Existence Theorems and Approximation Algorithms for Generalized Network Security Games

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Outline

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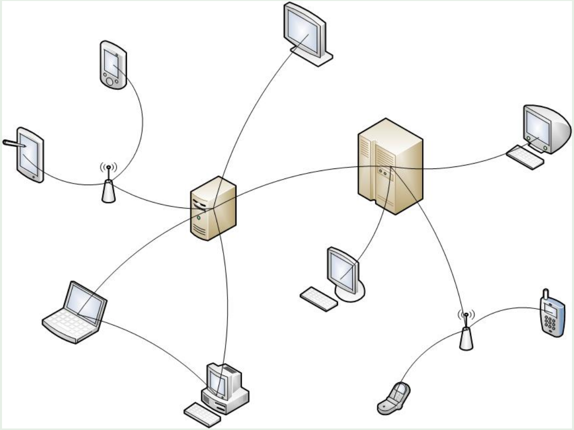
1. Introduction
   - Motivation and examples
   - Model and definitions
2. Our results
   - Existence of pure NE
   - Approximating the social optimum
   - Simulation study
3. Concluding remarks
   - Future work

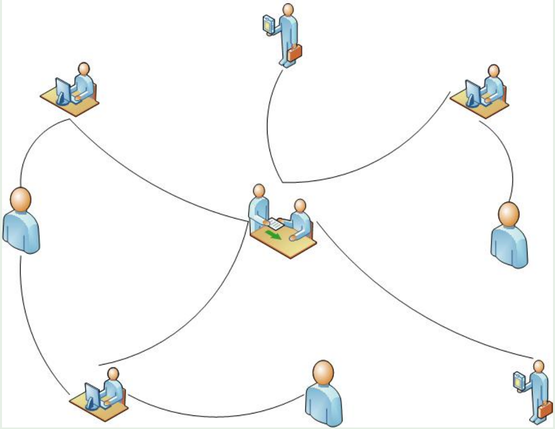
Motivation

Computer network



Motivation

Human contact network



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Game-theoretic model

- Contact graph: G(V,E).
- Strategies: install anti-virus software or not, $a_i \in \{0, 1\}$.
- Security cost/infection cost: C_i, L_i.
- Individual cost: $cost_i(\bar{a}) = a_i C_i + (1 - a_i) L_i p_i(\bar{a})$.
- Social cost: $\sum_i cost_i(\bar{a})$.
- Infection model: we assume infection is initiated at a node (picked with probability propotional to its weight w_i), and transmits over at most d (which is a parameter) hops in the contact graph.
- Generalized Network Security Game: GNS(d).

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List of results

	$d = 1$	$1 < d < \infty$	$d = \infty$
existence of pure NE	Yes	No/NP-complete	Yes
price of anarchy	$\Delta + 1$		$O(1/\alpha(G))$
approximating social	2	2d	$O(\log n)$

- Δ is the max degree in the contact graph.
- $\alpha(G)$ is the vertex expansion of the contact graph.

Related work

- Aspnes et al 2006 introduced a basic model for $d = \infty$ case that we have generalized here.
  - Show existence of pure NE in a uniform version.
  - Give an $O(\log^{3/2} n)$-approximation for social optimum.
- Interdependent security games [Kearns-Ortiz 2004].
  - Similar to our model for special case of $d = 1$.
  - Crucial difference in assumption about initial infection.
- n-intertwined games [Omic et al 2009].
  - Based on SIS model for worm spread.
- Considerable work in SIR and SIS models in epidemiology.

Existence of pure NE when $d = \infty$

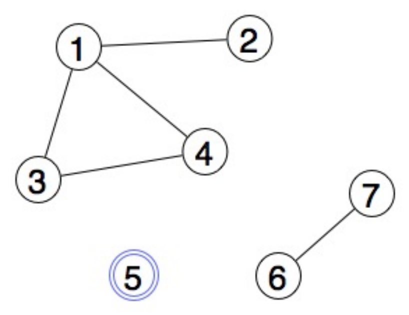
Theorem

Every GNS(∞) instance has a pure NE.

- The existence proof is a potential function argument.
- Define Threshold of a node, t_i: Bound on number of reachable nodes that would make the node want to secure itself.
- w.l.o.g., assume $t_1 \geq t_2 \geq \cdots \geq t_m$.
- Define potential function: $\hat{\Phi}(\vec{a}) = \left(\Phi_1(\vec{a}), \Phi_2(\vec{a}), \ldots, \Phi_m(\vec{a})\right)$ where $\Phi_i(\vec{a})$ is 0 if i is secure, -1 if i is insecure and happy, and 1 otherwise.

Example of potential function

Example



- $t_1 = 5, t_2 = 4, t_3 = 3, t_4 = 2, t_5 = 1, t_6 = 1, t_7 = 0.$
- 5 is secured.
- Potential function for this configuration is $(-1, -1, -1, 1, 0, -1, 1)$.

$\hat{\Phi}(\vec{a})$ lexicographically decreases

- Case 1: unhappy insecure → happy secure. One component decreases by 1, while none of the other components increases.
- Case 2: unhappy secure → happy insecure. All the happy insecure nodes with bigger thresholds are still happy. Happy insecure nodes with smaller thresholds may become unhappy. But the function still decreases lexicographically.

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Approximation algorithm for social optimum

LP formulation

- Let P_{ij}^d denote the set of all simple paths from i to j of length at most d.
- $\forall v \in V$, $x_v = 1$ if v is secure; $x_v = 0$ otherwise.
- $\forall i, j \in V$, $y_{ij} = 1$ is there is no $p \in P_{ij}^d$ consisting entirely of insecured nodes.

$$\begin{array}{ll} \min & \sum_v C_v \cdot x_v + \sum_{j \in V} L_j \sum_{i \in V} w_i (1 - y_{ij}) \\ \text{s.t.} & \sum_{v \in p} x_v \geq y_{ij} \;\; p \in P_{ij}^d \\ & x_v \in \{0, 1\} \;\; \forall v \in V \\ & y_{ij} \in \{0, 1\} \;\; \forall i, j \in V \end{array}$$

Objective function of LP

$$\begin{aligned} \min \quad & \sum_v C_v \cdot x_v + \sum_{j \in V} L_j \sum_{i \in V} w_i (1 - y_{ij}) \\ \text{s.t.} \quad & \sum_{v \in p} x_v \geq y_{ij} \; p \in P_{ij}^d \\ & x_v \in \{0, 1\} \; \forall v \in V \\ & y_{ij} \in \{0, 1\} \; \forall i, j \in V \end{aligned}$$

- First part of the objective function corresponds to the cost of securing nodes.
- Second part corresponds to the infection cost. For node j, its infection cost is L_j times the sum of the probabilities of all nodes that have a path to j of length at most d consisting entirely of insecure nodes.

Constraints of LP

$$\begin{array}{ll} \min & \sum_v C_v \cdot x_v + \sum_{j \in V} L_j \sum_{i \in V} w_i (1 - y_{ij}) \\ \text{s.t.} & \sum_{v \in p} x_v \geq y_{ij} \; p \in P_{ij}^d \\ & x_v \in \{0, 1\} \; \forall v \in V \\ & y_{ij} \in \{0, 1\} \; \forall i, j \in V \end{array}$$

- Constraint says, in order to separate a pair of nodes *i* and *j*, we need to secure at least one node in every path between these two.

Solving LP

- d is a constant:
  - Number of paths of length at most d is polynomial.
  - So LP is poly-size and can be solved in poly-time.
- d is not a constant:
  - Number of paths superpolynomial; still LP solvable using ellipsoid method.
  - Can also solve an equivalent LP of polynomial size.

Partial rounding

LP objective

$$\min \quad \sum_v C_v \cdot x_v + \sum_{j \in V} L_j \sum_{i \in V} w_i (1 - y_{ij})$$

- Let (x, y) denote an optimal solution.
- Round each y_{ij} to nearest integer.
  - So values at least 1/2 are rounded up to 1 and less than 1/2 rounded down to 0.
- Scale up each x_{ij} by a factor of 2.
  - If scaled value exceeds 1, set it to 1.
- New solution (x, y) is still feasible and new cost at most twice that before.

Final rounding

$$\sum_{v \in p} x_v \geq y_{ij} \quad p \in P_{ij}^d$$

- It remains to round the x-values.
- Simple approach: Each x_{ij} that is at least $1/d$ is rounded up to 1, other x_{ij}s rounded down to 0.
  - Yields $2d$-approximation.
  - Perhaps acceptable for small d.
- For $d = \infty$:
  - Need to select a set of nodes to secure such that all pairs of nodes i, j with $y_{ij} = 1$ are separated.
  - This is precisely a vertex multicut problem for which x-values give a fractional optimum.
  - Use algorithm of Garg-Vazirani-Yannakakis to round the x-values and obtain an $O(\log n)$-approximation.

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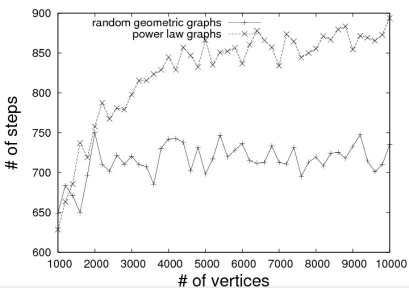
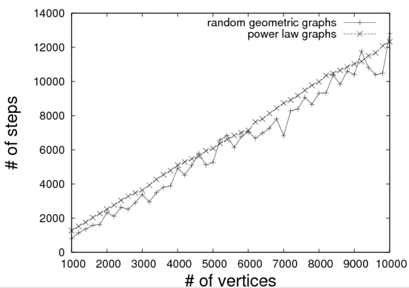
- Study the convergence time for best response strategies.
- Study the performance of our approximation algorithms.
- Simulate on 2 types of graphs.
  - Random geometric graphs: distributing n^2 nodes uniformly at random in an $n \times n$ square, and add an edge between a pair of nodes if there distance is no more than 1.
  - Preferential attachment graphs.

Convergence time

$d = 1$



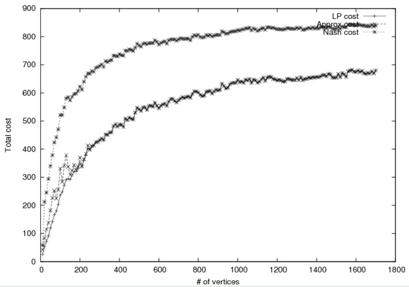
$d = \infty$



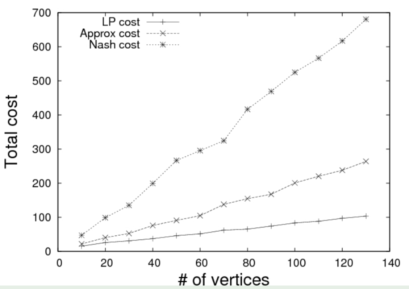
Performance of approx algorithms

Results for preferential attachment graphs.

$d = 1$



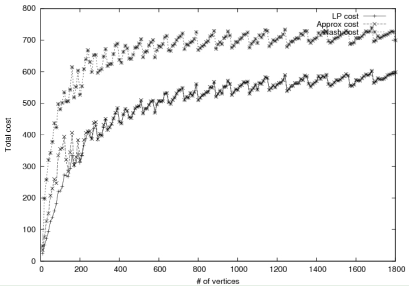
$d = \infty$



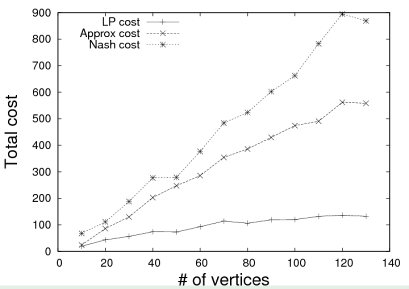
Performance of approx algorithms

Results for random geometric graphs.

$d = 1$



$d = \infty$



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Future work

- Incorporate unreliable anti-virus software/vaccination.
  - For a natural probabilistic model, we can show pure NE may not exist.
- Consider other virus/disease transmission models.
  - Again, pure NE may not exist in a probabilistic transmission model.
- In probabilistic models, approximating social optimum is challenging since even estimating the epidemic size given secure nodes is #P-hard.
- Stackelberg equilibrium: Given a budget of k secure nodes, find low-cost pure NE
- For $d = \infty$, bridge gap between upper ($O(logn)$) and lower bound ($O(1)$) on approximation ratio achievable in poly-time.