Control-Flow Graphs & Dataflow Analysis

CS4410: Spring 2013

Past Few Lectures:

High-level Intermediate Languages:

- Monadic Normal Form

Optimization as algebraic transformations:

- $3+4 \rightarrow 7$, $(\lambda x.e)\ v \rightarrow e[v/x]$, $\text{fst}\ (e_1,e_2) \rightarrow e_1$

Correctness issues:

- limiting ourselves to "pure" (valuable) expressions when we duplicate or eliminate.
- avoiding variable capture by keeping bound variables unique.

Today:

- Imperative Representations
  - Like MIPS assembly at the instruction level.
    - except we assume an infinite # of temps
    - and abstract away details of the calling convention
  - But with a bit more structure.
- Organized into a Control-Flow graph (ch 8)
  - nodes: labeled *basic blocks* of instructions
    - single-entry, single-exit
    - i.e., no jumps, branching, or labels inside block
  - edges: jumps/branches to basic blocks
- Dataflow analysis (ch 17)
  - computing information to answer questions about data flowing through the graph.

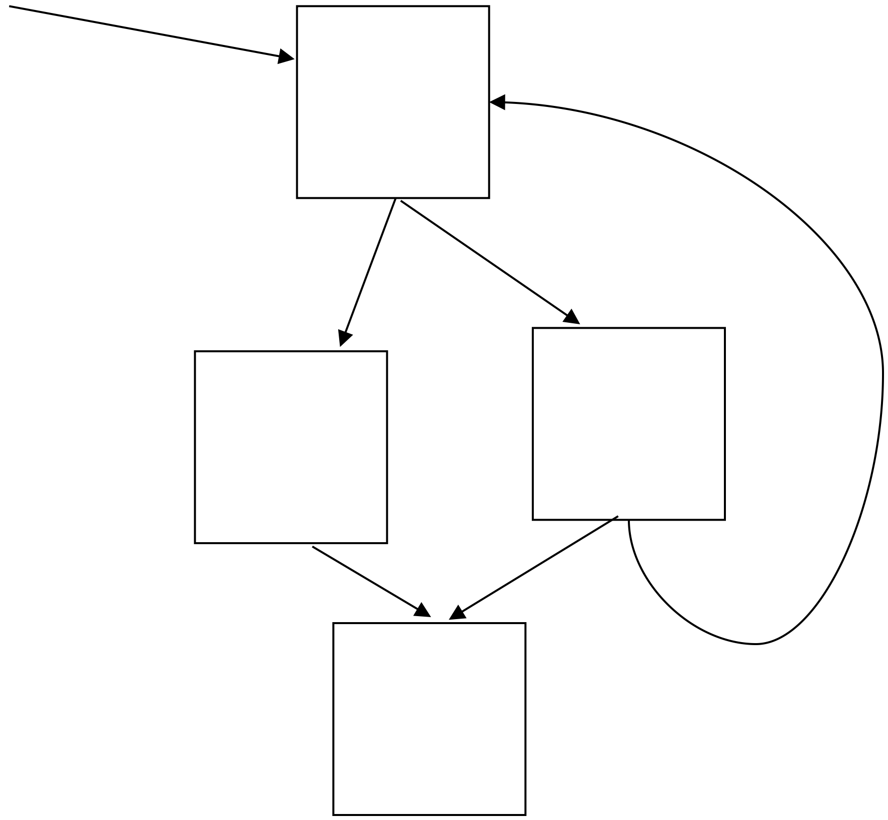
A CFG Abstract Syntax

```
Operands  w ::=  i | x | L   (* ints, vars, labels *)
Cmp-op    c ::= < | > | = | …   (* comparison *)
Blocks   B ::= return w |  jump L
              | if w1 c  w2 then L1 else L2
              | x := w; B             (* move *)
              | y := *(x + i); B   (* load *)
              | *(x + i) := y; B   (* store *)
              | x := p (w1,…,wn); B   (* arith op *)
              | x := f (w1,…,wn); B    (* call *)
```

A CFG Abstract Syntax:

```
type operand =
    | Int of int | Var of var | Label of label
type block =
| Return of operand
| Jump of label
| If of operand * cmp * operand * label * label
| Move of var * operand * block
| Load of var * operand * int * block
| Store of var * int * operand * block
| Arith of var * primop * (operand list) * block
| Call of var * operand * (operand list) * block
type proc = { vars : var list,
              prologue: label, epilogue: label,
              blocks : (label * block) list }
```

Conceptually



Differences with Monadic Form

```
datatype block =
  Return of operand
| Jump of label
| If of operand * test * operand * label * label
| Move of var * operand * block
| Load of var * operand * int * block
| Store of var * int * operand * block
| Arith of var * primop * (operand list) * block
| Call of var * operand * (operand list) * block
```

- Essentially MIPS assembly with an infinite # of registers.
- No lambdas, so easy to translate to MIPS modulo register allocation and assignment.
  - Monadic form requires extra pass to eliminate lambdas and make closures explicit. (Closure Conversion)
- Unlike Monadic Form, variables are *mutable*.

Let's Revisit Optimizations

- constant folding

  t := 3+4 → t := 7

- constant propagation

  t := 7;B; u:=t+3 → t := 7; B;u:=7+3

  – problem: B might assign a fresh value to t.

- copy propagation

  t:=u;B; v:=t+3 → t:=u;B;v:=u+3

  – problems: B might assign a fresh value to t or a fresh value to u!

More Optimizations:

- Dead code elimination

  x:=e; B; jump L → B; jump L

  – problem: the block L might use x.

  $x:=e_1;B_1;\ x:=e_2;B_2 \rightarrow B_1;x:=e2;B_2$ (x not in B_1)
- Common sub-expression elimination

  $x:=y+z;B_1;w := y+z;B_2 \rightarrow x:=y+z;B_1;w:=x;B_2$

  – problem: B_1 might change x,y, or z.

Point:

Optimization on a functional representation:

- we only had to worry about variable capture.
- we could avoid this by renaming all of the variables so that they were unique.
- then: let $x=p(v_1,\ldots,v_n)$ in e == $e[p(v_1,\ldots,v_n)/x]$

Optimization in an imperative representation:

- we have to worry about intervening updates.
  - for defined variable, similar to variable capture.
  - but we must also worry about *free* variables.
  - $x:=p(v_1,\ldots,v_n);B$ == $B[p(v_1,\ldots,v_n)/x]$ only when B doesn't modify x nor modifies any of the v_i !
- on the other hand, a graph representation makes it possible to be more precise about the *scope* of a variable.

Consider:

```
let k(x,y) = let z=x+1 in … c(z,y)
in let a = x+1 in
  if b then ... k(x,a)
  else ... k(x,a)
```

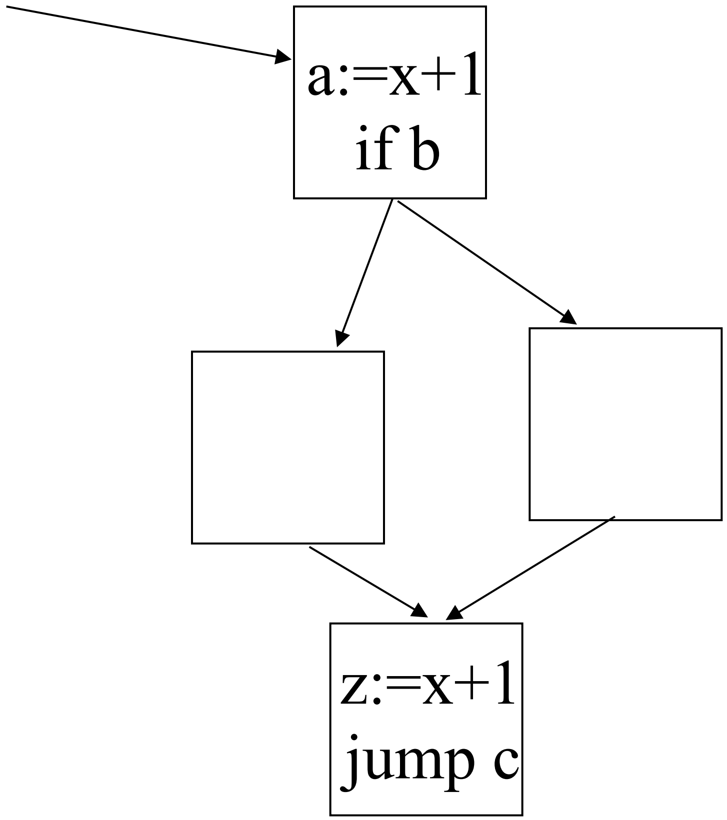
If we inline the function k, we get:

```
let a=x+1 in
  if b then … let z=x+1 in …c(z,y)
  else … let z=x+1 in …c(z,y)
```

so we can do CSE on x+1, eliminating z.

But the price paid is that we had to duplicate the function body. Can we do this *without* inlining?

In the Graph World:



Monadic terms only let you build trees, and the scoping rules follow the tree.

To localize scope, we end up copying sub-trees.

What we need is some way to accommodate "scope" across paths in a graph.

(CPS & SSA get best of both)

Constant Propagation: Try #1

```
type env = var -> operand
val init_env = fun (x:var) => Var x
val subst : env -> operand -> operand
val extend : env -> var -> operand -> env

let rec cp (env:env) (b:block) : block =
  match b with
  | Return v -> Return (subst env v)
  | Jump L -> Jump L
  | If(v1,t,v2,L1,L2) ->
      If(subst env v1,t,subst env v2,L1,L2)
  | Move(x,v,b) ->
      let v' = subst env v
      in cp (extend env x v') b
  | Arith(x,p,vs,b) ->
      Arith(x,p,map (subst env) vs, cp env b)
```

Problem:

```
L1:   x := 3;
      j L2;

L2:   return x
```

Constant Propagation: Try #2

```
let rec cp (env:env) (b:block) : block =
  match b with
   | Return v -> Return (subst env v)
   | Jump L ->
      (setblock L (cp env (getblock L));
       Jump L)
   | If(v1,t,v2,L1,L2) ->
       If(subst env v1,t,subst env v2,L1,L2)
   | Move(x,v,b) ->
       let v' = subst env v
       in cp (extend env x v') b
   | Arith(x,p,vs,b) ->
       Arith(x,p,map (subst env) vs, cp env b)
   | ...
```

Problem:

```
L1:   x := 3;
      j L2

L2:   y := x;
      j L1
```

Constant Propagation: Try #3

```
let rec cp (env:env) (b:block) : block =
  match b with
   | Return v -> Return (subst env v)
   | Jump L -> Jump L
   | If(v1,t,v2,L1,L2) ->
       If(subst env v1,t,subst env v2,L1,L2)
   | Move(x,v,b) ->
       let v' = subst v env
       in Move(x,v',cp (extend env x v') b)
   | Arith(x,p,vs,b) ->
       Arith(x,p,map (subst env) vs, cp env b)
   | ...
```

Problem

`x := 3;`	`{ x -> 3}`	`x := 3;`
`y := x+1;`		`y := 3+1;`
`x := x-1;`		`x := 3-1;`
`z := x+2;`		`z := 3+2;`

Constant Propagation: Try #4

```
let rec cp (env:env) (b:block) : block =
  match b with
   | Return v -> Return (subst env v)
   | Jump L -> Jump L
   | If(v1,t,v2,L1,L2) ->
       If(subst env v1,t,subst env v2,L1,L2)
   | Move(x,v,b) ->
       let v' = subst env v
       in Move(x,v',cp (extend env x v') b)
   | Arith(x,p,vs,b) ->
       Arith(x,p,map (subst env) vs,
             cp (extend env x (Var x)) b)
   | ...
```

Moral:

- Can't just hack this up with simple substitution.
- To extend across blocks, we have to be careful about termination.

Available Expressions:

A definition "`x := e`" reaches a program point p if there is no intervening assignment to `x` or to the free variables of `e` on any path leading from the definition to p. We say `e` is *available* at p.

If "`x:=e`" is available at p, we can use `x` in place of `e` (i.e., for common sub-expression elimination.)

How do we compute the available expressions at each program point?

Gen and Kill

- Suppose D is a set of assignments that reaches the program point p.
- Suppose p is of the form "$x := e_1$; B"
- Then the statement "$x:=e_1$"
  - *generates* the definition "$x:=e_1$", and
  - *kills* any definition "$y:= e_2$" in D such that either x=y or x is in FV(e_2).
- So the definitions that reach B are:

  D - { $y:=e_2$ | x=y or x in FV(e_2)} + {$x:=e_1$}

More Generally:

statement	gen's	kill's
$x:=v$	$x:=v$	$\{y:=e \mid x=y \text{ or } x \text{ in } e\}$
$x:=v_1\ p\ v_2$	$x:=v_1\ p\ v_2$	$\{y:=e \mid x=y \text{ or } x \text{ in } e\}$
$x:=*(v+i)$	{}	$\{y:=e \mid x=y \text{ or } x \text{ in } e\}$
$*(v+i):=x$	{}	{}
jump L	{}	{}
return v	{}	{}
if v_1 r v_2 goto L1 else goto L2	{}	{}
x := call $v(v_1,\ldots,v_n)$	{}	$\{y:=e \mid x=y \text{ or } x \text{ in } e\}$

Flowing through the Graph:

- Given the available expressions Din[L] that flow into a block labeled L, we can compute the definitions Dout[L] that flow out by just using the gen & kill's for each statement in L's block.
- For each block L, we can define:
  - succ[L] = the blocks L might jump to.
  - pred[L] = the blocks that might jump to L.
- We can then flow Dout[L] to all of the blocks in succ[L].
- They'll compute new Dout's and flow them to their successors and so on.

Algorithm Sketch:

initialize Din[L] to be the empty set.

initialize Dout[L] to be the available expressions that flow out of block L, assuming Din[L] are the set flowing in.

```
loop until no change {
  for each L:
    In := intersection(Dout[L']) for all L' in pred[L]
    if In == Din[L] then continue to next block.
    Din[L] := In.
    Dout[L] := flow Din[L] through L's block.
}
```

Termination and Speed:

- We're ensured that this will terminate because Din[L] can at worst grow to the set of all assignments in the program.
  - If Din[L] doesn't change, neither will Dout[L].
- There are a number of tricks used to speed up the analysis:
  - can calculate gen/kill for a whole block before running the algorithm.
  - can keep a work queue that holds only those blocks that have changed.

Gen/Kill Available Expressions:

statement	gen's	kills
x:=v	{x:=v}	{y:=e | x=y or x in e}
x:=p(v_1,v_2)	{x:=v_1 p v_2}	{y:=e | x=y or x in e}
x:=*(v+i)	{}	{y:=e | x=y or x in e}
*(v+i):=x	{}	{}
x := v(…)	{}	{y:=e | x=y or x in e}

Extending to Basic Blocks

Gen[B]:

- Gen[s; B] = (Gen[s] - Kill[B]) ∪ Gen[B]
- Gen[return v] = {}
- Gen[jump L] = {}
- Gen[if $r(v_1,v_2)$ then L_1 else L_2] = {}

Kill[B]:

- Kill[s; B] = Kill[s] ∪ Kill[B]
- Kill[return v] = {}
- Kill[jump L] = {}
- Kill[if $r(v_1,v_2)$ then L_1 else L_2] = {}

Equational Interpretation:

We need to solve the following equations:

- $Din[L] = Dout[L_1] \cap \ldots \cap Dout[L_n]$
  where $pred[L] = \{L_1,\ldots,L_n\}$
- $Dout[L] = (Din[L] - Kill[L]) \cup Gen[L]$

Note that for cyclic graphs, this isn't a definition, it's an equation.

- e.g., $x*x = 2y$ is not a definition for x.
- must solve for x.
- might have 0 or > 1 solution.

Solving the Equations

initialize Din[L] to be the empty set.

initialize Dout[L] to be Gen[L].

loop until no change {

  for each L:

    In := $Dout[L_1] \cap \ldots \cap Dout[L_n]$
        where $pred[L] = \{L_1, \ldots, L_n\}$

    if In == Din[L] then continue to next block.

    Din[L] := In.

    Dout[L] := (Din[L] - Kill[L]) ∪ Gen[L]

}

Recap:

Control-flow graphs:

- nodes are basic blocks
  - single-entry, single-exit sequences of code
  - statements are imperative
  - variables have no nested scope
- edges correspond to jumps/branches

Dataflow analysis:

- Example: available expressions
- Iterative solution

Next: Another dataflow analysis - Liveness

Liveness Analysis

- A variable x is *live* at a point p if there is some path from p to a use of x that does not go through a definition of x.
  - Liveness is backwards: flows from uses backwards
  - Available expressions forwards: flows from definitions.
- We would like to calculate the set of live variables coming into and out of each statement.
  - dead code: x:=e; B if x is not live coming out of B, then we can delete the assignment.
  - register allocation: if x and y are live at the same point p, then they can't share a register.

Gen & Kill for Liveness

A *use* of x generates liveness, while a definition kills it.

statement	gen's	kills
x:=y	{y}	{x}
x:=p(y,z)	{y,z}	{x}
x:=*(y+i)	{y}	{x}
*(v+i):=x	{x}	{}
$x := f(y_1,\ldots,y_n)$	$\{f,y_1,\ldots,y_n\}$	{x}

Extending to blocks:

Gen[B]:

- Gen[s; B] = (Gen[B] - Kill[s]) ∪ Gen[s]
- Gen[return x] = {x}
- Gen[jump L] = {}
- Gen[if r(x,z) then L1 else L2] = {x,z}

Kill[B]:

- Kill[s; B] = Kill[s] ∪ Kill[B]
- Kill[return v] = {}
- Kill[jump L] = {}
- Kill[if v1 r v2 then L1 else L2] = {}

Equations for graph:

We need to solve:

- LiveIn[L] = Gen[L] ∪ (LiveOut[L] - Kill[L])
- LiveOut[L] = LiveIn[L_1] ∪ … ∪ LiveIn[L_n]
  where succ[L] = {L_1,…,L_n}

So if LiveIn changes for some successor, our LiveOut changes, which then changes our LiveIn, which then propagates to our predecessors…

Liveness Algorithm

initialize LiveIn[L] := Gen[L].

initialize LiveOut[L] := { }.

loop until no change {

  for each L:

    Out := LiveIn[L_1] ∪ … ∪ LiveIn[L_n]
      where succ[L] = {L_1,…,L_n}

    if Out == LiveOut[L] then continue to next block.

    LiveOut[L] := Out.

    LiveIn[L] := Gen[L] ∪ (LiveOut[L] - Kill[L]).

}

Speeding up the Analysis

- For liveness, flow is backwards.
  - so processing successors before predecessors will avoid doing another loop.
  - of course, when there's a loop, we have to just pick a place to break the cycle.
- For available expressions, flow is forwards.
  - so processing predecessors before successors will avoid doing another loop.
- Only need to revisit blocks that change.
  - keep a priority queue, sorted by flow order

Representing Sets (See Appel)

- Consider liveness analysis:
  - need to calculate sets of variables.
  - need efficient union, subtraction.
- Usual solution uses bitsets
  - use bitwise operations (e.g., &, |, ~, etc.) to implement set operations.
  - note: this solution scales well, but has bad asymptotic complexity compared to a sparse representation.
- Complexity of whole liveness algorithm?
  - worst case, $O(n^4)$ assuming set ops are $O(n)$
  - in practice it's roughly quadratic.

Coming up…

- Register allocation [ch. 11]
  - seen first part: liveness analysis
  - next: construct interference graph
  - then: graph coloring & simplification
- Loop-oriented optimizations [ch. 18]
  - e.g., loop-invariant removal
- CPS & SSA