid angle.



Color

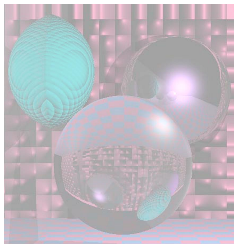
Given a detector, e.g. eye or camera,

$$\text{response} = k \int w(\lambda) L(\lambda) d\lambda$$

The eye has three type of sensors, *cones*, for daytime color vision.

This was verified in the 1800's.

Wyszecki & Stiles, 1992 show how this was done.



Tristimulus Color Theory

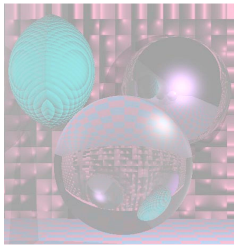
Assume the eye has three independent sensors. Then the response of the sensors to a spectral radiance $A(\lambda)$ is

Blue receptors = Short $S = \int s(\lambda) A(\lambda) d\lambda$

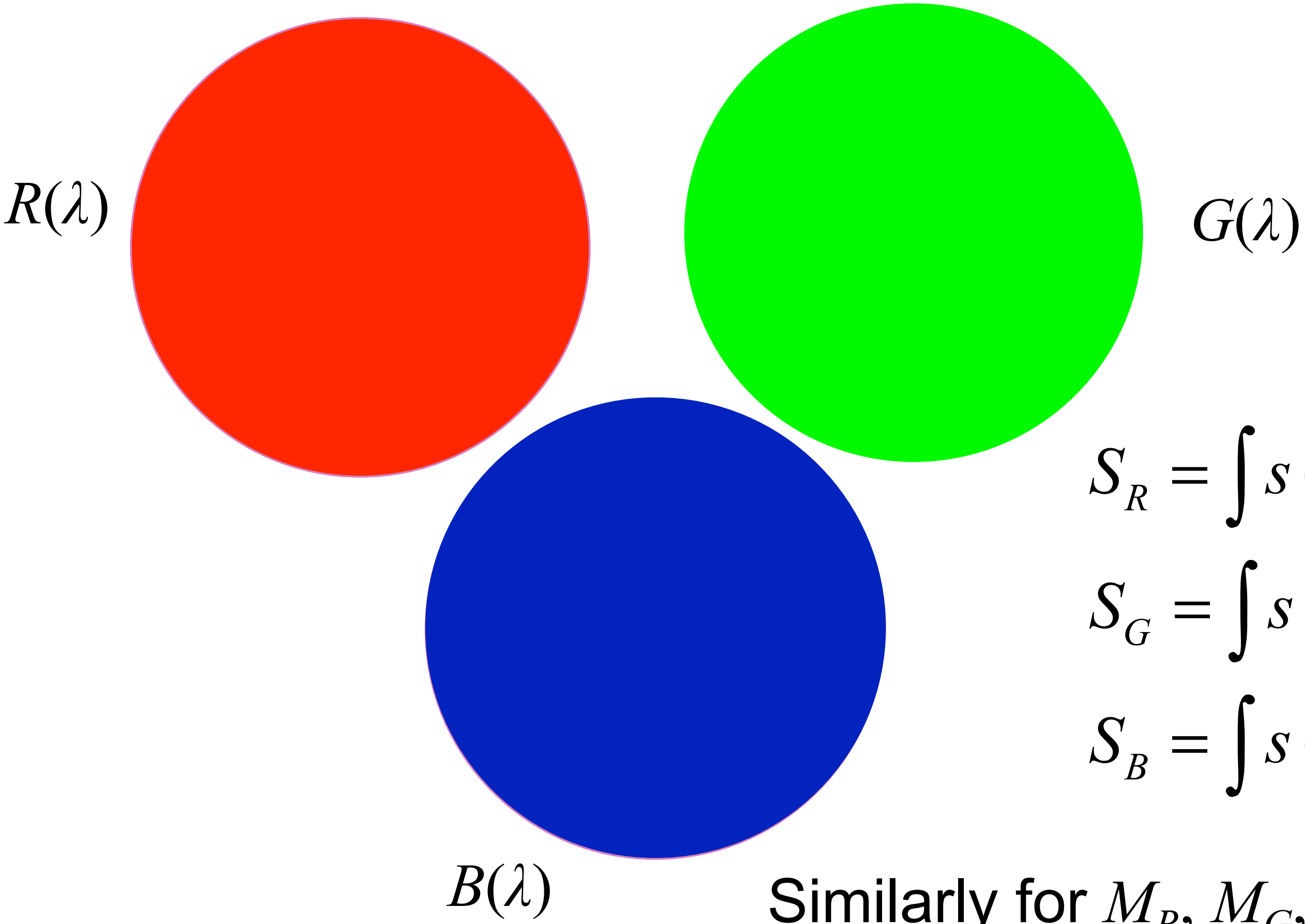
Green receptors = Medium $M = \int m(\lambda) A(\lambda) d\lambda$

Red Receptors = Long $L = \int l(\lambda) A(\lambda) d\lambda$

If two spectral radiances A_1 and A_2 produce the same (S, M, L), they are indistinguishable and called *metamers*.



Three Spotlights



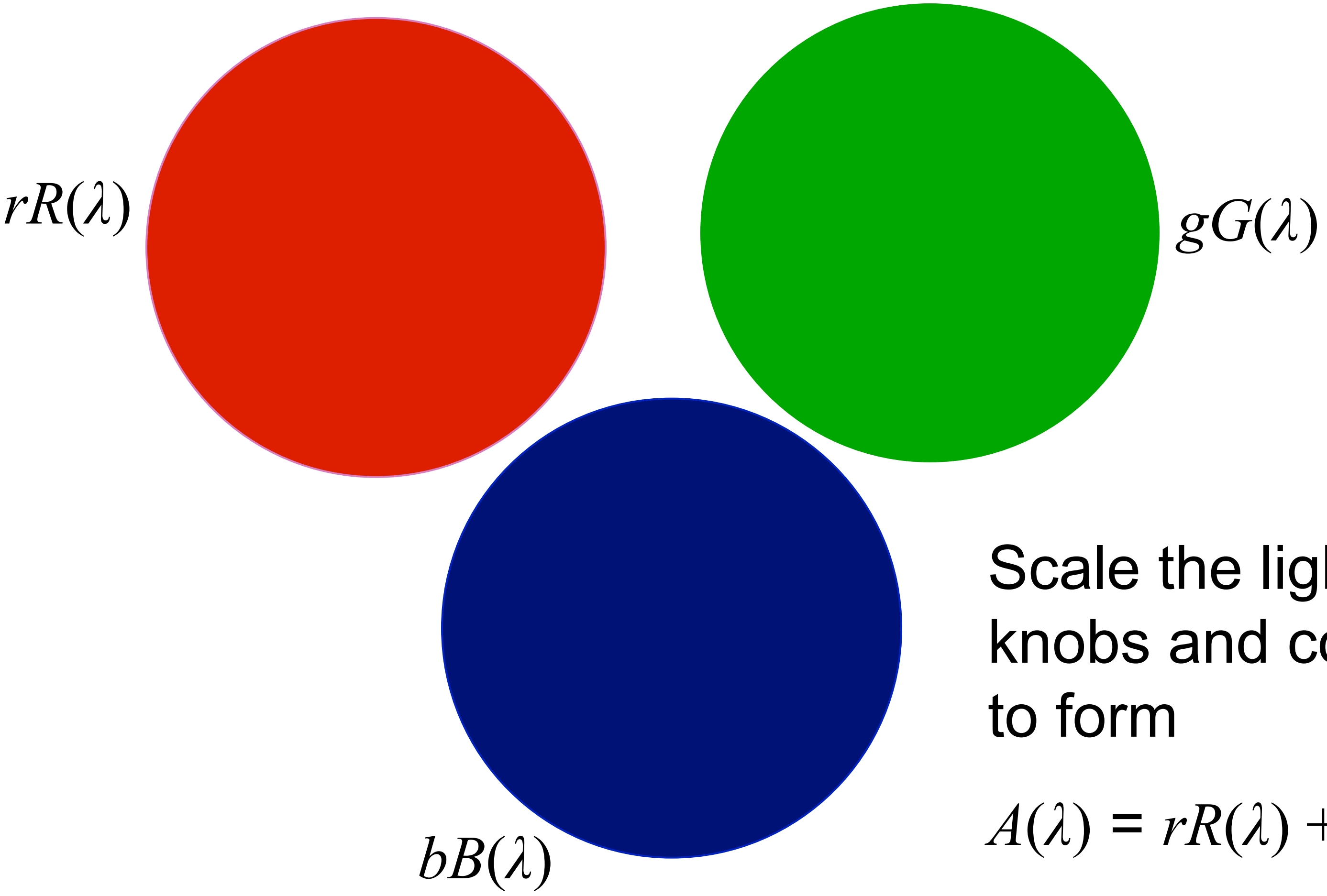
$$S_R = \int s(\lambda) R(\lambda) d\lambda$$

$$S_G = \int s(\lambda) G(\lambda) d\lambda$$

$$S_B = \int s(\lambda) B(\lambda) d\lambda$$

Similarly for M_R, M_G, M_B, L_R, L_G, L_B.

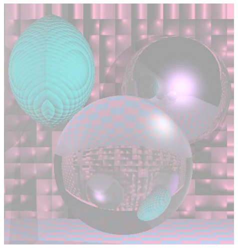
Response to a Mixed Light



Scale the lights with control knobs and combine them to form

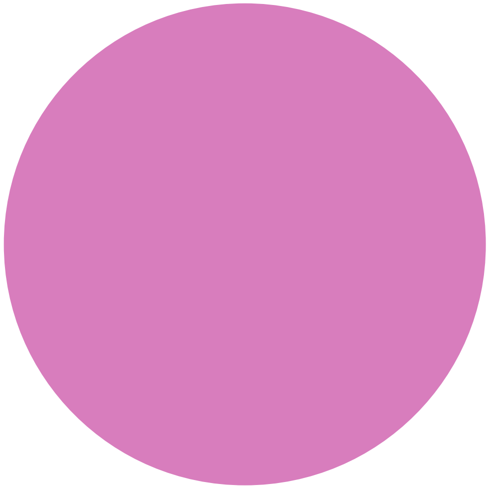
$$A(\lambda) = rR(\lambda) + gG(\lambda) + bB(\lambda)$$

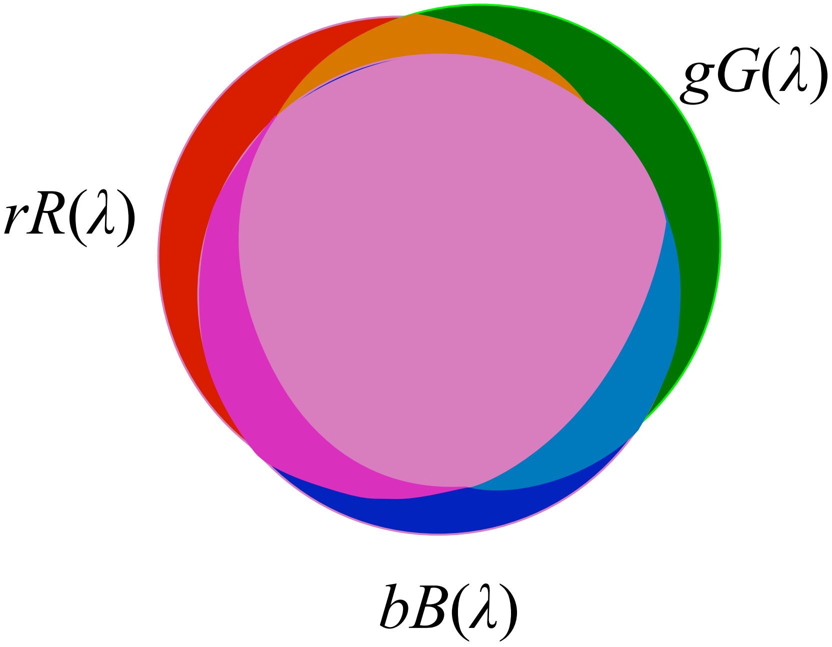
The S response to $A(\lambda)$ is $rS_R + gS_G + bS_B$.



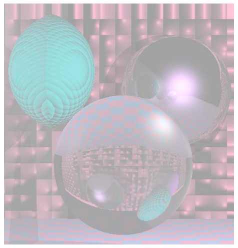
Matching Lights

Given a light with spectral radiance $C(\lambda)$,





a subject uses control knobs to set the fraction of $R(\lambda)$, $G(\lambda)$, and $B(\lambda)$ to match the given color.



Matching Lights

Assume the sensor responses to $C(\lambda)$ are

(S_C, M_C, L_C), then

$$S_C = rS_R + gS_G + bS_B$$

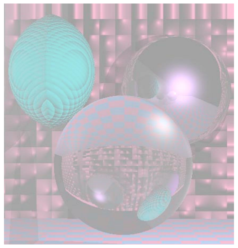
$$M_C = rM_R + gM_G + bS_B$$

$$L_C = rL_R + gL_G + bL_B$$

Users could make the color matches.

So there really are three sensors.

But, there is no guarantee in the equations that r, g, and b are positive or less than 1.



Matching Lights

Not all test lights can be matched with positive r, g, b.

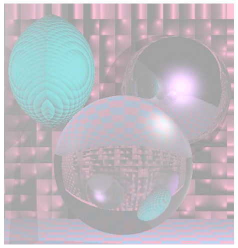
Allow the subject to mix combinations of $R(\lambda)$, $G(\lambda)$, and $B(\lambda)$ with the test color.

If $C(\lambda) + 0.3R(\lambda)$ matches $0 \cdot R(\lambda) + gG(\lambda) + bB(\lambda)$ then $r = -0.3$.

Two different spectra can have the same r, g, b.

Any three independent lights can be used to specify a color.

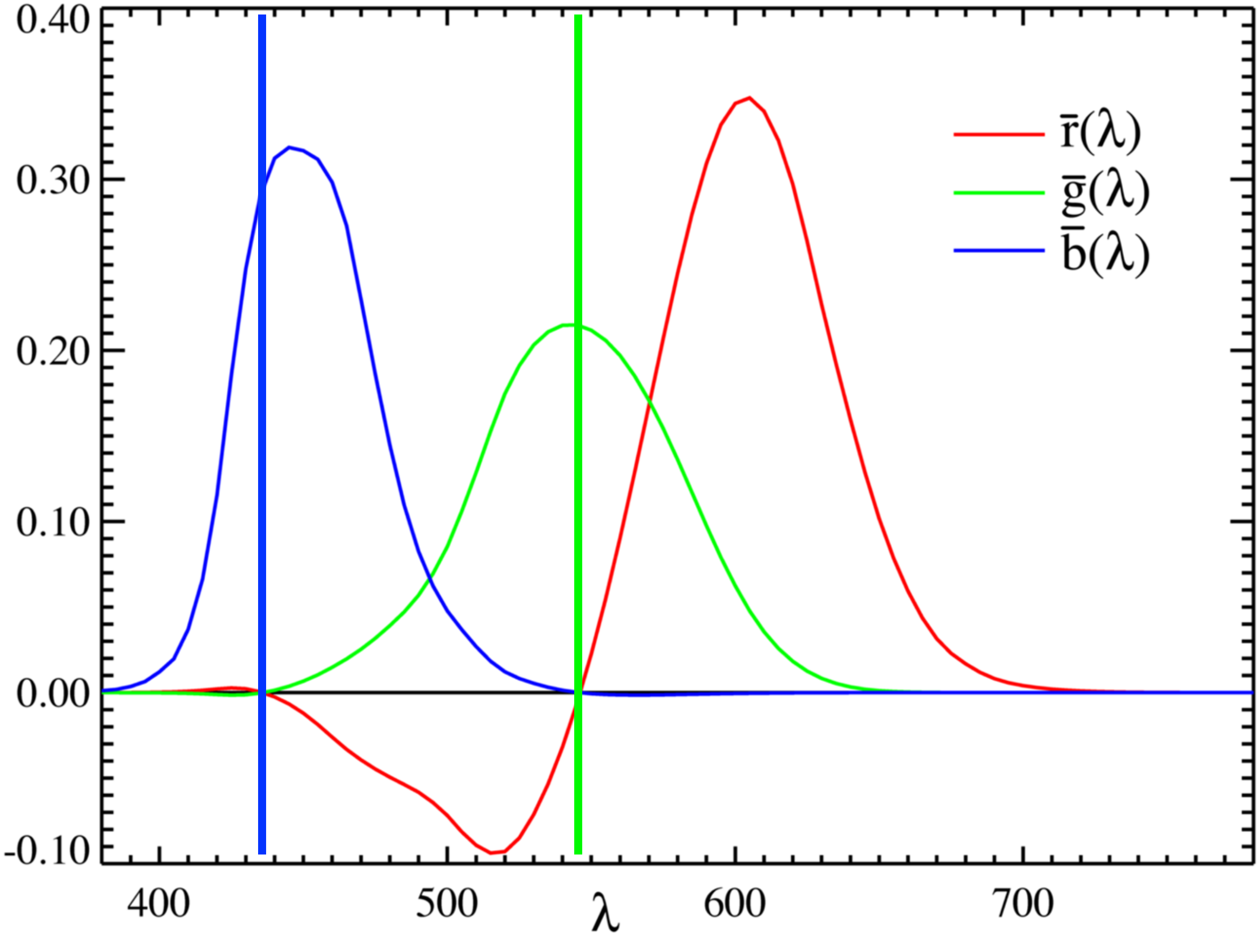
What are the best lights to use for standardizing color matching?



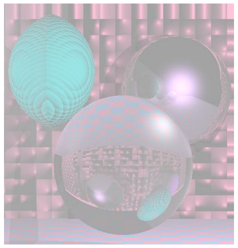
The Monochromatic Primaries

- The three monochromatic primaries are at standardized wavelengths of
  - ▪ 700 nm (red)
    - Hard to reproduce as a monchromatic beam, resulting in small errors.
    - Max of human visual range.
  - ▪ 546.1 nm (green)
  - ▪ 435.8 nm (blue).
    - The last two wavelengths are easily reproducible monochromatic lines of a mercury vapor discharge.
      - – http://en.wikipedia.org/wiki/CIE_1931_color_space

CIE 1931 RGB Color Matching Functions



How much of r, g, b was needed to match each λ.



CIE Tristimulus Values ala Shirley

- The CIE defined the XYZ system in the 1930s.
- The lights are imaginary.
- One of the lights is grey – no hue information.
- The other two lights have zero luminance and provide only hue information, *chromaticity*.

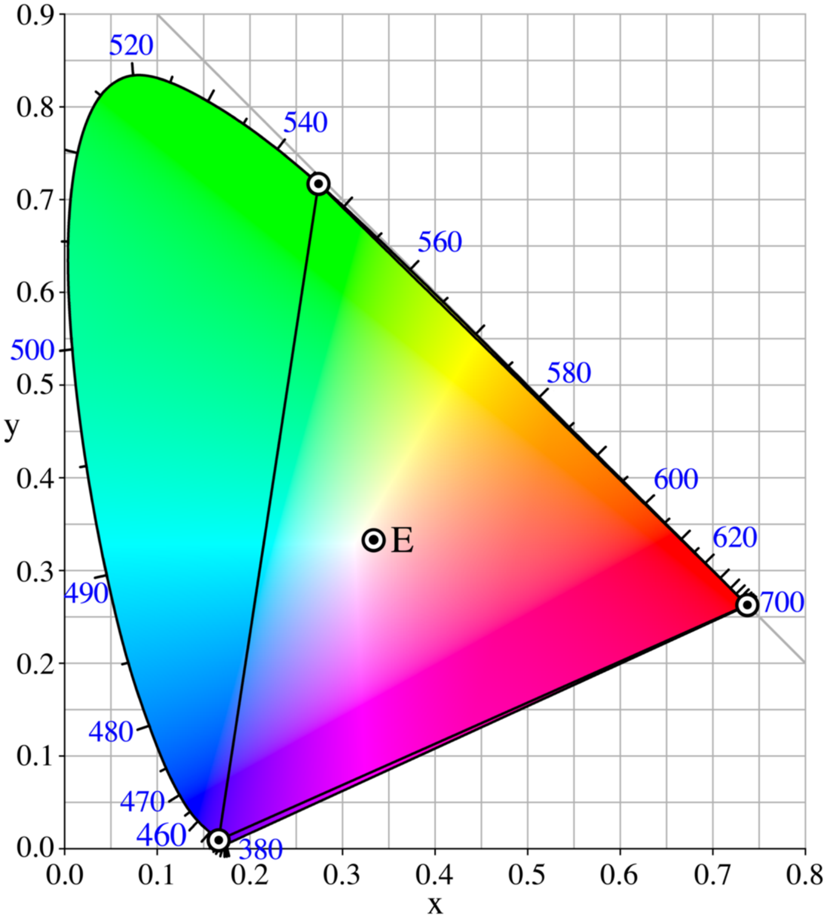
Chromaticity and Luminance

Luminance

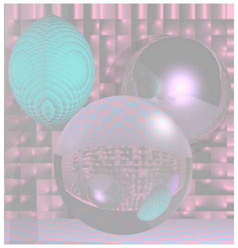
Chromaticity

CIE 1931 xy Chromaticity Diagram

Gamut and Location of the CIE RGB primaries



represents all of the chromaticities visible to the average person



CIE XYZ color space

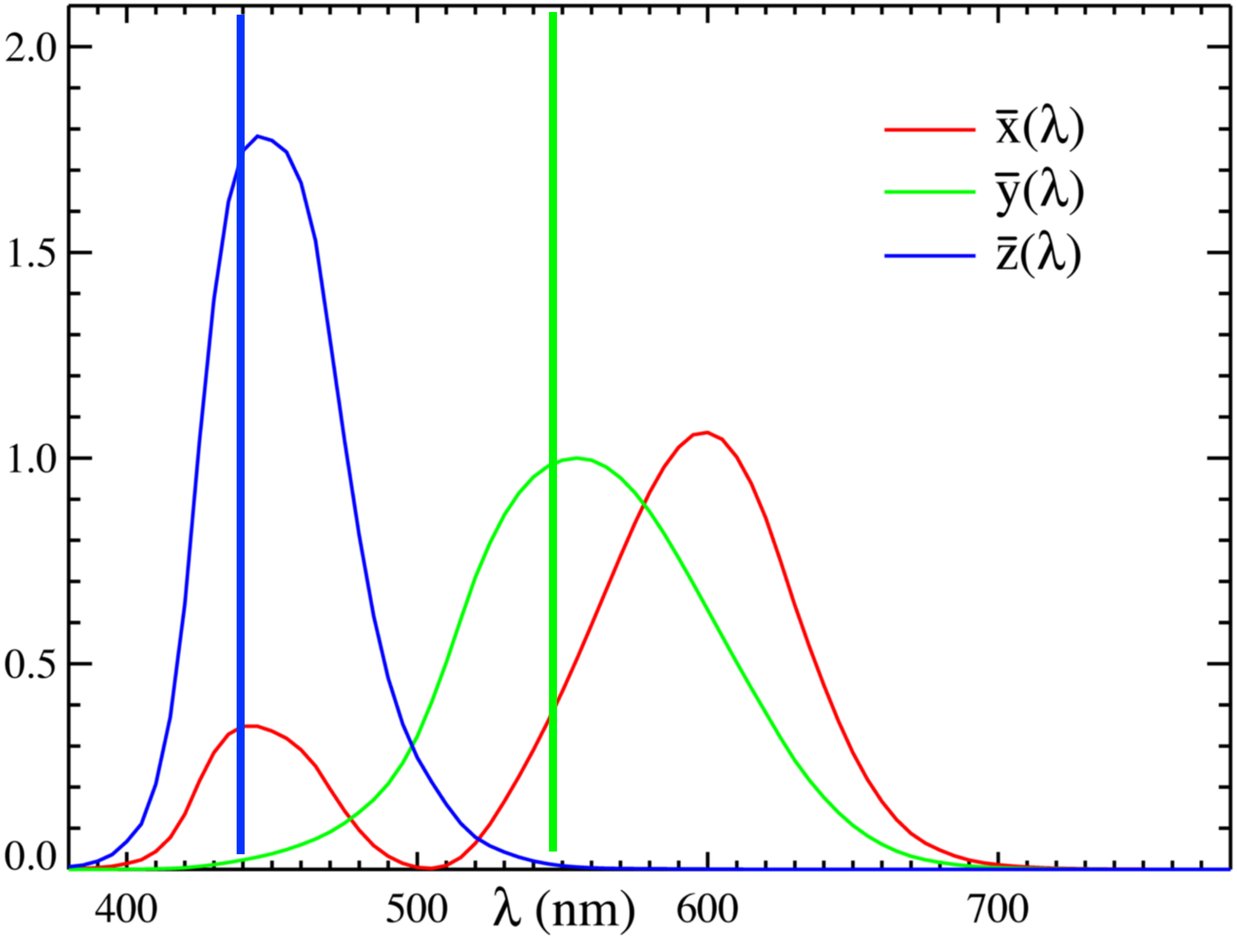
color matching functions $\bar{x}(\lambda), \bar{y}(\lambda), \bar{z}(\lambda)$

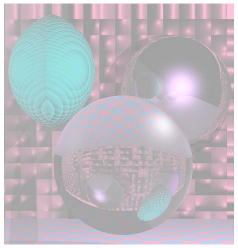
1. Color matching functions were to be everywhere greater than or equal to zero.
2. The $\bar{y}(\lambda)$ color matching function = the photopic luminous efficiency function.
3. x=y=1/3 is the the *white point*.
4. Gamut of all colors is inside the triangle [1,0], [0,0], [0,1].
5. $\bar{z}(\lambda)$ = zero above 650 nm.

http://en.wikipedia.org/wiki/CIE_1931_color_space

CIE 1931 Standard Observer Colorimetric XYZ Functions

between 380 nm and 780 nm





XYZ Tristimulus Values for a Color

with Spectral Distribution *I(λ)*

$$X = \int_0^\infty I(\lambda)\bar{x}(\lambda)$$

$$Y = \int_0^\infty I(\lambda)\bar{y}(\lambda)$$

$$Z = \int_0^\infty I(\lambda)\bar{z}(\lambda)$$

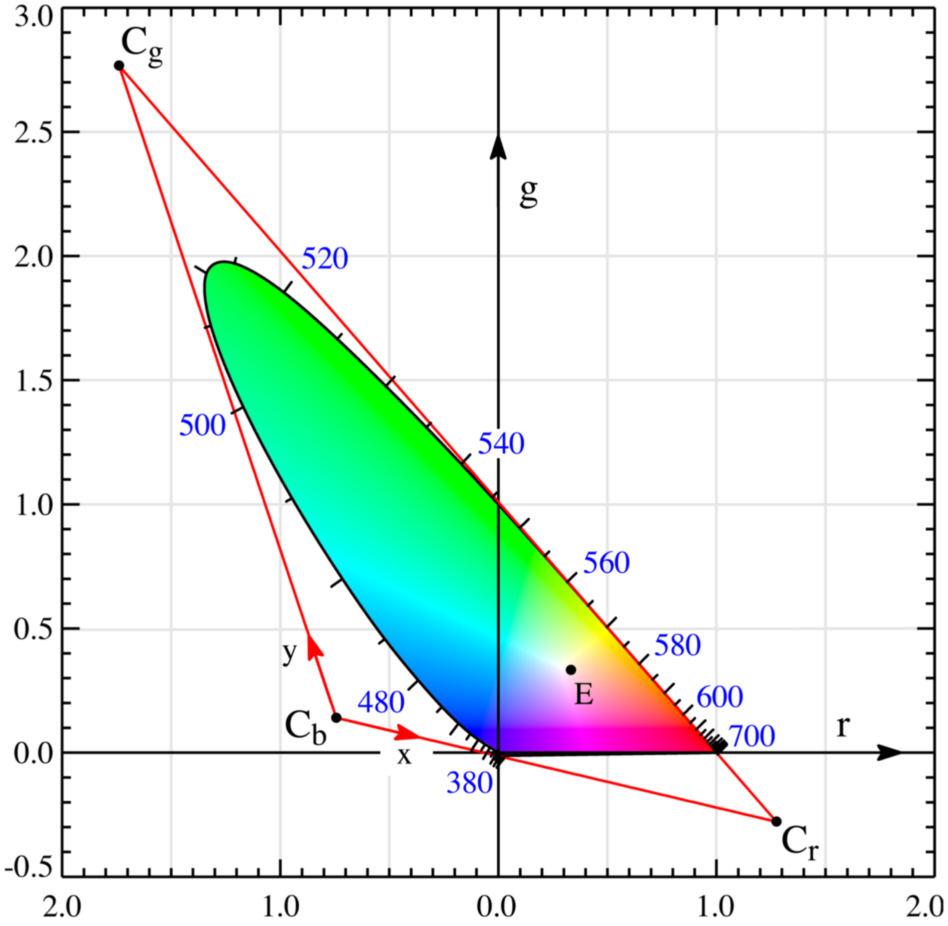
$$\text{Chromaticity} = (x, y) = \left(\frac{X}{X+Y+Z}, \frac{Y}{X+Y+Z}\right)$$

$$\text{Luminance} = Y$$

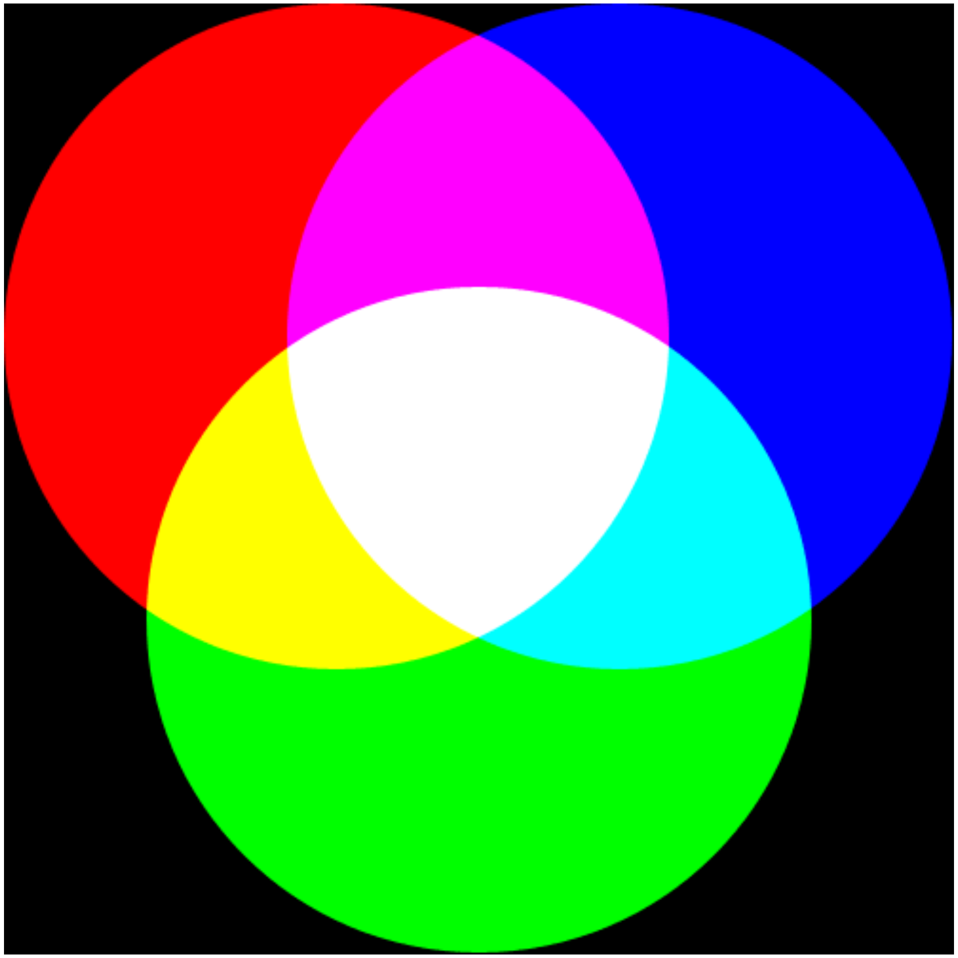
$$(X,Y,Z) = \left(\frac{xY}{y}, Y, \frac{(1-x-y)Y}{y}\right)$$

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \frac{1}{b_{21}} \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix} \begin{bmatrix} R \\ G \\ B \end{bmatrix} = \frac{1}{0.17697} \begin{bmatrix} 0.49 & 0.31 & 0.20 \\ 0.17697 & 0.81240 & 0.01063 \\ 0.00 & 0.01 & 0.99 \end{bmatrix} \begin{bmatrix} R \\ G \\ B \end{bmatrix}$$

CIE XYZ color space

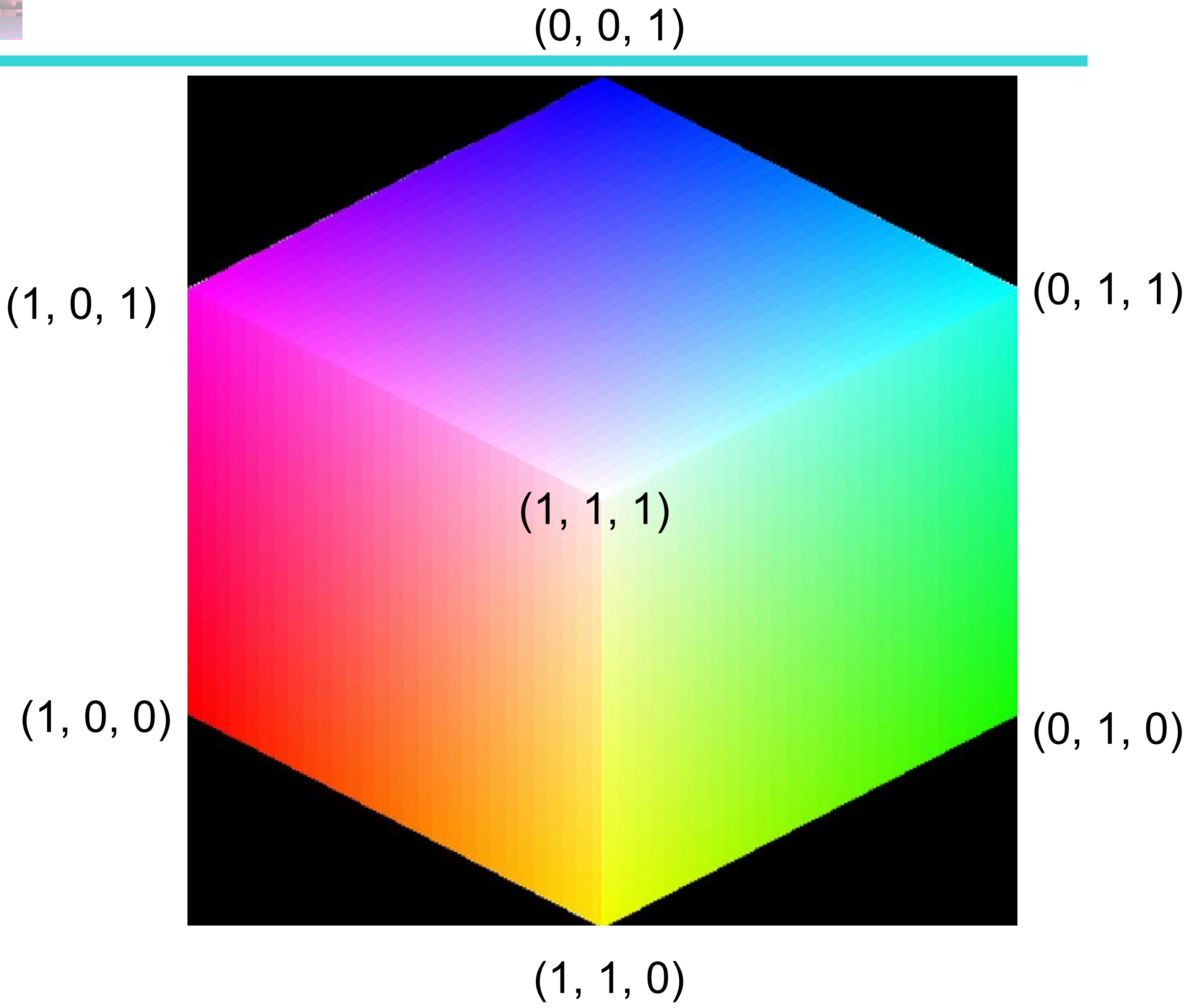


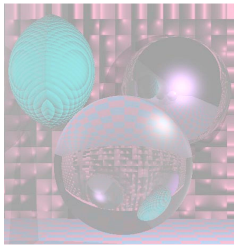
Adding R, G, and B Values



http://en.wikipedia.org/wiki/RGB

RGB Color Cube

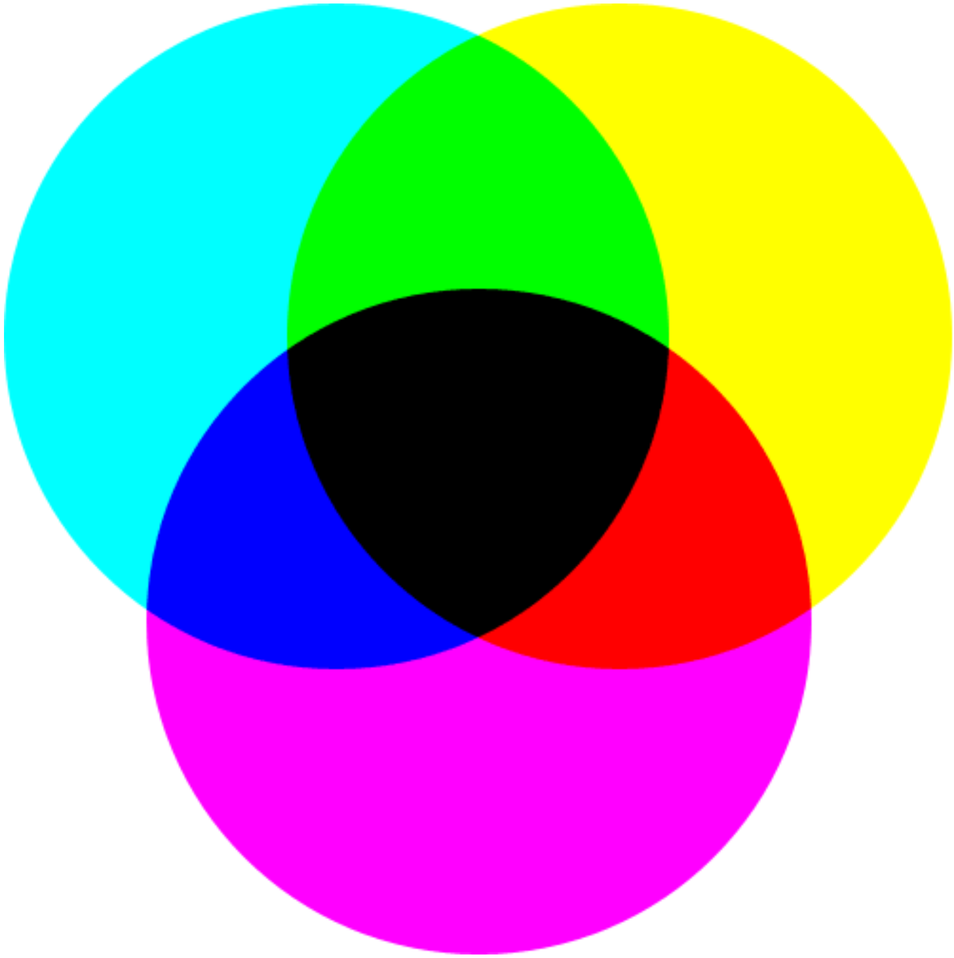


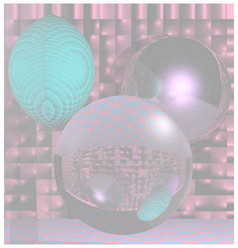


CMY Complements of RGB

- CMYK are commonly used for inks.
- They are called the subtractive colors.
- Yellow ink removes blue light.

Subtractive Color Mixing





CMYK → CMY → RGB

in Theory

$C_{CMYK} = (C, M, Y, \mathbf{K})$

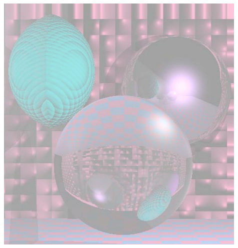
$\Downarrow$

$C_{CMY} = (C \quad , M \quad , Y \quad) = (C(1 - \mathbf{K}) + \mathbf{K}, M(1 - \mathbf{K}) + \mathbf{K}, Y(1 - \mathbf{K}) + \mathbf{K})$

$\Downarrow$

$C_{RGB} = (R, G, B) = (1 - C \quad , 1 - M \quad , 1 - Y \quad)$

$= (1 – (C(1 - \mathbf{K}) + \mathbf{K}), 1 – (M(1 - \mathbf{K}) + \mathbf{K}), 1 – (Y(1 - \mathbf{K}) + \mathbf{K}))$



RGB → CMY → CMYK
in Theory

RGB → CMYK is not unique.

$C_{RGB} = (R, G, B)$

$\downarrow$

$C_{CMY} = (C, M, Y) = (1 - R, 1 - G, 1 - B)$

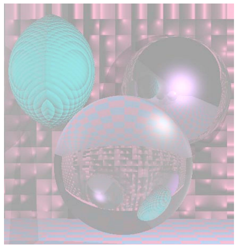
$\downarrow$

if $\min(C, M, Y) == 1$ then $C_{CMYK} = (0, 0, 0, 1)$

else $\mathbf{K} = \min(C, M, Y)$

$C_{CMYK} = ((C - \mathbf{K})/(1 - \mathbf{K}), (M - \mathbf{K}) /(1 - \mathbf{K}), (Y - \mathbf{K})/(1 - \mathbf{K}), \mathbf{K})$

This uses as much black as possible.

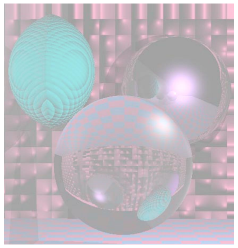


CMYK → CMY → RGB

in Practice

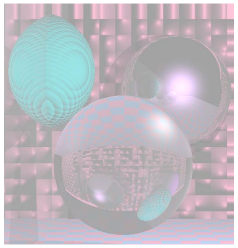
- RGB is commonly used for displays.
- CMYK is commonly used for 4-color printing.
- CMYK or CMY can be used for displays.
  - CMY colours mix more naturally than RGB colors for people who grew up with crayons and paint.
- Printing inks do not have the same range as RGB display colors.

Time for a Break

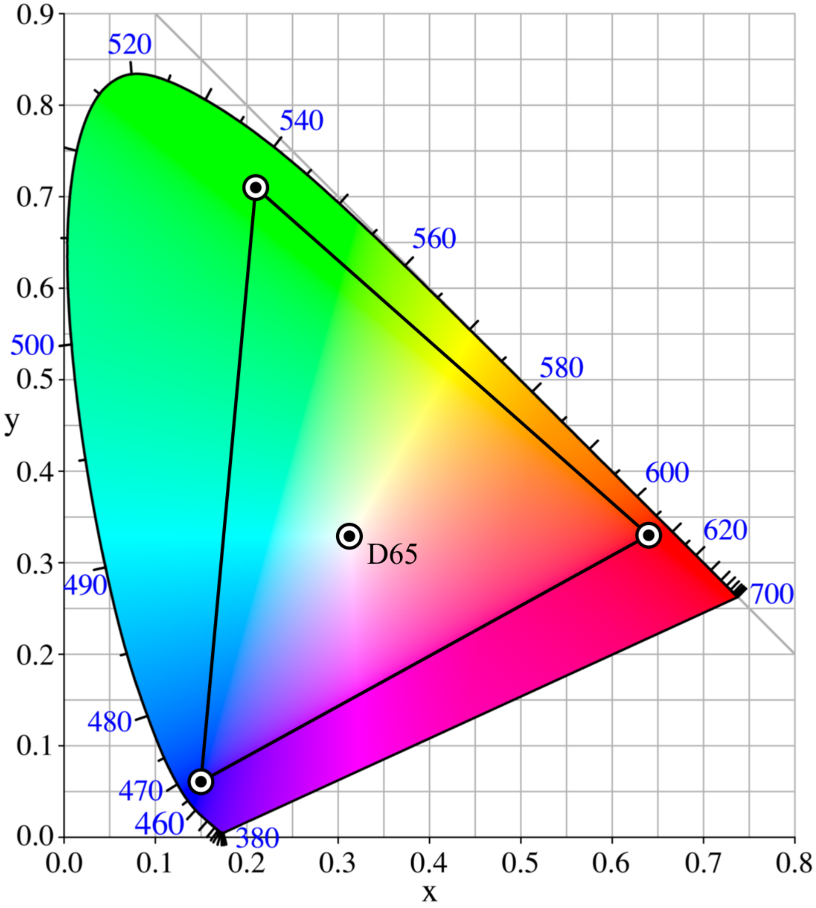


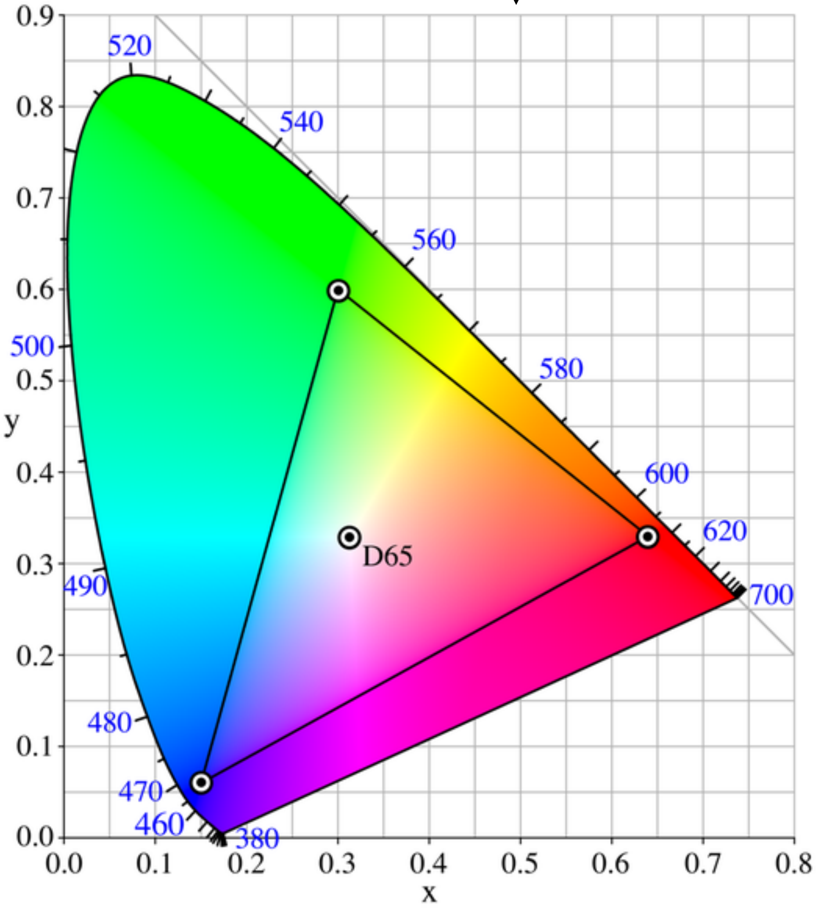
Color Spaces

- RGB and CMYK are color models.
- A mapping between the color model and an *absolute reference color space* results a gamut, defines a new color space.



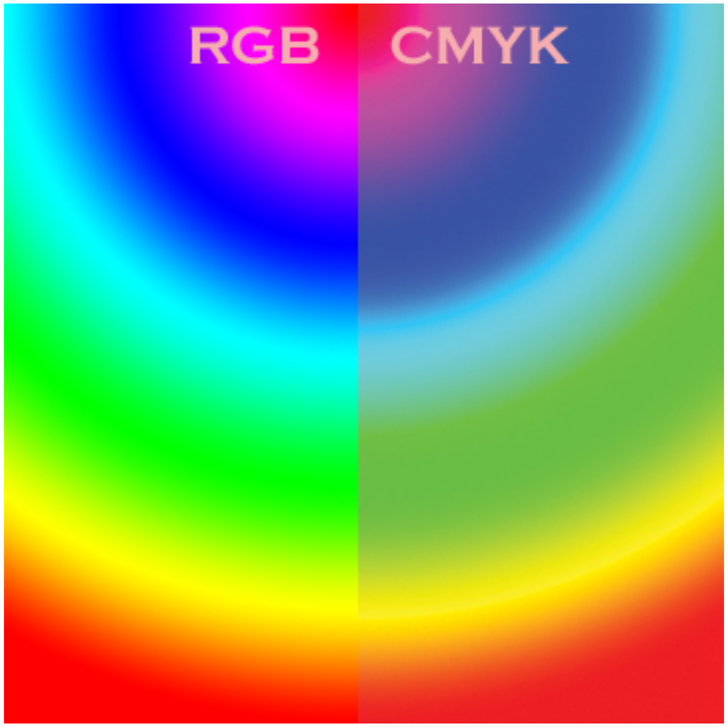
ADOBE RGB and RGBs





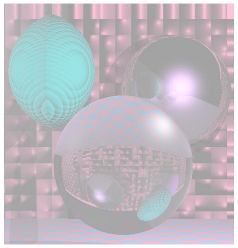
CIE 1931 xy chromaticity diagram showing the gamut of the sRGB color space and location of the primaries.

RGB vs CMYK Space



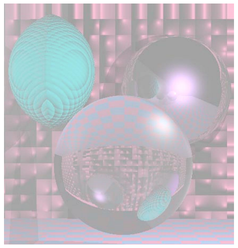
Blue

RGB(0, 0, 255) converted in Photoshop to CMYK becomes CMYK(88, 77, 0, 0) = RGB(57, 83, 164).



Color Spaces for Designers

- Mixing colors in RGB is not natural.
- Mixing colors in CMY is a bit more natural but still not very intuitive.
- How do you make a color paler?
- How do you make a color brighter?
- How do you make this **color**?
- How do you make this **color**?
- HSV (HSB) and HSL (HSI) are systems for designers.

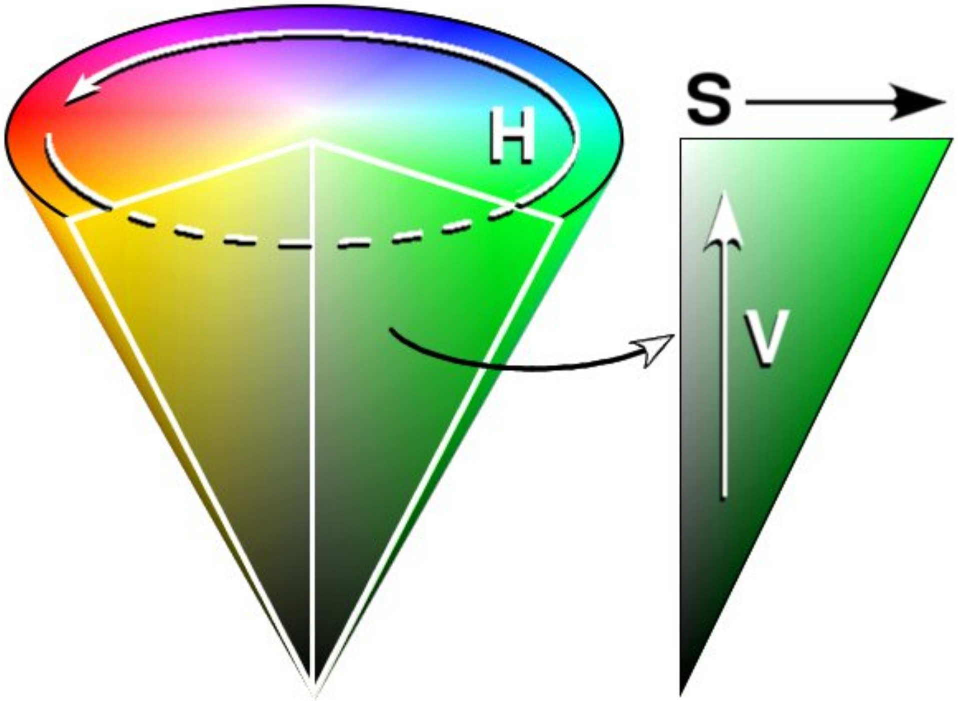


HSV (Hue, Saturation, Value)
HSB (Hue, Saturation, Brightness)

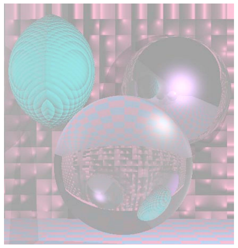
- Hue (e.g. red, blue, or yellow):
  - Ranges from 0-360
- Saturation, the "vibrancy" or “purity” of the color:
  - Ranges from 0-100%
  - The lower the saturation of a color, the more "grayness" is present and the more faded or pale the color will appear.
- Value, the brightness of the color:
  - Ranges from 0-100%

HSV

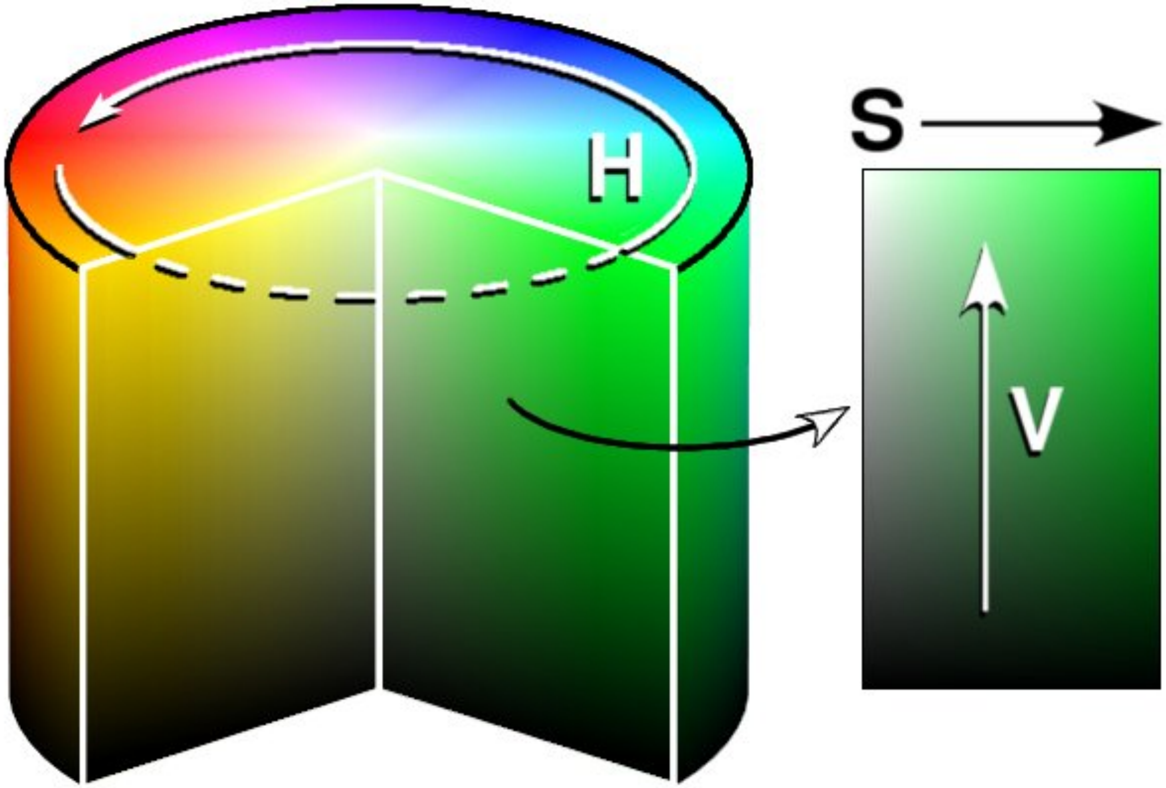
http://en.wikipedia.org/wiki/HSV_color_space



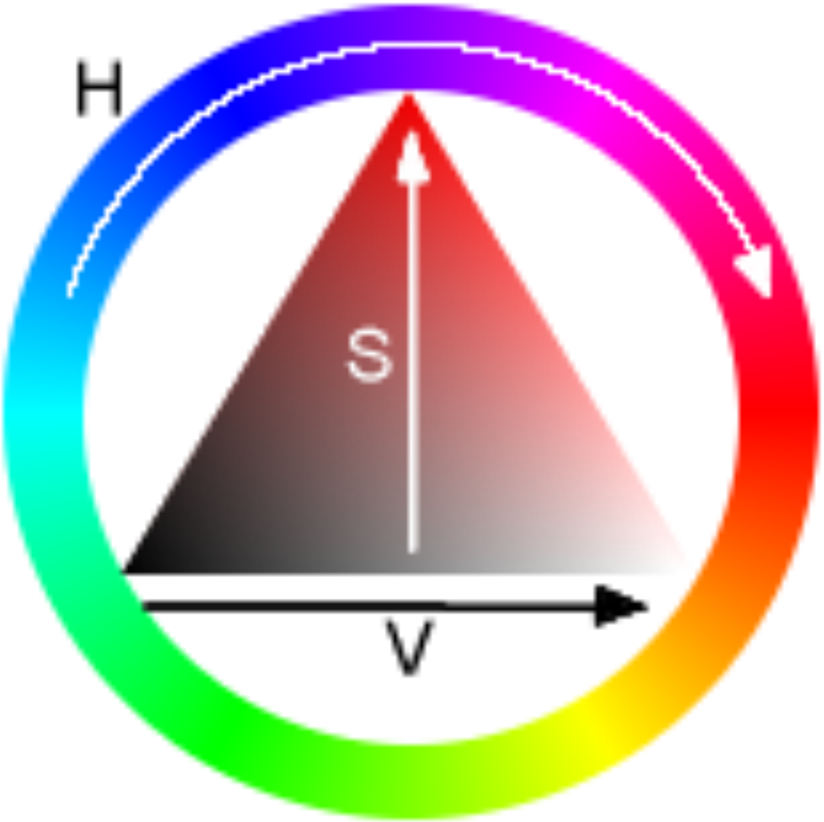
Created in the GIMP by Wapcaplet



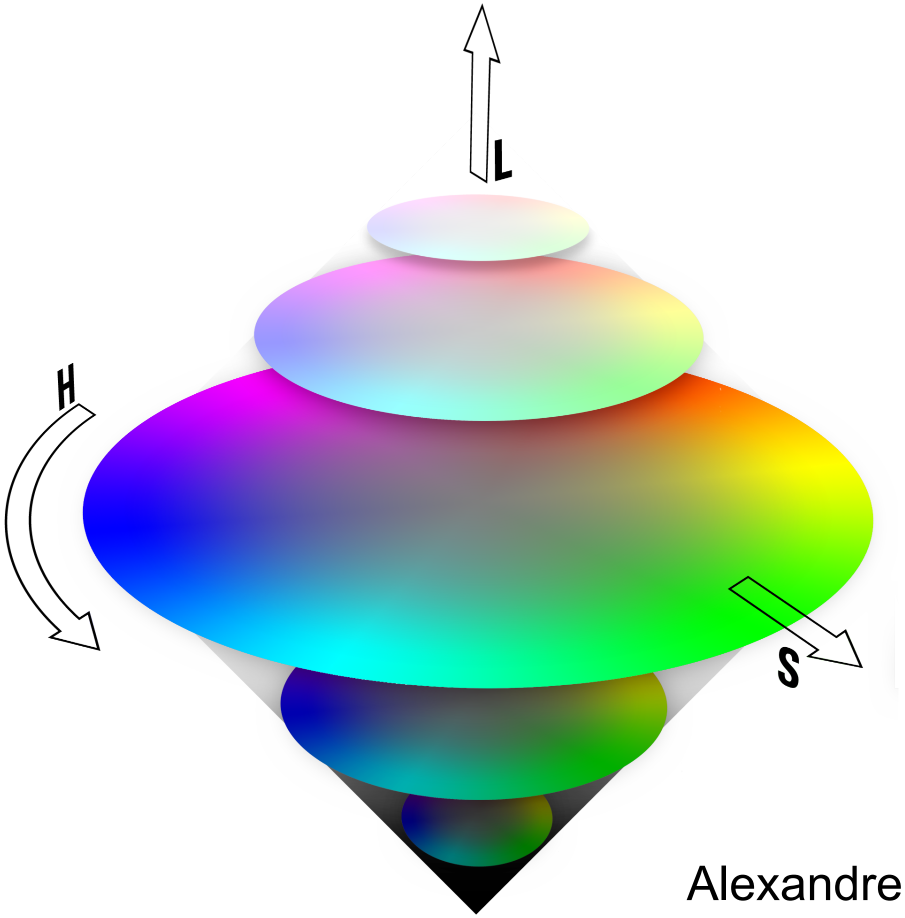
HSV Cylinder



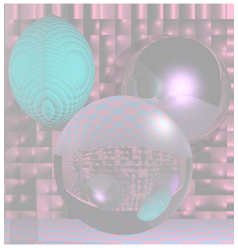
HSV Annulus



HSL



Alexandre Van de Sander



RGB → HSV

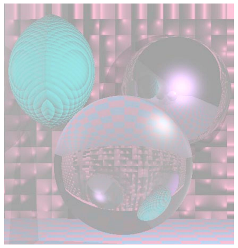
Given (R, G, B) $0.0 \le R, G, B \le 1.0$

$MAX = \max(R, G, B)$ $MIN = \min(R, G, B)$

$$H = \begin{cases} 60 \times \frac{G-B}{MAX-MIN} + 0 & \text{if } MAX = R \text{ and } G \ge B \\ 60 \times \frac{G-B}{MAX-MIN} + 360 & \text{if } MAX = R \text{ and } G < B \\ 60 \times \frac{B-R}{MAX-MIN} + 120 & \text{if } MAX = G \\ 60 \times \frac{R-G}{MAX-MIN} + 240 & \text{if } MAX = B \end{cases}$$

$$S = \frac{MAX - MIN}{MAX}$$

$$V = MAX$$



HSV→RGB

Given color (H, S, V) $0.0 \le H \le 360.0$, $0.0 \le S, V \le 1.0$

if $S == 0.0$ then $R = G = B = V$ and H and S don't matter.

else
$$H_i = \left\lfloor \frac{H}{60} \right\rfloor \bmod 6 \qquad f = \frac{H}{60} - H_i$$

$$p = V(1-S) \qquad q = V(1-fS) \qquad t = V\left(1-(1-f)S\right)$$

$$\text{if } H_i == 0 \rightarrow R = V, G = t, B = p$$

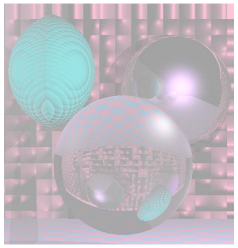
$$\text{if } H_i == 1 \rightarrow R = q, G = V, B = p$$

$$\text{if } H_i == 2 \rightarrow R = p, G = V, B = t$$

$$\text{if } H_i == 3 \rightarrow R = p, G = q, B = V$$

$$\text{if } H_i == 4 \rightarrow R = t, G = p, B = V$$

$$\text{if } H_i == 5 \rightarrow R = V, G = p, B = q$$



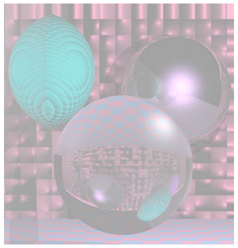
YIQ

NTSC Television YIQ is a linear transformation of RGB.

- exploits characteristics of human visual system
- maximizes use of fixed bandwidth
- provides compatibility with B&W receivers

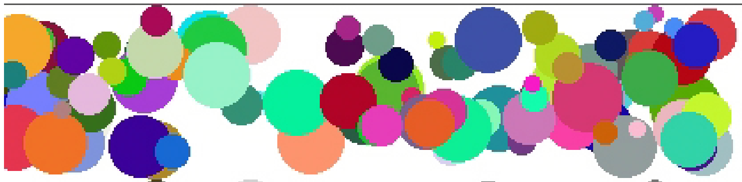
- $Y = 0.299R + 0.587G + 0.114B$ luminance
- $I = 0.74(R - Y) - 0.27(B - Y)$ } chrominance
- $Q = 0.48(R - Y) + 0.41(B - Y)$

- See http://en.wikipedia.org/wiki/YIQ and discussion

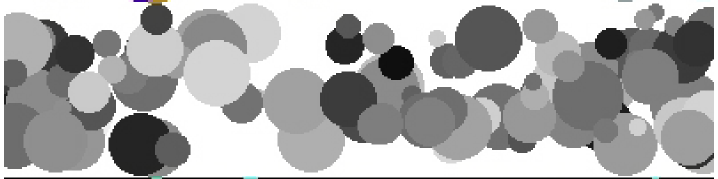


YIQ

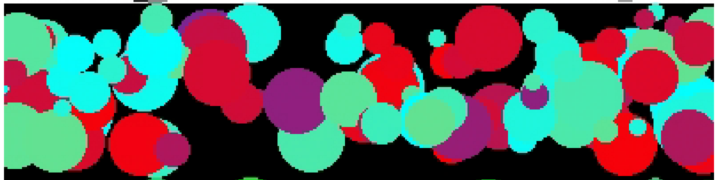
- Y is all that is used for B&W TV
- B-Y and R-Y small for dark and low saturation colors
- Y is transmitted at bandwidth 4.2 MHz
- I at 1.3 MHz
- Q at .7 MHz.



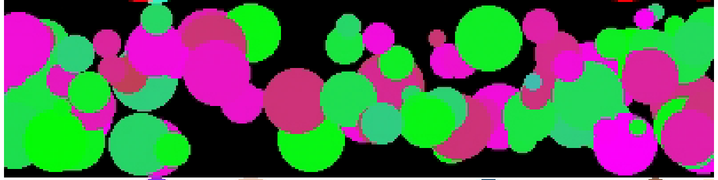
Original



Y = 0.299*R + 0.587*G + 0.114*B
luminance

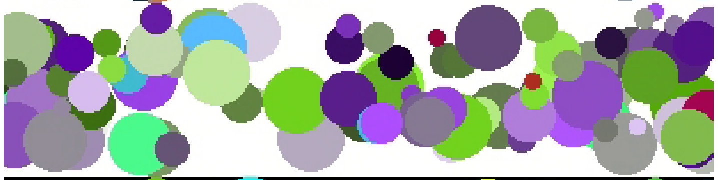


I = 0.596*R - 0.274*G - 0.322*B
orange-cyan - caucasian flesh tones

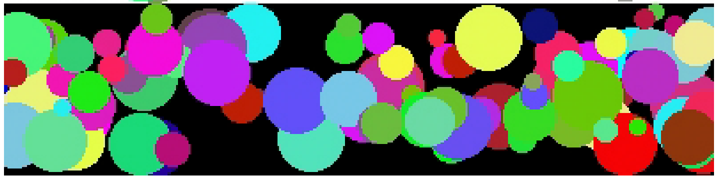


Q = 0.212*R - 0.523*G + 0.311*B
green-magenta

YI



YQ



IQ