Heuristic Search

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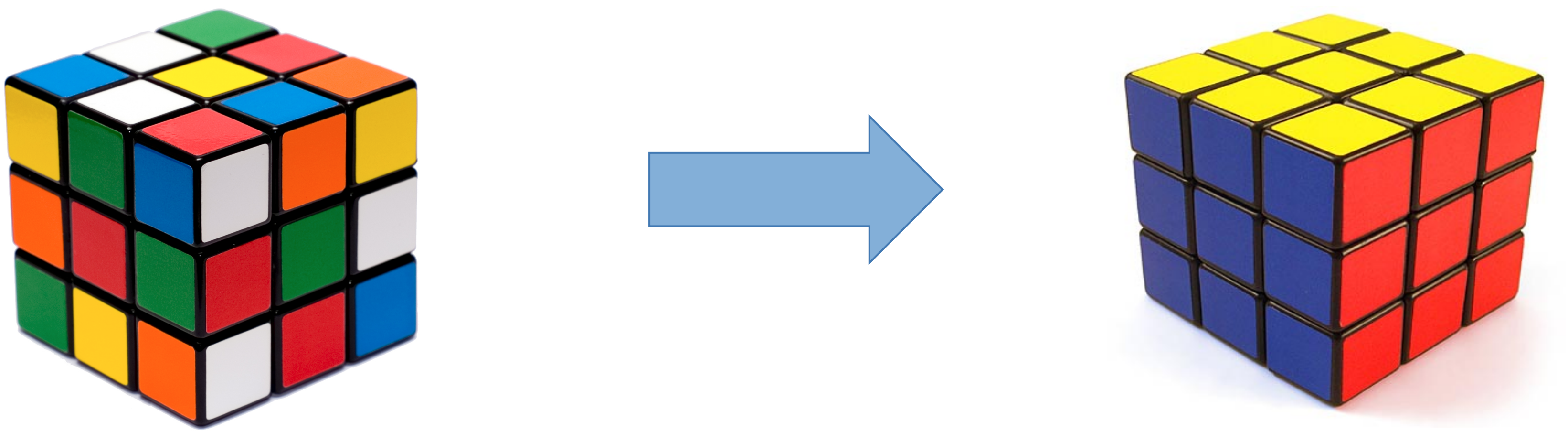
Some images and slides are used from: Rob Platt, CS188 UC Berkeley, AIMA

Recap: What is graph search?



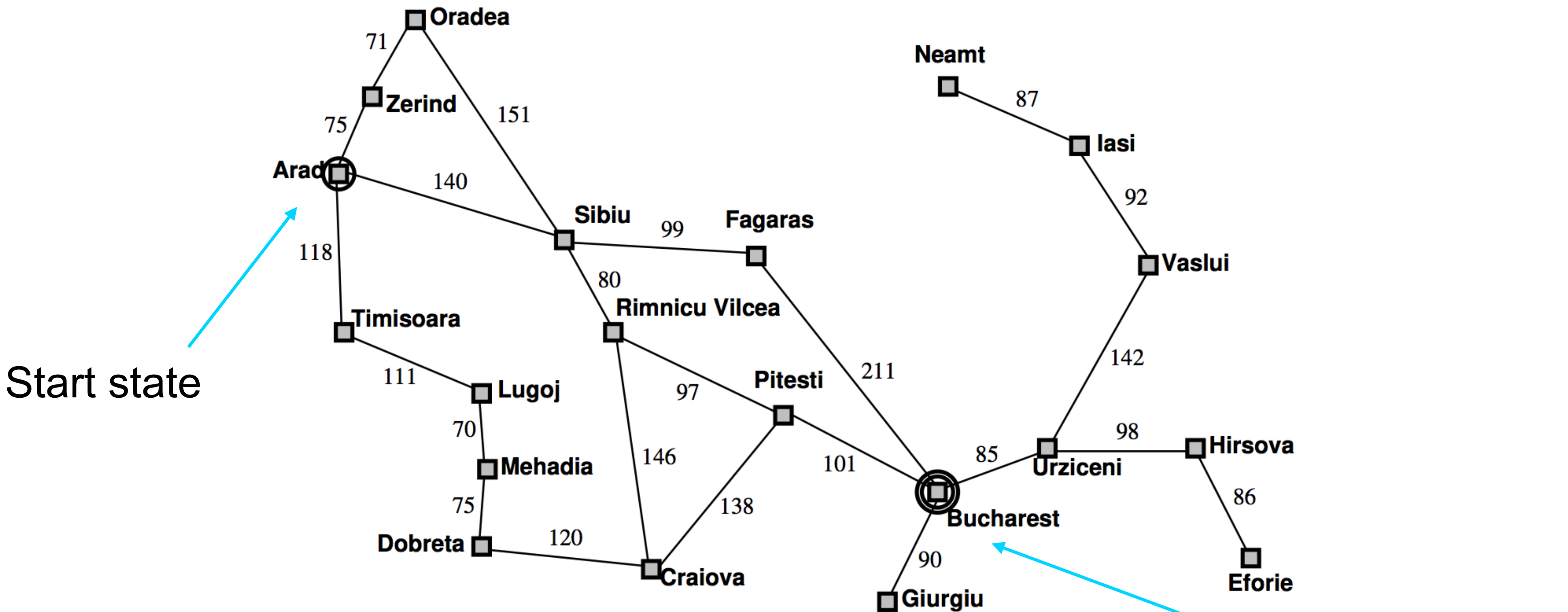
Start state

Goal state

Graph search: find a path from start to goal

- what are the states?
- what are the actions (transitions)?
- how is this a graph?

Recap: What is graph search?



Graph search: find a path from start to goal

– what are the states?

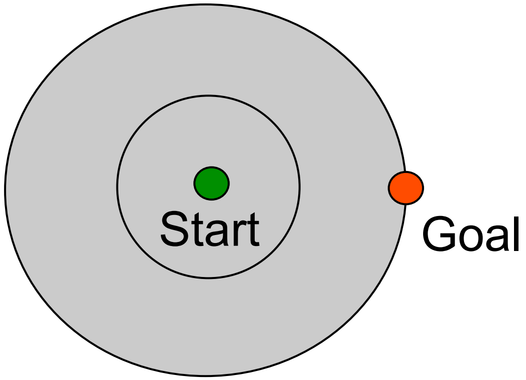
– what are the actions (transitions)?

– how is this a graph?

Recap: BFS/UCS

Notice that we search equally far in all directions...

It's like this



UCS in Pacman Small Maze

Idea

Is it possible to use additional information to decide which direction to search in?

Idea

Is it possible to use additional information to decide which direction to search in?

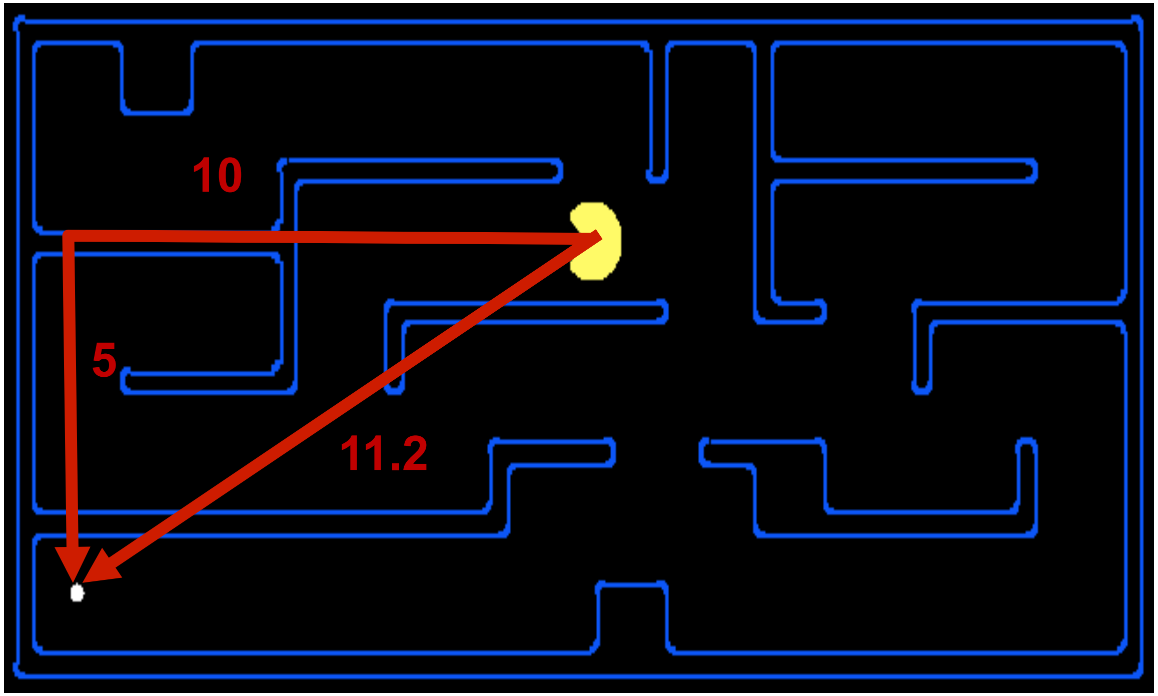
Yes!

Instead of searching in all directions, let's bias search in the direction of the goal.

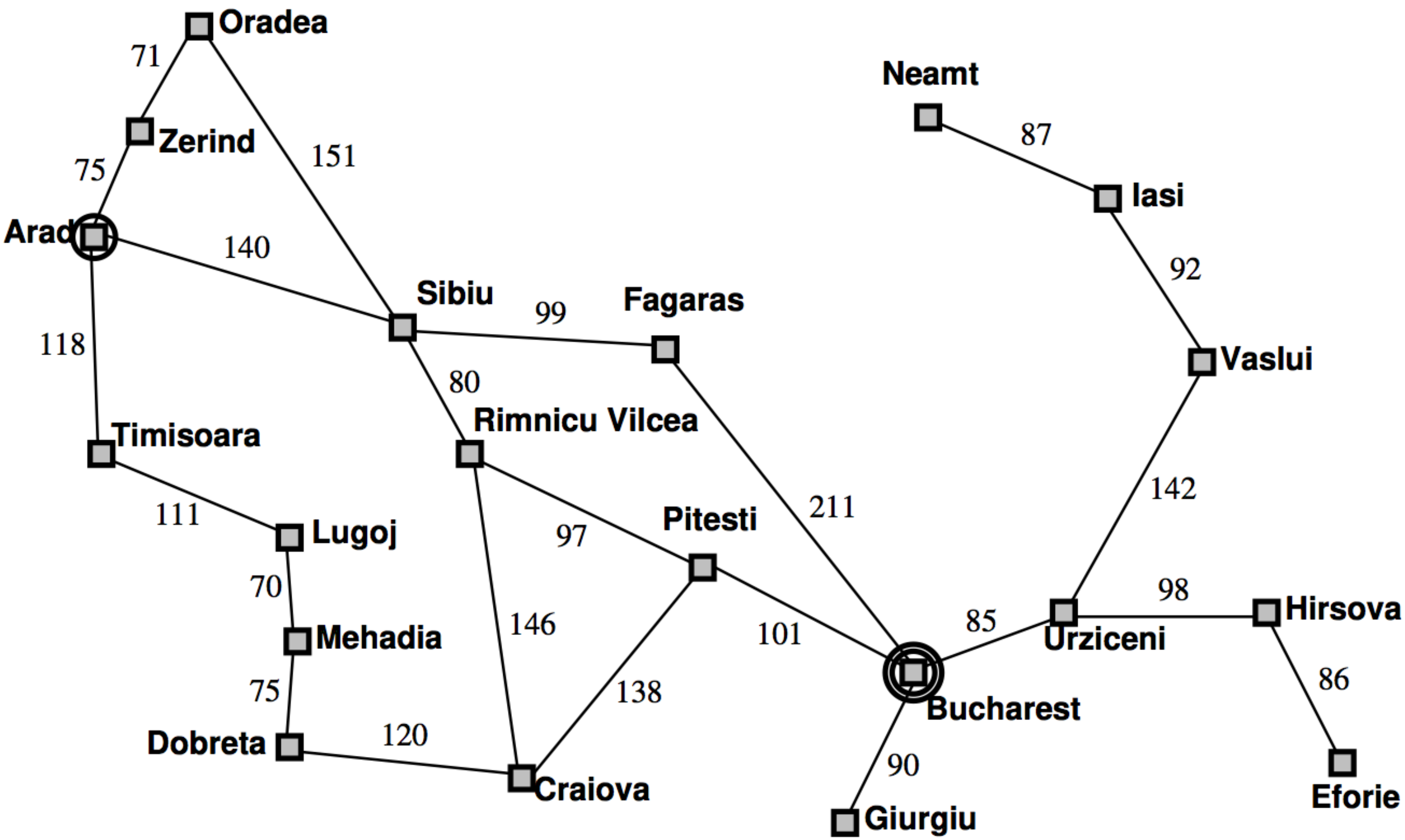
Search heuristics

A heuristic is:

- A function that estimates how close a state is to a goal
- Designed for a particular search problem
- Examples: Manhattan distance, Euclidean distance for path finding



Example



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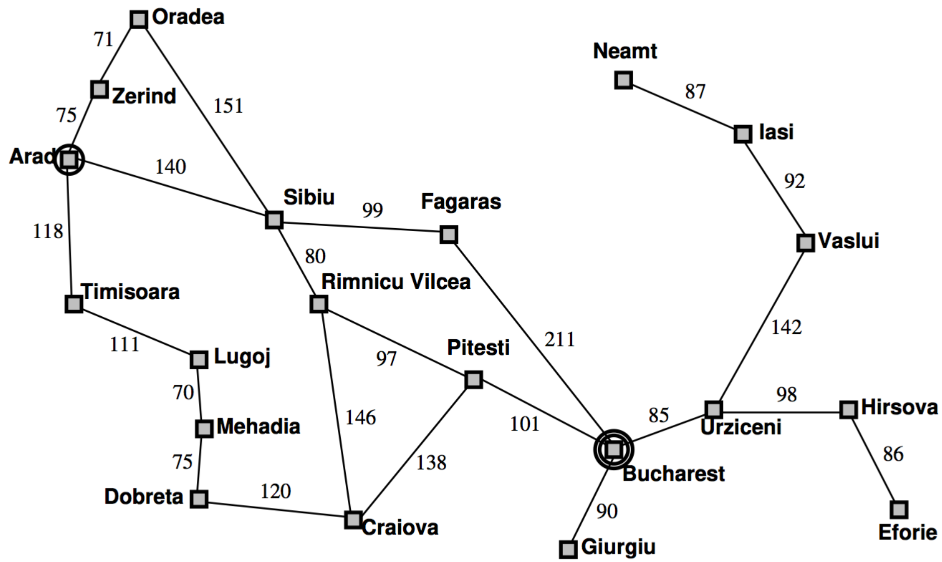
<u>Stright-line distances to Bucharest</u>

Example

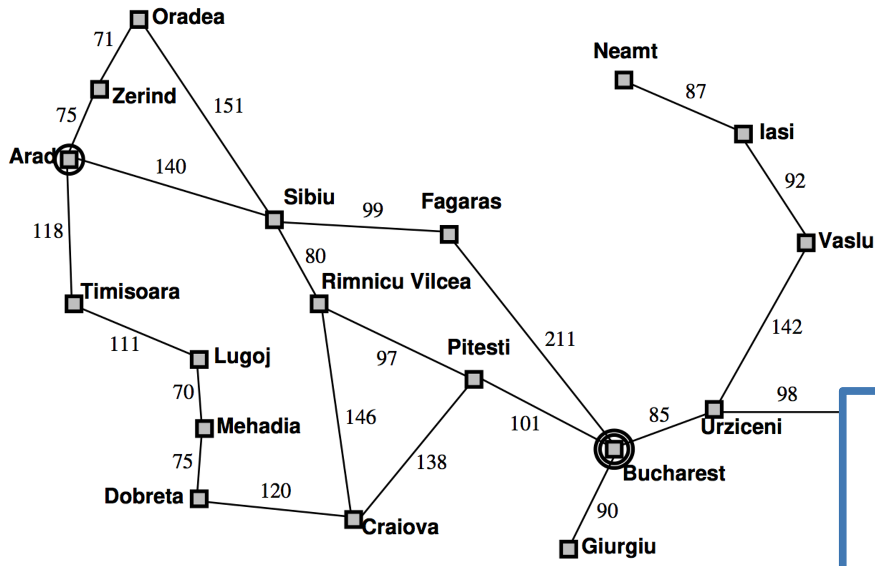


Expand states in order of their distance to the goal

– for each state that you put on the fringe: calculate straight-line distance to the goal

– expand the state on the fringe closest to the goal

Example



Heuristic:

Expand states in order of their distance to the goal

– for each state that you put on the fringe: calculate straight-line distance to the goal

– expand the state on the fringe closest to the goal

Greedy search

Greedy Search

Greedy Search

Each time you expand a state, calculate the heuristic for each of the states that you add to the fringe.

– heuristic: $h(s)$ i.e. distance to Bucharest

– on each step, choose to expand the state with the lowest heuristic value.

Greedy Search

This is like a guess about how far the state is from the goal

Each time you expand a state, calculate the heuristic for each of the states that you add to the fringe.

– heuristic: $h(s)$ ⇦ i.e. distance to Bucharest

– on each step, choose to expand the state with the lowest heuristic value.

Example: Greedy Search

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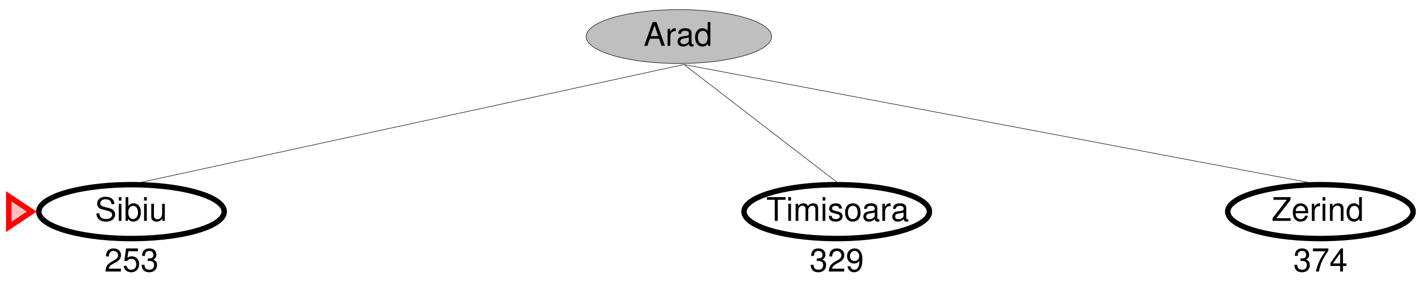
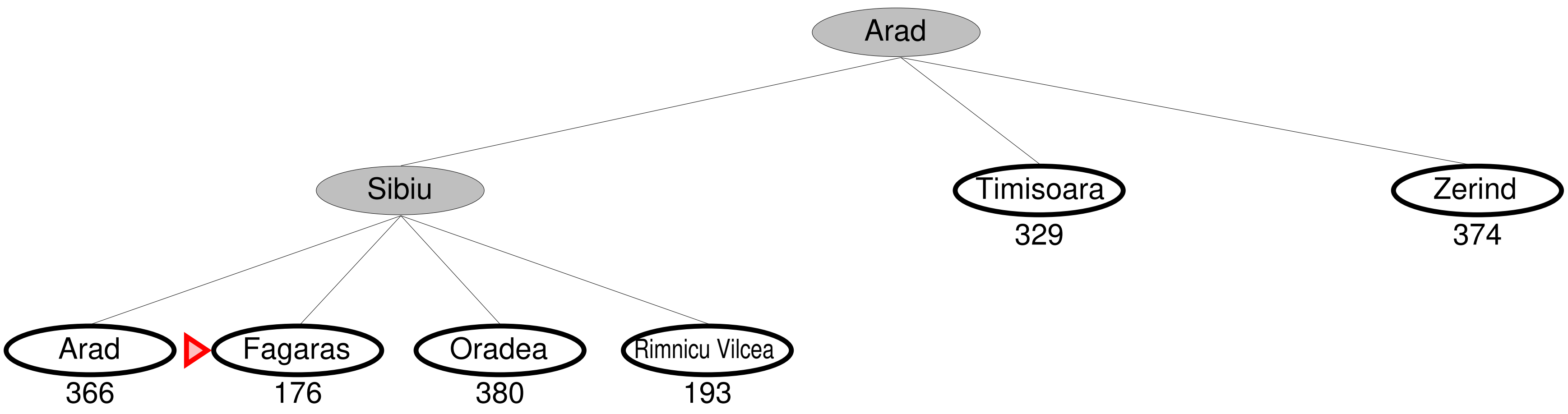


Example: Greedy Search



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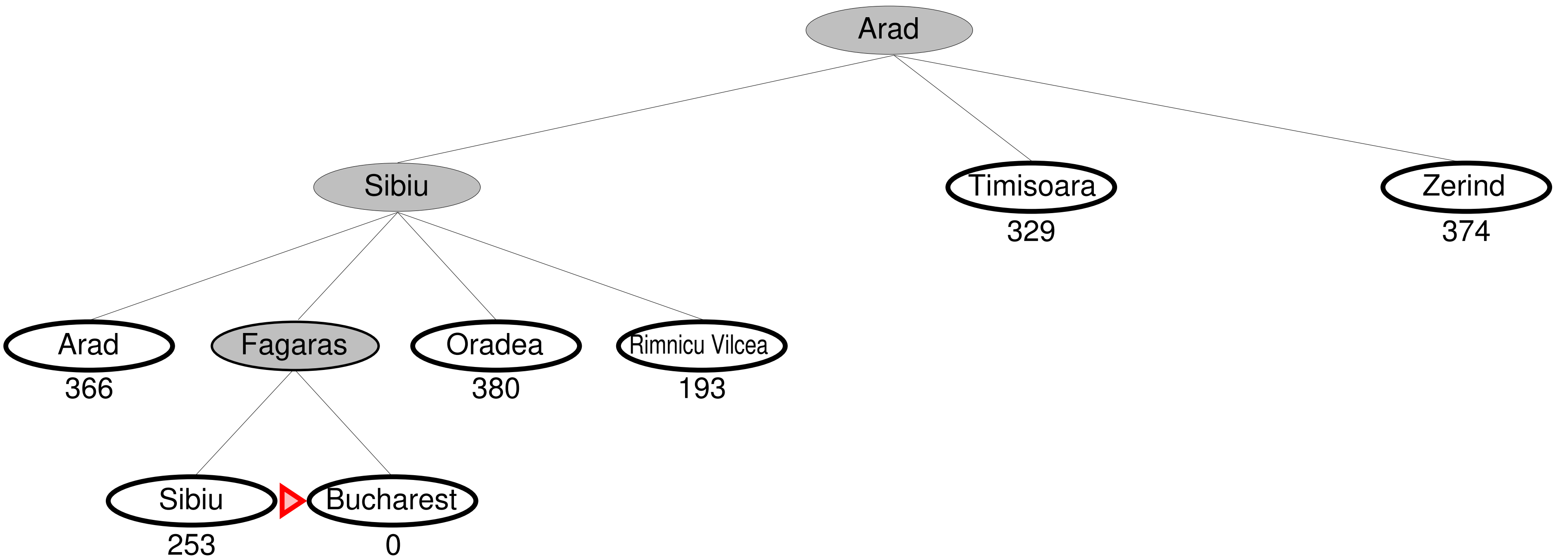

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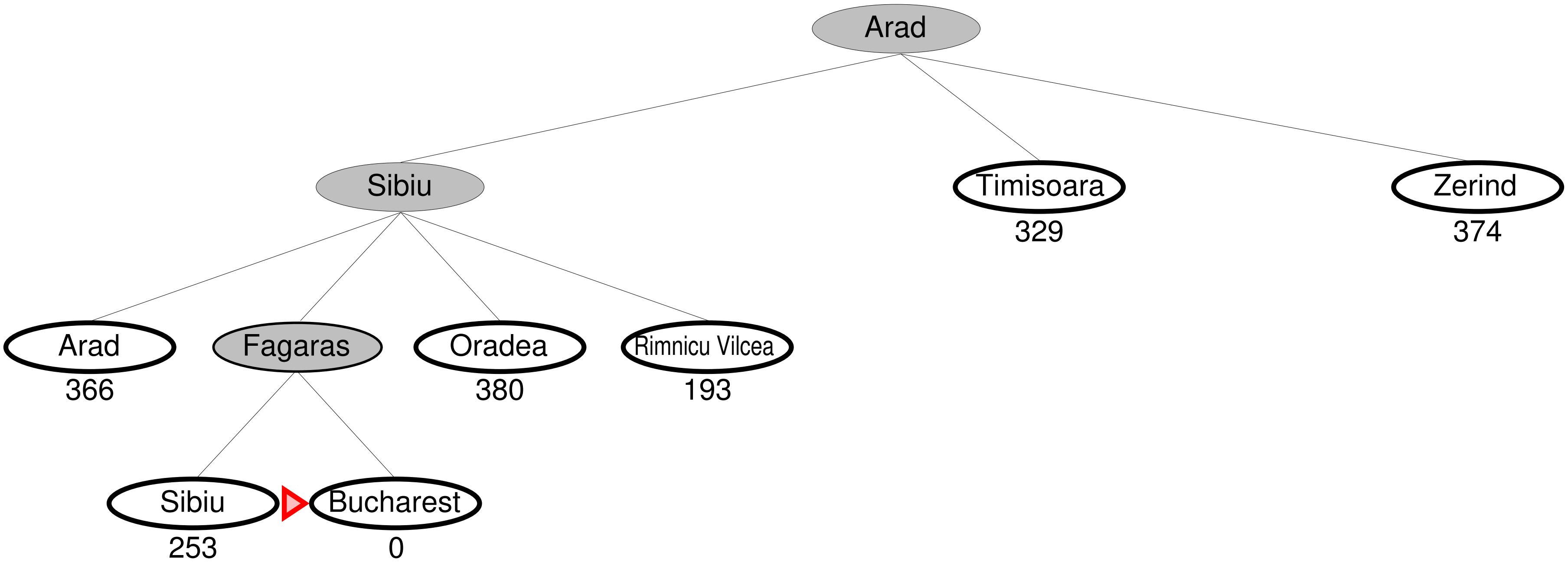
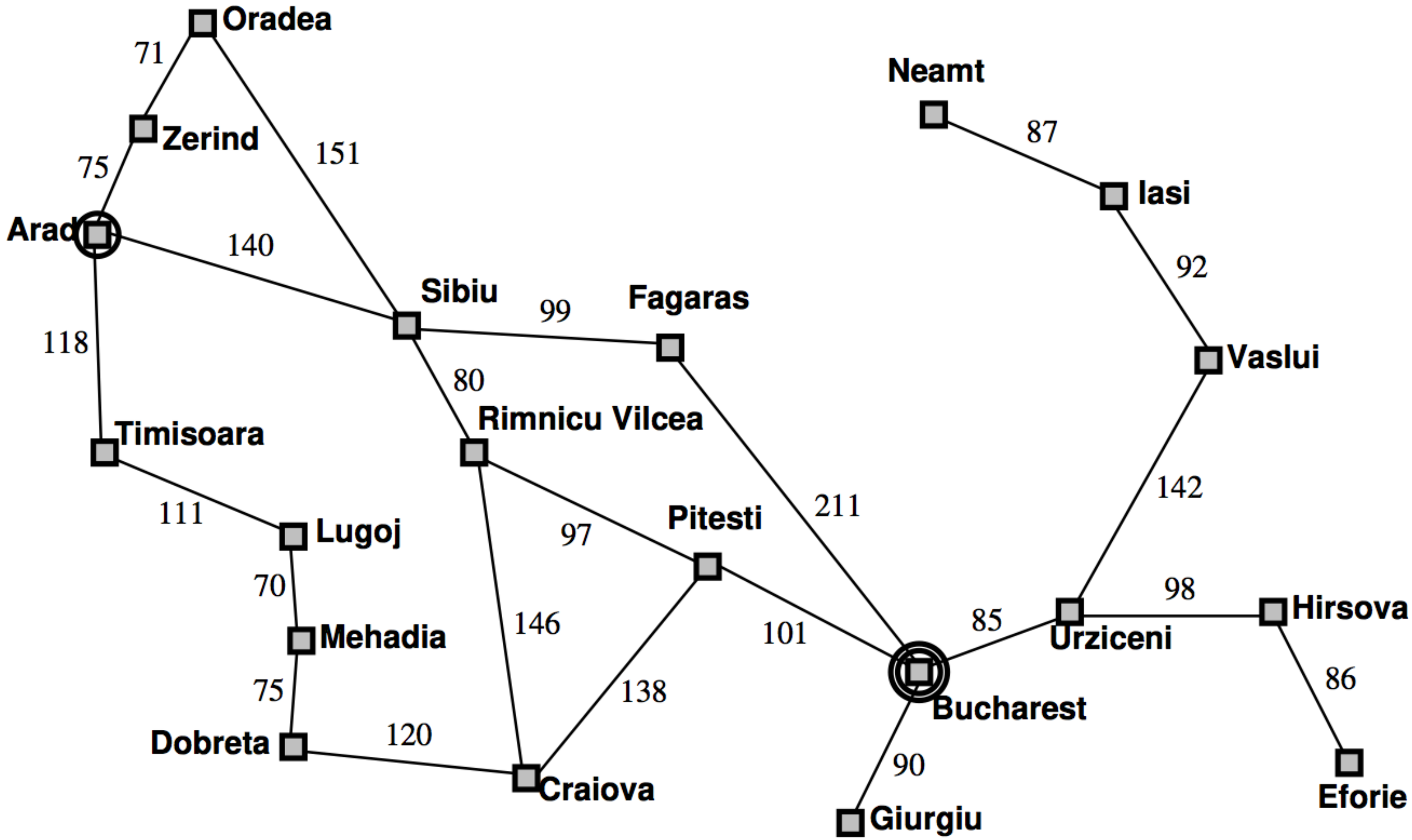


Path: A-S-F-B

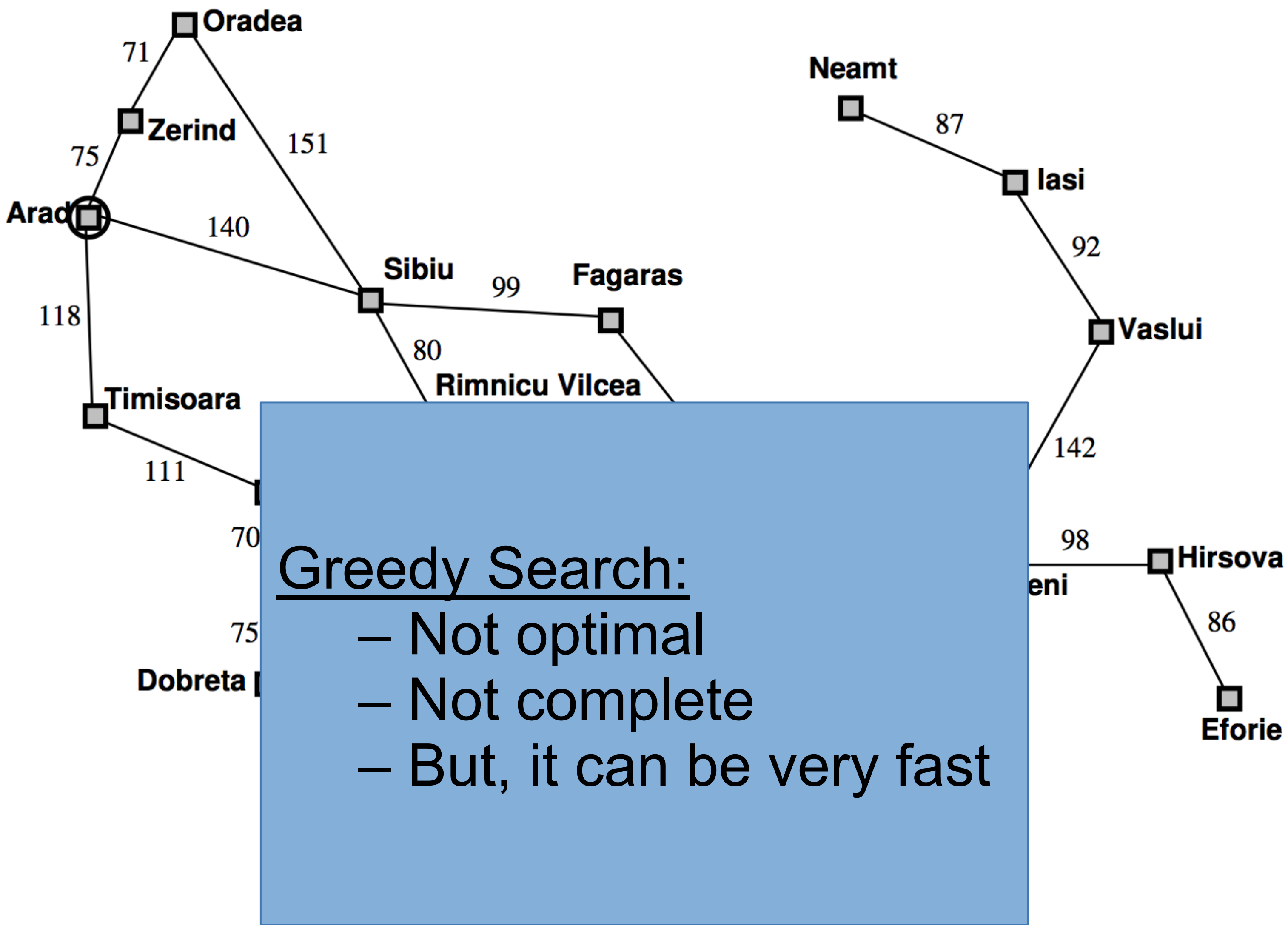
Example: Greedy Search



Path: A-S-F-B

<u>Notice that this is not the optimal path!</u>

Example: Greedy Search



Greedy Search:

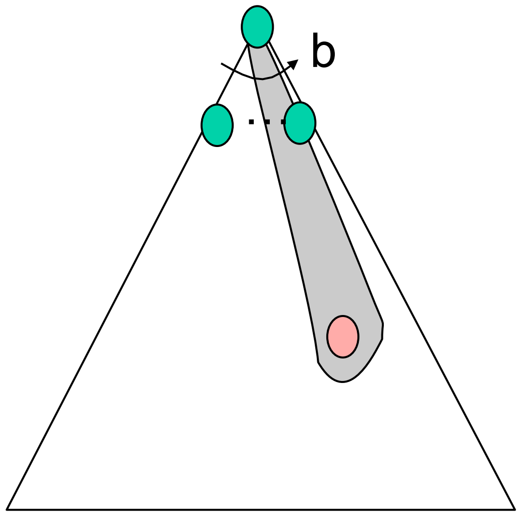
- Not optimal
- Not complete
- But, it can be very fast

Notice that this is not the optimal path!

Greedy search

Strategy: expand a node that you think is closest to a goal state

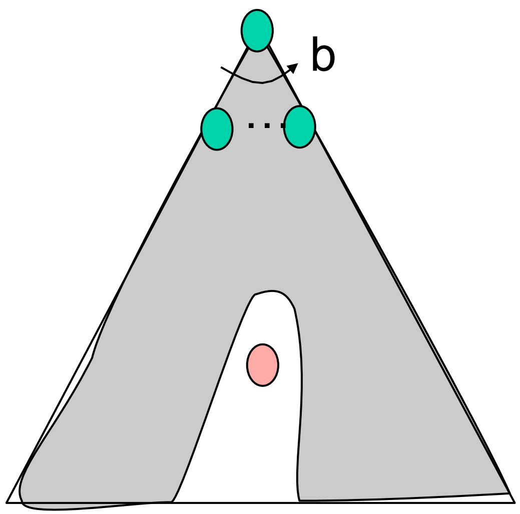
- Heuristic: estimate of distance to nearest goal for each state



A common case:

- Takes you straight to the (wrong) goal

Worst-case: like a badly-guided DFS



Greedy in Pacman Small Maze

Greedy vs UCS

Greedy Search:

- Not optimal
- Not complete
- But, it can be very fast

UCS:

- Optimal
- Complete
- Usually very slow

Greedy vs UCS

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Can we combine greedy and UCS???

Greedy vs UCS

<u>Greedy Search:</u>
- Not optimal
- Not complete
- But, it can be very fast

<u>UCS:</u>
- Optimal
- Complete
- Usually very slow

Can we combine greedy and UCS???

<u>YES: A*</u>

Greedy vs UCS

UCS

Greedy vs UCS

UCS

Greedy

Greedy vs UCS

UCS

Greedy

A*

A*

A*

s : a state

$g(s)$: minimum cost from start to s

$h(s)$: heuristic at s (*i.e.* an estimate of remaining cost-to-go)

UCS: expand states in order of $g(s)$

Greedy: expand states in order of $h(s)$

A*: expand states in order of $f(s) = g(s) + h(s)$

A*

What is “cost-to-go”?

s : a state

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UCS: expand states in order of $g(s)$

Greedy: expand states in order of $h(s)$

A*: expand states in order of $f(s) = g(s) + h(s)$

A*

What is "cost-to-go"?
– minimum cost required to reach a goal state

s : a state

$g(s)$: minimum cost from start to s

$h(s)$: heuristic at s (*i.e.* an estimate of remaining $\boxed{\text{cost-to-go}}$)

UCS: expand states in order of $g(s)$

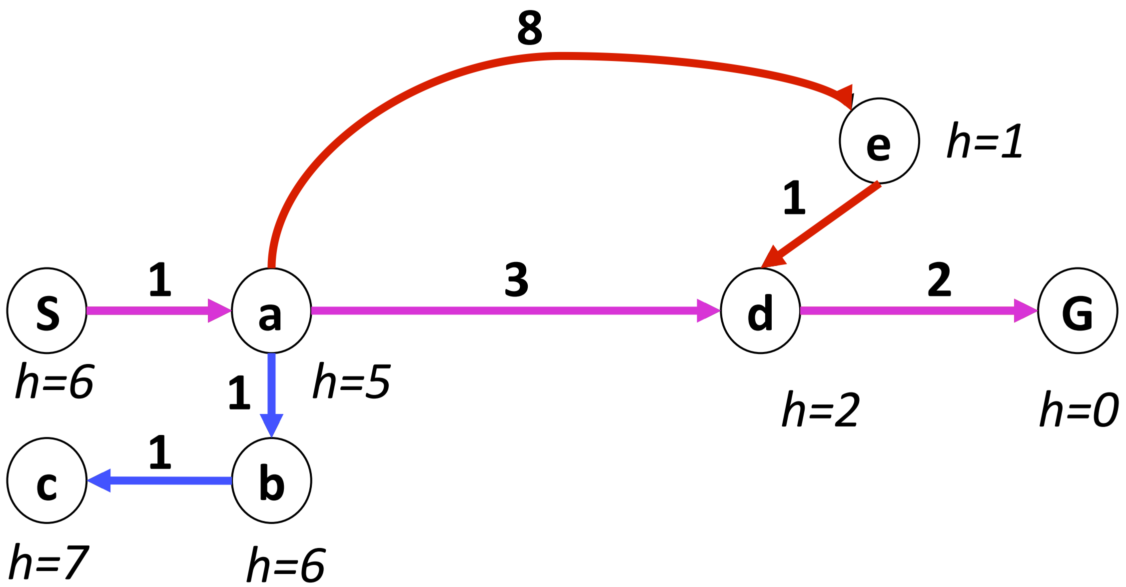
Greedy: expand states in order of $h(s)$

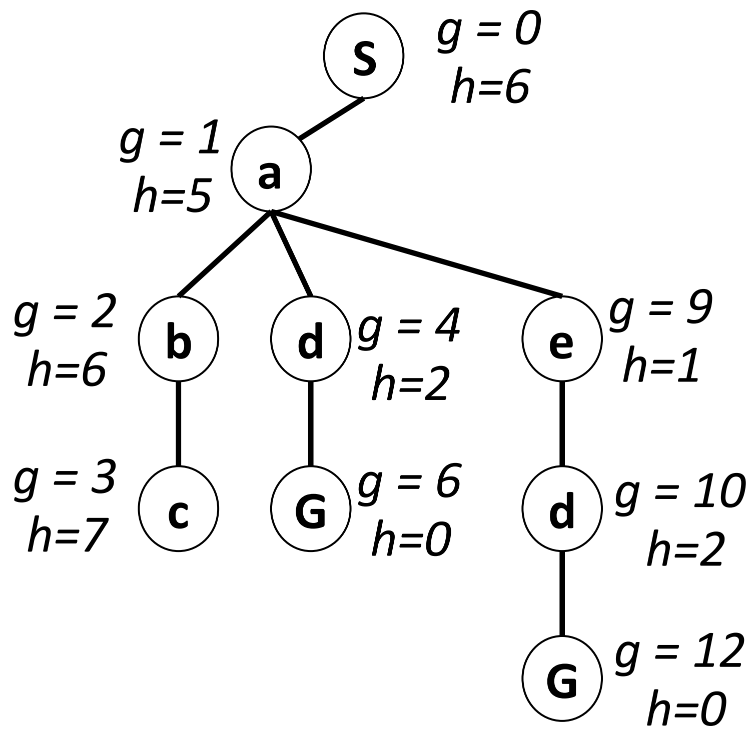
A*: expand states in order of $f(s) = g(s) + h(s)$

A*

Uniform-cost orders by path cost, or *backward cost* $g(s)$

Greedy orders by goal proximity, or *forward cost* $h(s)$



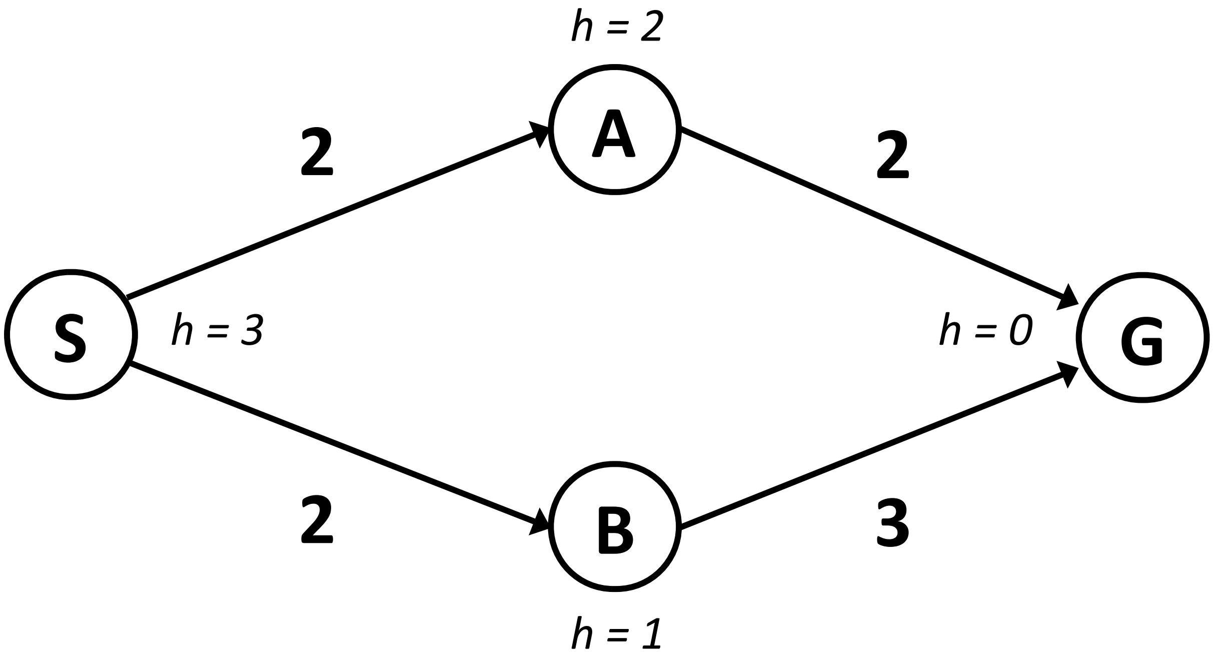


Example: Teg Grenager

A* Search orders by the sum: $f(s) = g(s) + h(s)$

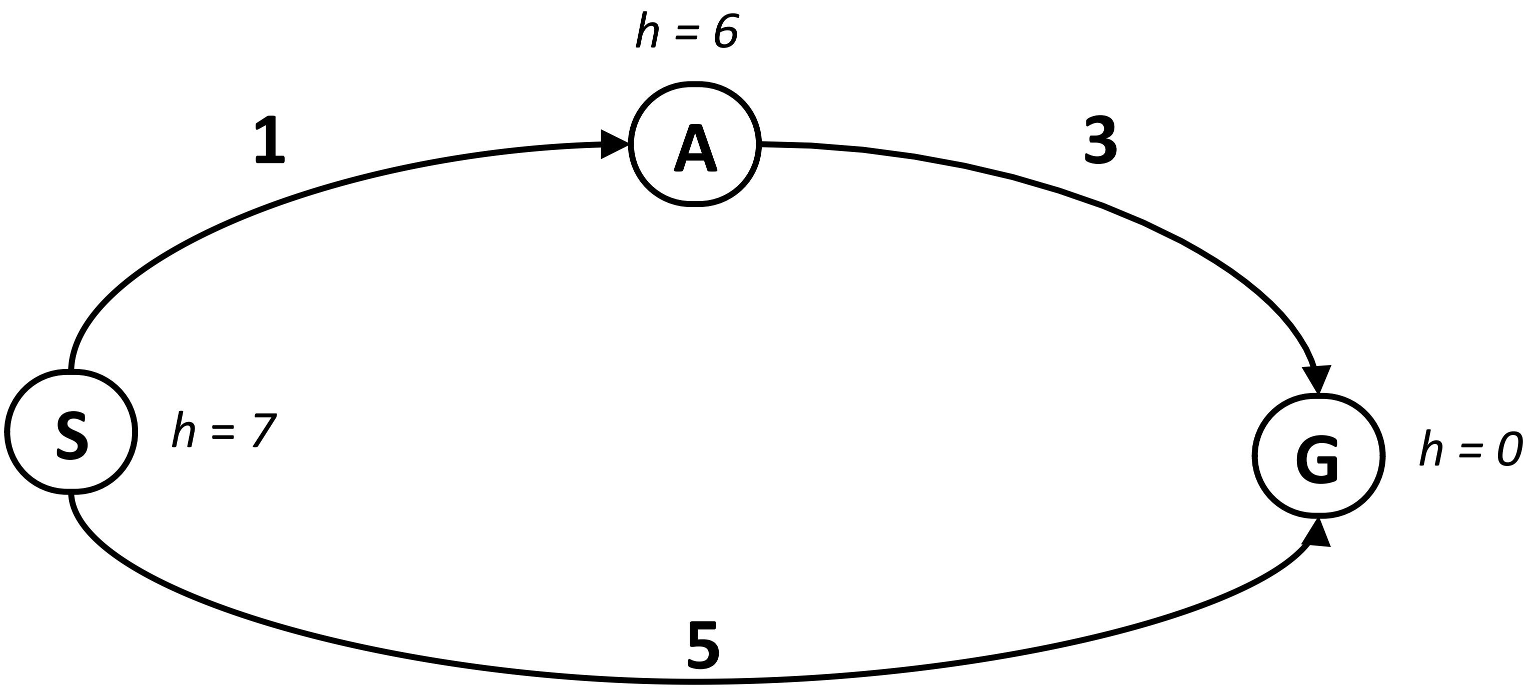
When should A* terminate?

Should we stop when we enqueue a goal?



No: only stop when we dequeue a goal

Is A* optimal?



What went wrong?
Actual cost-to-go < heuristic
The heuristic must be less than the actual cost-to-go!

When is A* optimal?

It depends on whether we are using the tree search or the graph search version of the algorithm.

Recall:

- in tree search, we do not track the explored set
- in graph search, we do

Recall: Breadth first search (BFS)

```
function BREADTH-FIRST-SEARCH(problem) returns a solution, or failure
  node ← a node with STATE = problem.INITIAL-STATE, PATH-COST = 0
  if problem.GOAL-TEST(node.STATE) then return SOLUTION(node)
  frontier ← a FIFO queue with node as the only element
  explored ← an empty set
  loop do
     if EMPTY?(frontier) then return failure
     node ← POP(frontier)  /* chooses the shallowest node in frontier */
     add node.STATE to explored
     for each action in problem.ACTIONS(node.STATE) do
        child ← CHILD-NODE(problem, node, action)
        if child.STATE is not in explored or frontier then
           if problem.GOAL-TEST(child.STATE) then return SOLUTION(child)
           frontier ← INSERT(child, frontier)
```

Figure 3.11 Breadth-first search on a graph.

What is the purpose of the *explored* set?

When is A* optimal?

It depends on whether we are using the tree search or the graph search version of the algorithm.

Optimal if h is consistent

Optimal if h is admissible

When is A* optimal?

It depends on whether we are using the tree search or the graph search version of the algorithm.

Optimal if h is consistent

– $h(s)$ is an underestimate of the cost of each arc.

Optimal if h is admissible

– $h(s)$ is an underestimate of the true cost-to-go.

When is A* optimal?

It depends on whether we are using the <u>tree search</u> or the <u>graph search</u> version of the algorithm.

Optimal if *h* is <u>consistent</u>

– *h*(*s*) is an underestimate of the cost of each arc.

Optimal if *h* is <u>admissible</u>

– *h*(*s*) is an underestimate of the true cost-to-go.

<u>What is “cost-to-go”?</u>

– minimum cost required to reach a goal state

When is A* optimal?

It depends on whether we are using the tree search or the graph search version of the algorithm.

Optimal if h is consistent

– $h(s)$ is an underestimate of the cost of each arc.

Optimal if h is admissible

– $h(s)$ is an underestimate of the true cost-to-go.

More on this later...

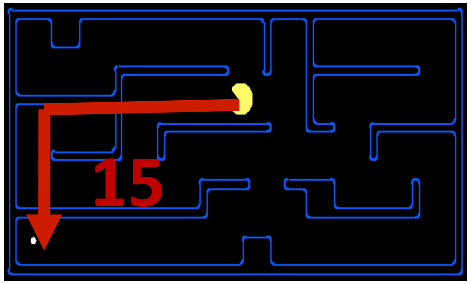
Admissible heuristics

A heuristic h is *admissible* (optimistic) if:

$$0 \leq h(n) \leq h^*(n)$$

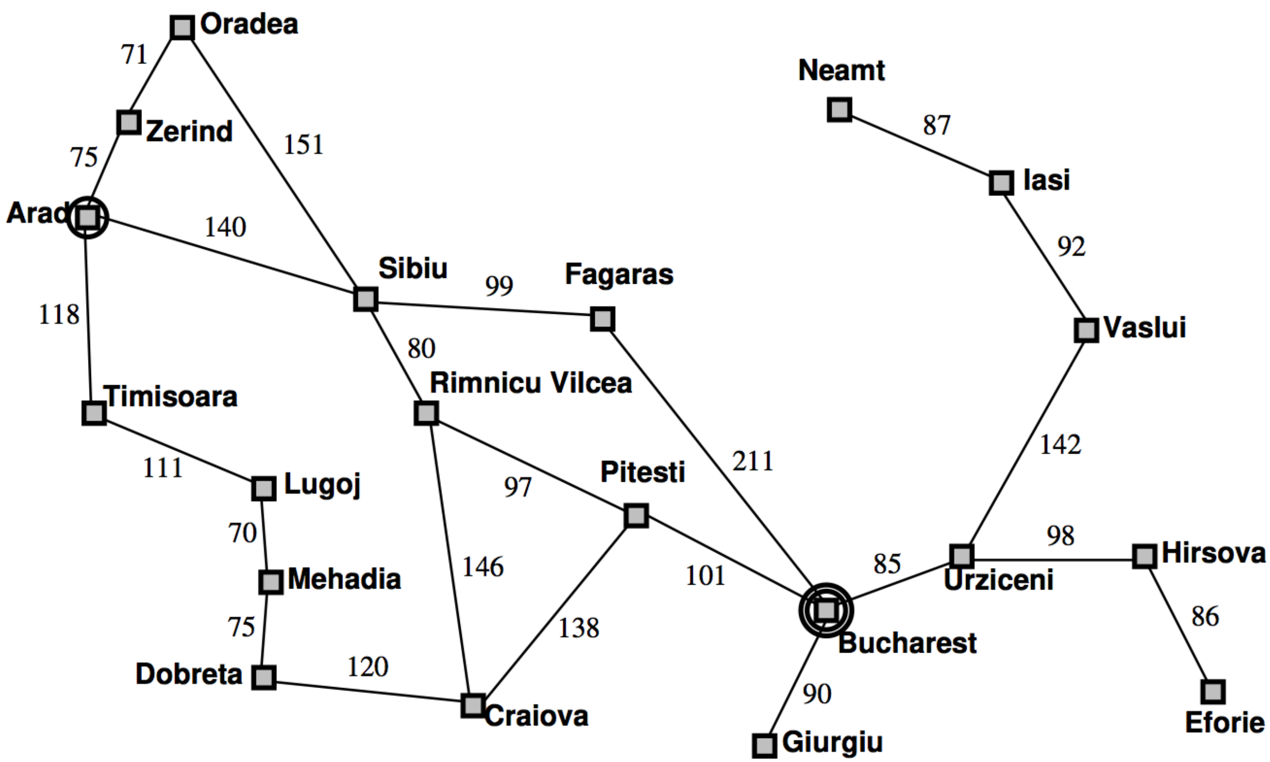
where $h^*(n)$ is the true cost to a nearest goal

Example:



Coming up with admissible heuristics is most of what's involved in using A* in practice.

Admissibility: Example



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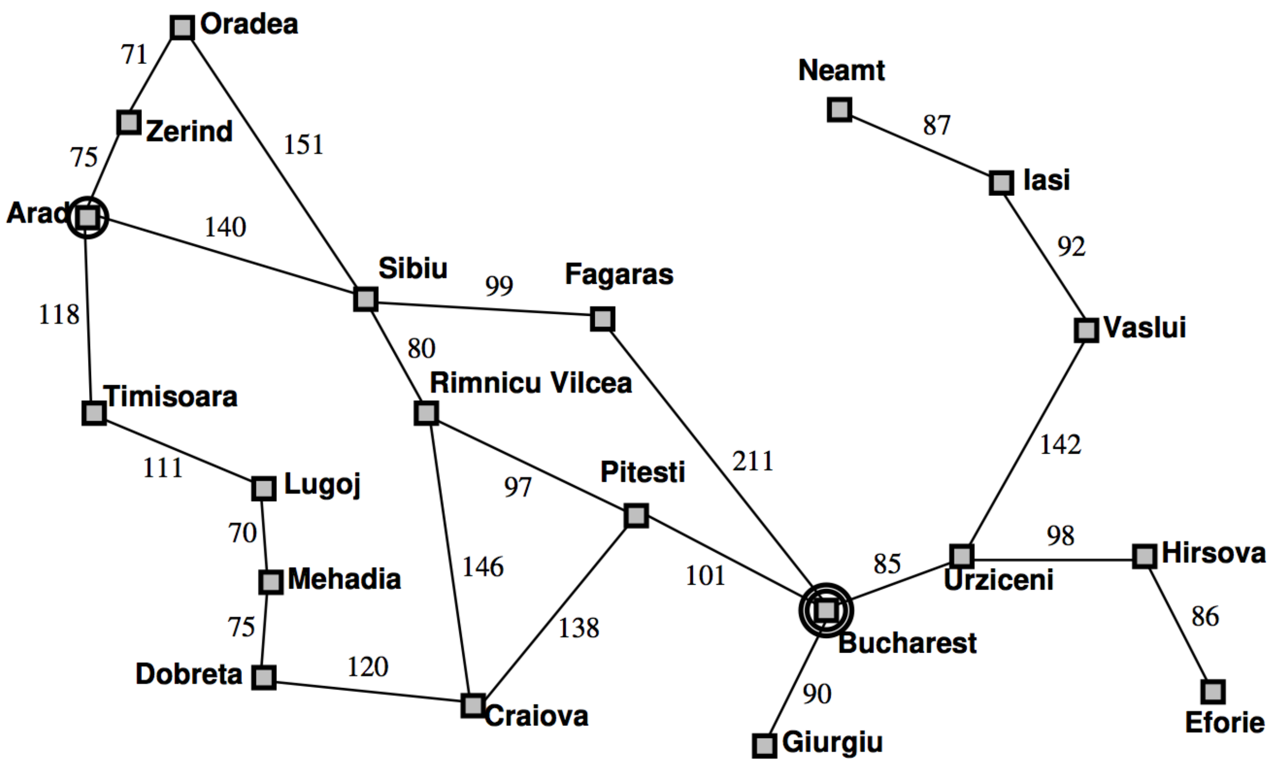
Stright-line distances to Bucharest

$h(s)$ = straight-line distance to goal state (Bucharest)

Admissibility



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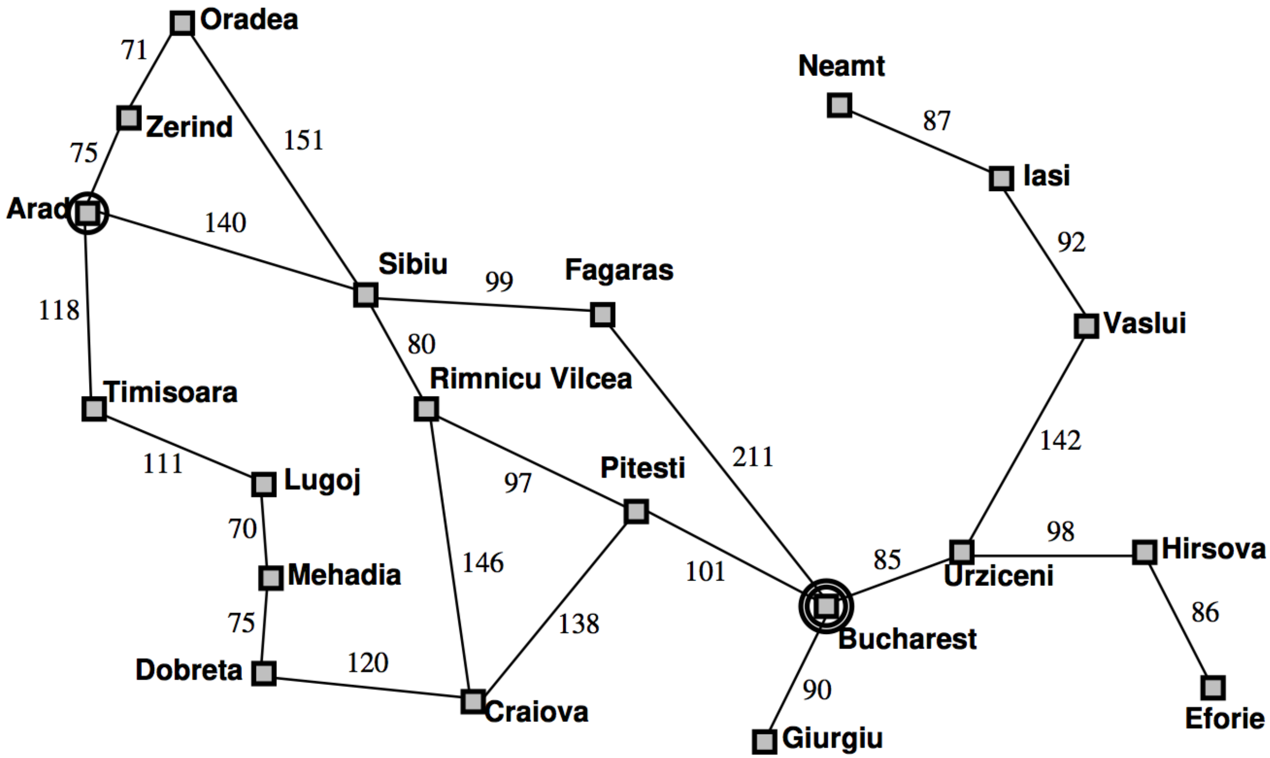
<u>Stright-line distances to Bucharest</u>

$h(s)$ = straight-line distance to goal state (Bucharest)

Is this heuristic admissible???

Admissibility



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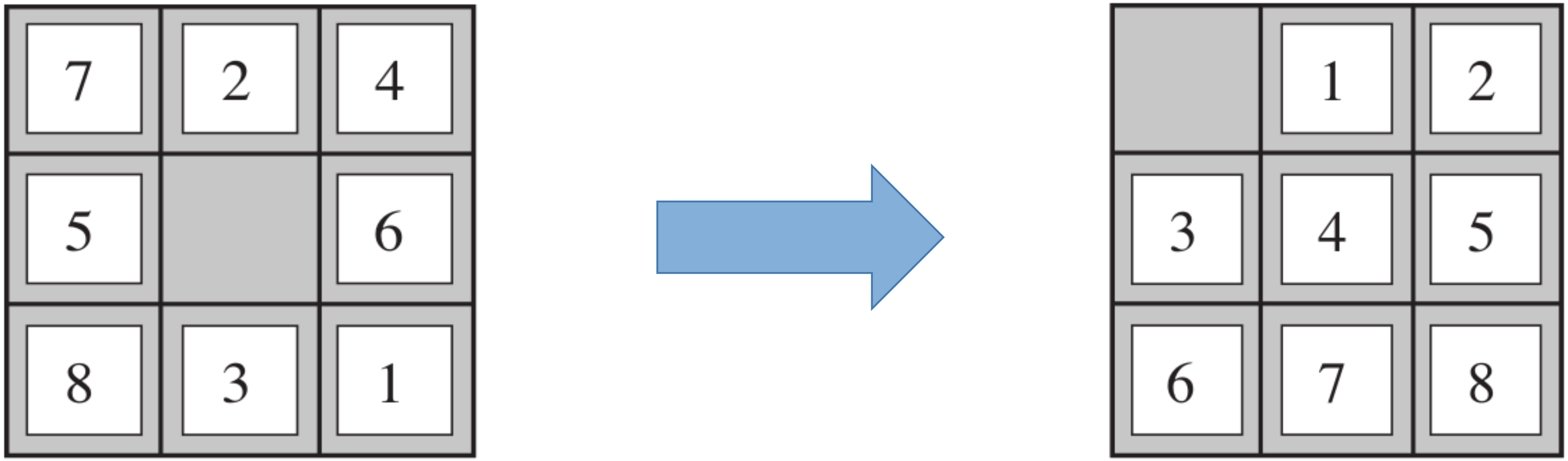
Stright-line distances to Bucharest

h(*s*) = straight-line distance to goal state (Bucharest)

Is this heuristic admissible???
YES! Why?

Admissibility: Example



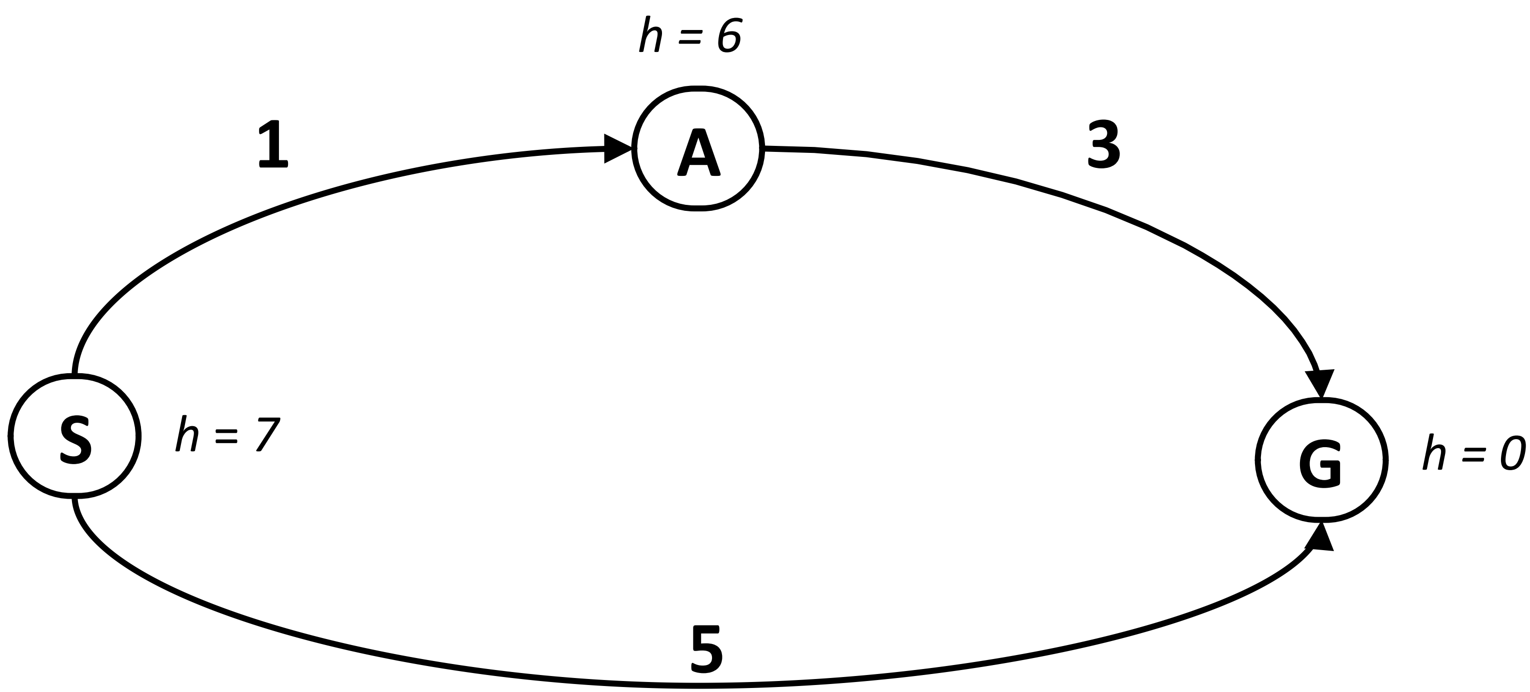
Start state

Goal state

$h(s)$ = ?

Can you think of an admissible heuristic for this problem?

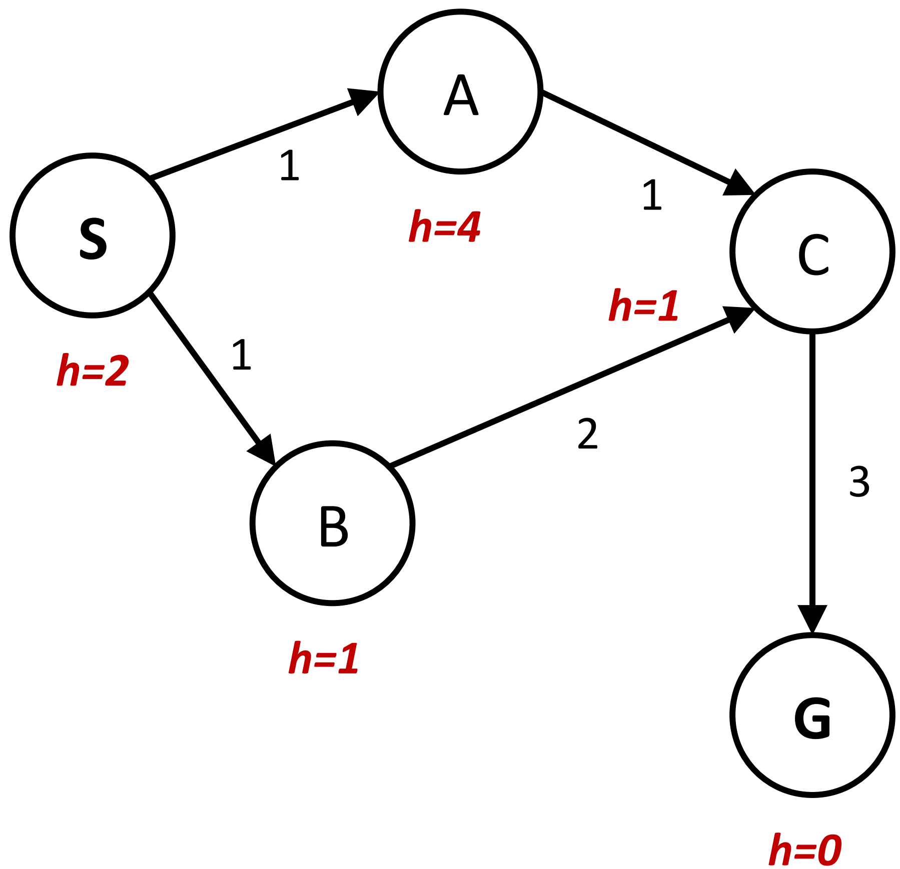
Admissibility



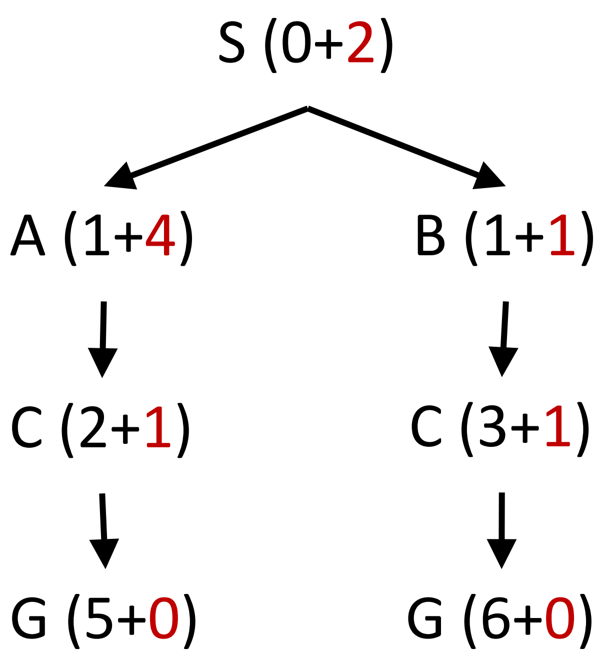
Why isn't this heuristic admissible?

Consistency

State space graph

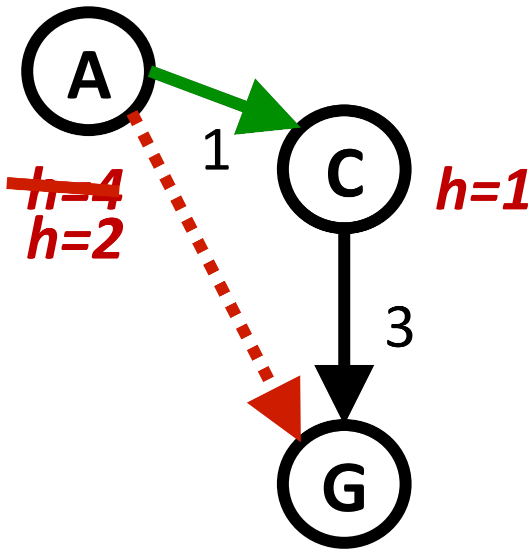


Search tree



What went wrong?

Consistency



Main idea: estimated heuristic costs ≤ actual costs

- Admissibility: heuristic cost ≤ actual cost to goal

  h(A) ≤ actual cost from A to G

- Consistency: heuristic "arc" cost ≤ actual cost for each arc

  h(A) – h(C) ≤ cost(A to C)

Consequences of consistency:

- The f value along a path never decreases

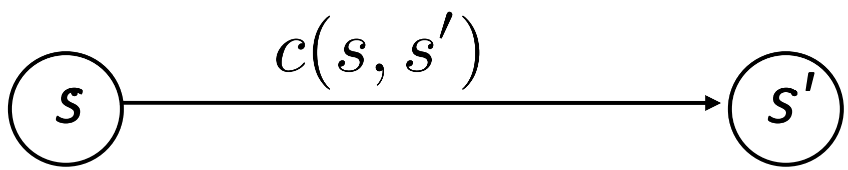
  h(A) ≤ cost(A to C) + h(C)

- A* graph search is optimal

Consistency

$$h(s) \leq c(s, s') + h(s')$$

Cost of going from *s* to *s*′



Consistency

$$h(s) \leq c(s, s') + h(s')$$

$$h(s) - h(s') \leq c(s, s')$$

Rearrange terms

Consistency

$$h(s) \leq c(s, s') + h(s')$$

$$\underbrace{h(s) - h(s')}_{\text{Cost of going from s to s' implied by heuristic}} \leq c(s, s')$$

Actual cost of
going from *s* to *s*′

Consistency

$$f(s) = g(s) + h(s)$$

Consistency implies that the “f-cost” never decreases along any path to a goal state.
– the optimal path gives a goal state its lowest f-cost.

A* expands states in order of their f-cost.

Given any goal state, A* expands states that reach the goal state optimally before expanding states the reach the goal state suboptimally.

Consistency implies admissibility

Suppose: $\forall s_t, s_{t+1} : h(s_t) \leq c(s_t, s_{t+1}) + h(s_{t+1})$

Then: $h(s_{T-1}) \leq c(s_{T-1}, s_T) + h(s_T)$

Consistency implies admissibility

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admissible

Consistency implies admissibility

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admissible

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Consistency implies admissibility

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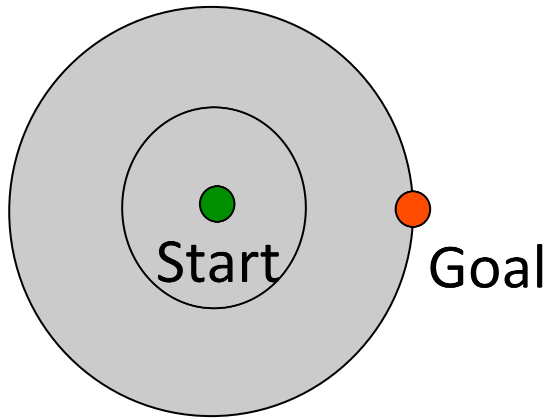
$h(s_{T-2}) \leq c(s_{T-2}, s_{T-1}) + h(S_{T-1})$

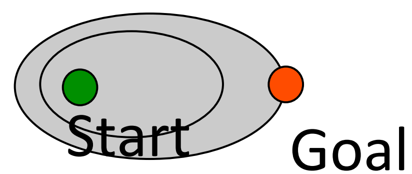
$\cdots$

A* vs UCS

Uniform-cost expands equally in all “directions”

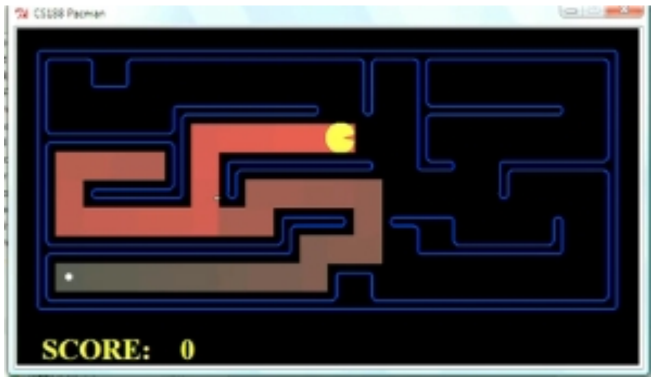


A* expands mainly toward the goal, but does hedge its bets to ensure optimality



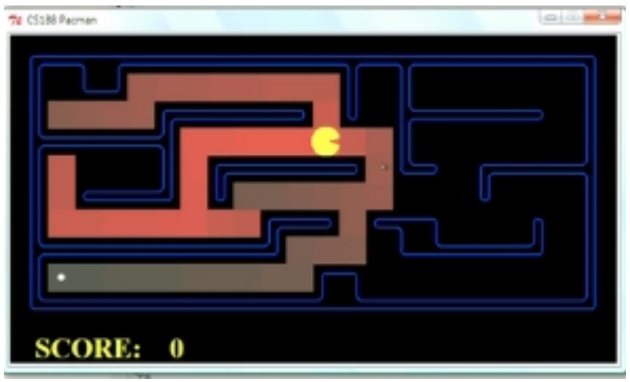
A* in Pacman Small Maze

A* vs UCS



Greedy

UCS



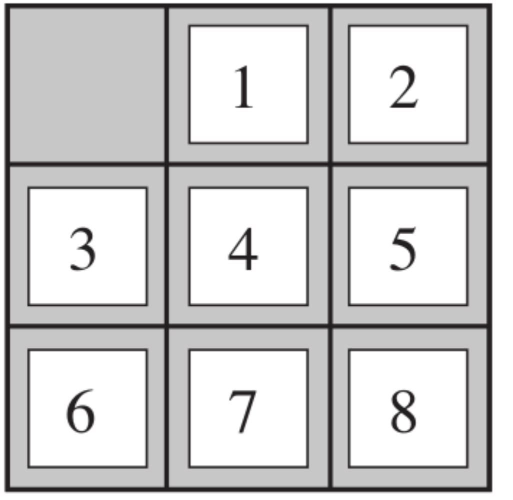
A*

Choosing a heuristic

The right heuristic is often problem-specific.

But it is very important to select a good heuristic!

Choosing a heuristic



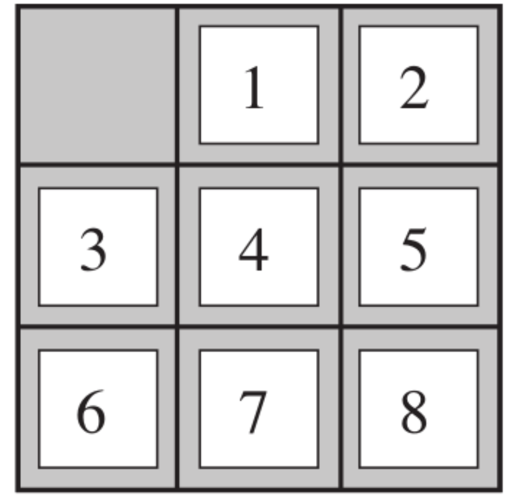
Consider the 8-puzzle:

h_1 : number of misplaced tiles

h_2 : sum of manhattan distances between each tile and its goal.

How much better is h_2 ?

Choosing a heuristic



Consider the 8-puzzle:

h_1 : number of misplaced tiles

h_2 : sum of manhattan distances between each tile and its goal.

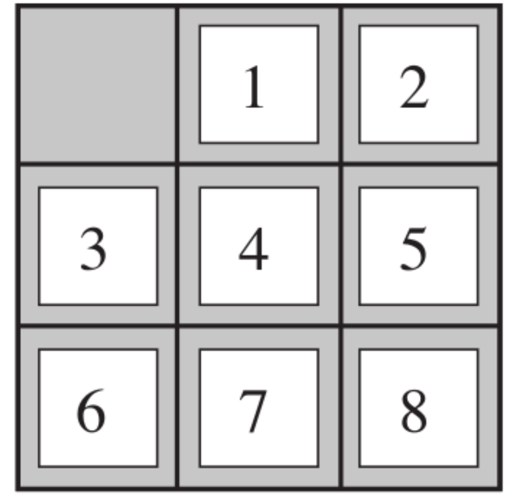
Average # states expanded on a random depth-24 puzzle:

$$A^*(h_1) = 39k$$

$$A^*(h_2) = 1.6k$$

$$IDS = 3.6M$$ (by depth 12)

Choosing a heuristic



Consider the 8-puzzle:

h_1 : number of misplaced tiles

h_2 : sum of manhattan distances between each tile and its goal.

zle:

So, getting the heuristic right can speed things up by multiple orders of magnitude!

$IDS = 3.6M$ (by depth 12)

Choosing a heuristic

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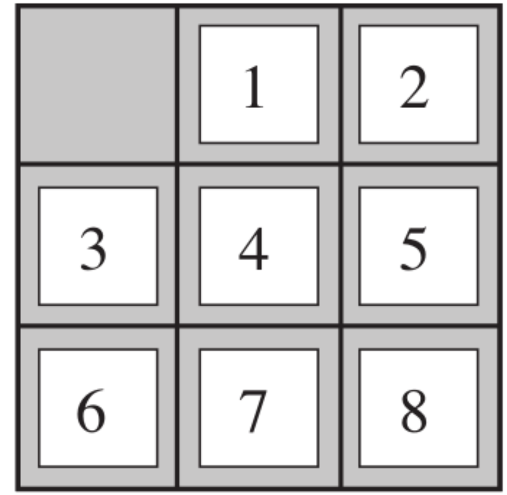

Why not use the actual cost to goal as a heuristic?

How to choose a heuristic?

Nobody has an answer that always works.

A couple of best-practices:
– solve a relaxed version of the problem
– solve a subproblem