

You can turn in handwritten solutions to this part of the assignment. Please write clearly and use standard-sized (8.5 by 11in) paper. Solutions should be submitted at the beginning of class on the due date.

If you choose to typeset your solutions using LaTeX, you may find the `mathpartir.sty` package useful.

Problem 2 Below is the syntax, call-by-value operational semantics, and typing rules for the simply-typed λ -calculus with booleans.

<i>Types</i>	$\tau ::= \text{Bool} \mid \tau_1 \rightarrow \tau_2$
<i>Terms</i>	$e ::= x \mid \lambda x:\tau. e \mid e_1 e_2 \mid \text{true} \mid \text{false} \mid \text{if } e \text{ then } e_1 \text{ else } e_2$
<i>Values</i>	$v ::= \text{true} \mid \text{false} \mid \lambda x:\tau. e$
<i>Evaluation contexts</i>	$E ::= [\cdot] \mid E e \mid v E \mid \text{if } E \text{ then } e_1 \text{ else } e_2$

Evaluation rules:

$E[\lambda x:\tau. e v]$	$\longrightarrow E[e[v/x]]$	(E-BETA)
$E[\text{if true then } e_1 \text{ else } e_2]$	$\longrightarrow E[e_1]$	(E-IFTRUE)
$E[\text{if false then } e_1 \text{ else } e_2]$	$\longrightarrow E[e_2]$	(E-IFFALSE)

Typing rules:

Term environments $\Gamma ::= \cdot \mid \Gamma, x:\tau$

$\boxed{\Gamma \vdash e:\tau}$

$\frac{x:\tau \in \Gamma}{\Gamma \vdash x:\tau}$ (T-VAR)	$\frac{\Gamma, x:\tau_1 \vdash e:\tau_2}{\Gamma \vdash \lambda x:\tau_1. e:\tau_1 \rightarrow \tau_2}$ (T-LAM)	$\frac{\Gamma \vdash e_1:\tau_1 \rightarrow \tau \quad \Gamma \vdash e_2:\tau_2}{\Gamma \vdash e_1 e_2:\tau}$ (T-APP)
$\overline{\Gamma \vdash \text{true}:\text{Bool}}$ (T-TRUE)	$\overline{\Gamma \vdash \text{false}:\text{Bool}}$ (T-FALSE)	
$\frac{\Gamma \vdash e:\text{Bool} \quad \Gamma \vdash e_1:\tau \quad \Gamma \vdash e_2:\tau}{\Gamma \vdash \text{if } e \text{ then } e_1 \text{ else } e_2:\tau}$ (T-IF)		

For each of the following expressions, say if the expression is well typed or not. If it is well typed, provide a typing derivation; if it is not well typed, explain why in no more than 30 words.

- $\lambda x:\text{Bool}. \lambda y:\text{Bool}. \text{if } y \text{ then } x \text{ else } y.$
- $\lambda x:\text{Bool}. \lambda y:\text{Bool} \rightarrow \text{Bool}. \text{if } (y x) \text{ then } x \text{ else } y.$
- $\lambda x:\text{Bool}. \lambda y:\text{Bool} \rightarrow \text{Bool}. \text{if } x \text{ then } (y x) \text{ else } y.$
- $\lambda x:\text{Bool}. \lambda y:\text{Bool} \rightarrow \text{Bool}. \text{if } x \text{ then } y \text{ else } \lambda z:\text{Bool}. x.$