SETS

A set is a collection of distinct elements

$B = \{\text{France, Portugal, Andorra}\}$ = countries bordering mainland Spain.

oops... Gibraltar?

$S = \{\text{basketball, soccer}\}$ = sports that I like.

Natural numbers : $\mathbb{N} = \{0, 1, 2, 3, \ldots\}$ → Some sources: no zero.

Integers : $\mathbb{Z} = \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$

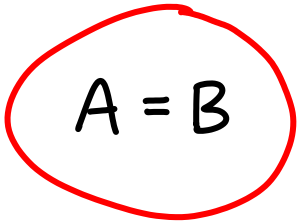
Positive integers : $\mathbb{Z}^+ = \{1, 2, 3, \ldots\}$

Rational numbers : $\mathbb{Q}$ e.g., $\frac{1}{3}, \frac{5}{7}, \frac{13}{2}, 8$ integer divided by integer

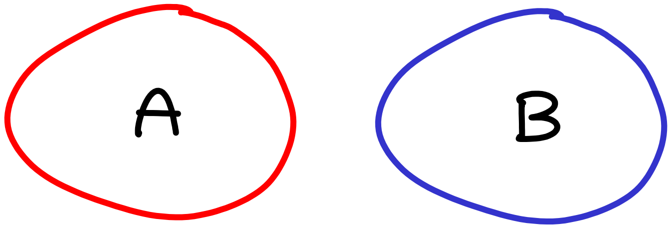
Irrational numbers e.g., $\pi, e, \sqrt{2}$

Real numbers : $\mathbb{R}$ all rational & irrational

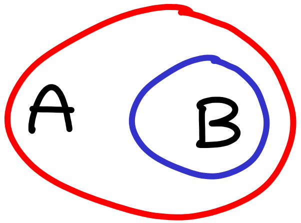
Venn diagrams



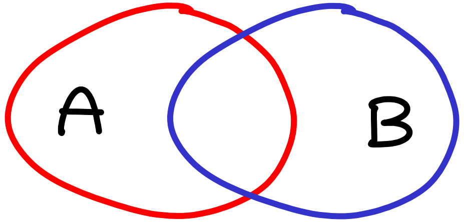
equal sets



disjoint sets



B subset of A



general picture

E : even natural numbers $= \{0, 2, 4, \dots\}$

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If S has n elements, number of subsets of $S = 2^n$.

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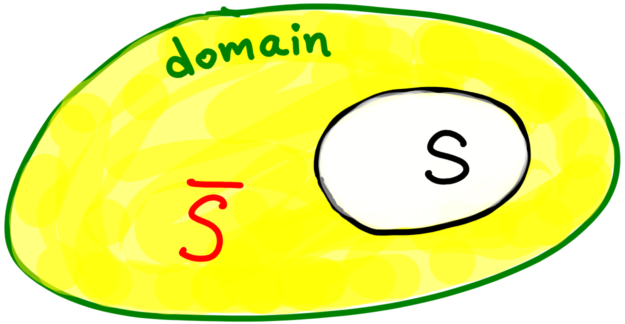

This requires context: a known domain.

Essentially S is a subset of the domain.

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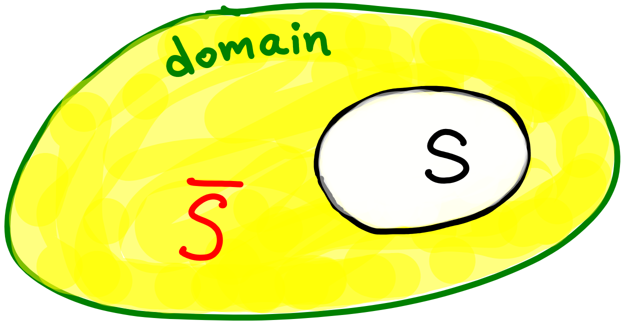

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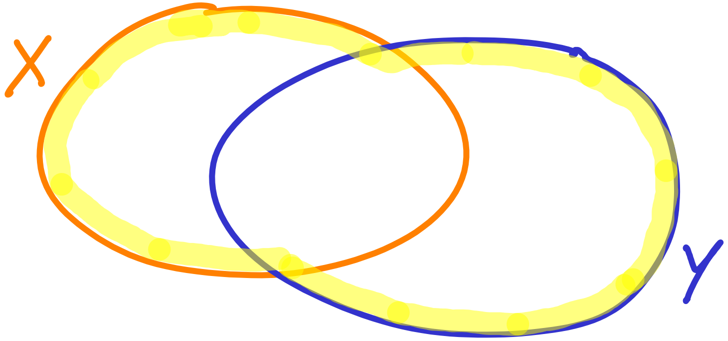
e.g., domain $= \mathbb{Z}^+$

S = powers of 2, $\overline{S} = \{3, 5, 6, 7, 9, \ldots\}$

Union of sets: include every element present.



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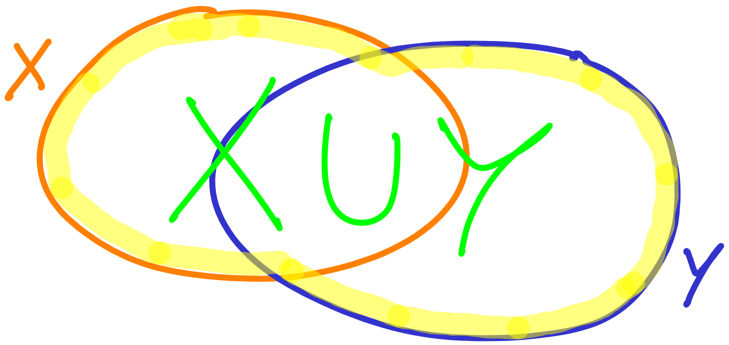

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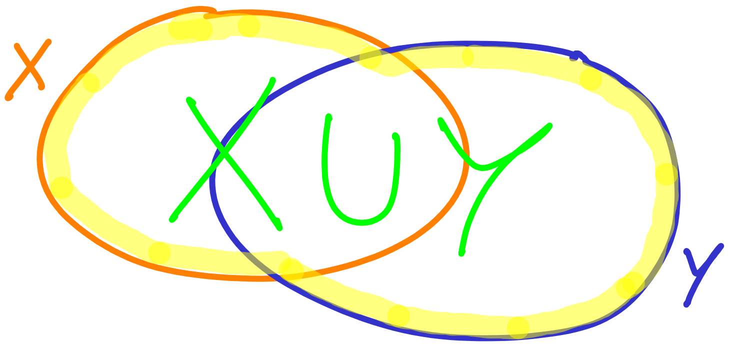

$p \in X \cup Y$ iff $p \in X$ OR $p \in Y$

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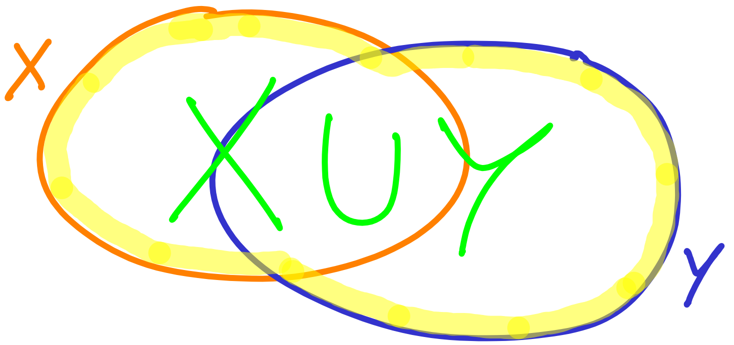
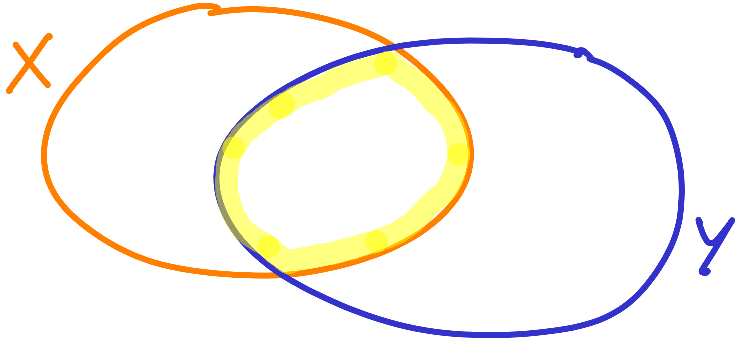


$\boxed{\text{Intersection}}$ of sets: include common elements.



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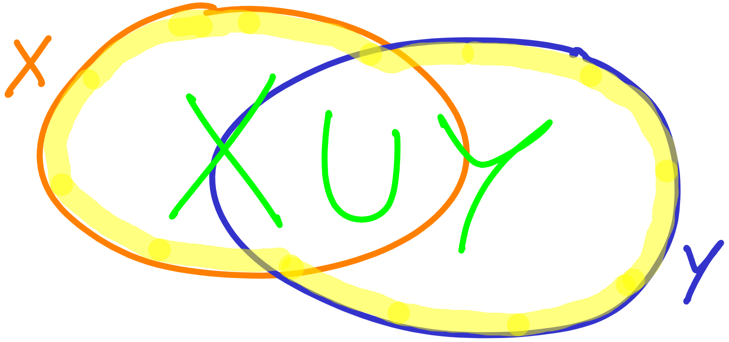

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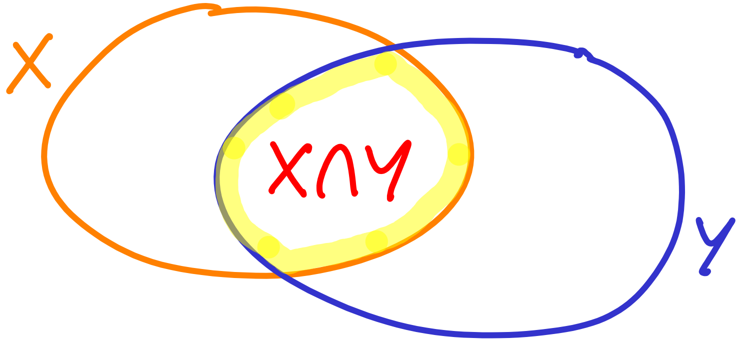
$X \cap Y = \{5\}$



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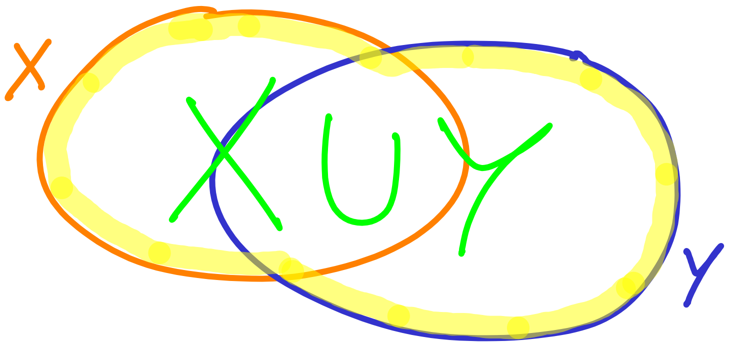

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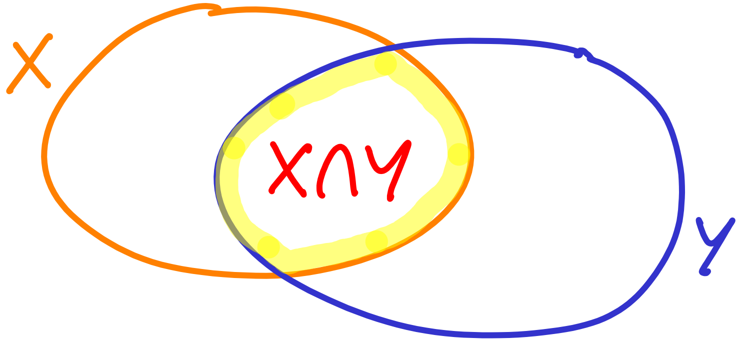
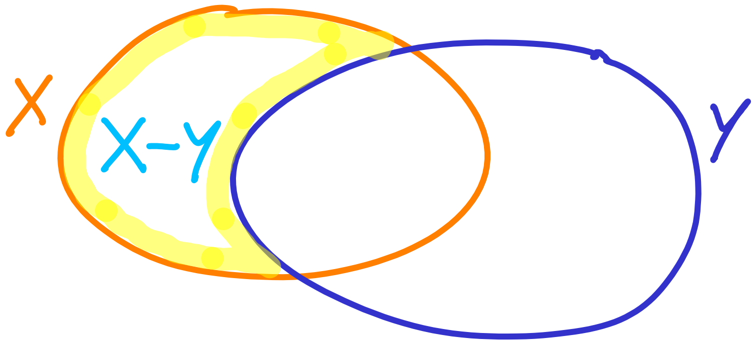


$p \in X \cap Y$ iff $p \in X$ <u>AND</u> $p \in Y$

Set difference: $X-Y$

$p \in X-Y$ iff $p \in X$ AND $p \notin Y$



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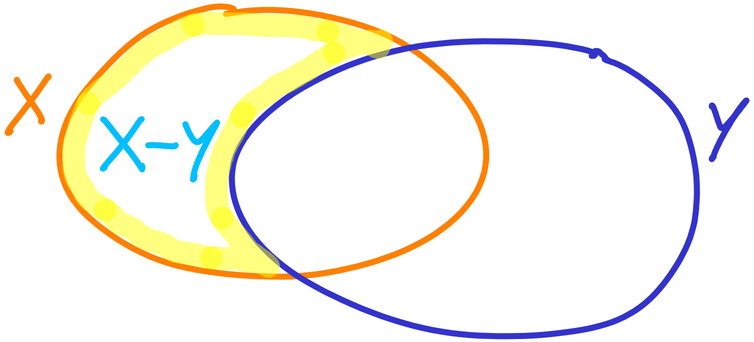
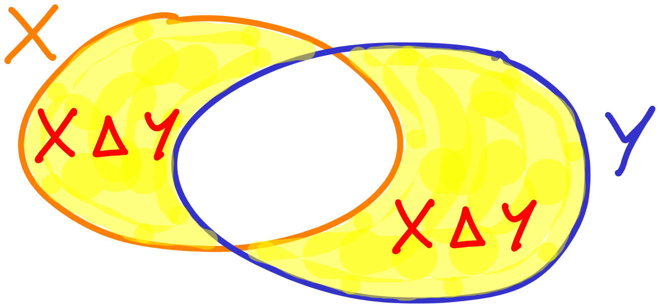
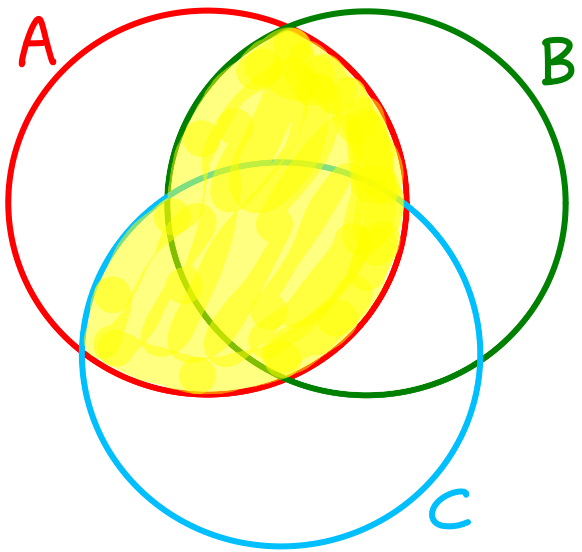


Symmetric difference: $X \Delta Y = (X-Y) \cup (Y-X)$



Distributive Laws for sets:

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$



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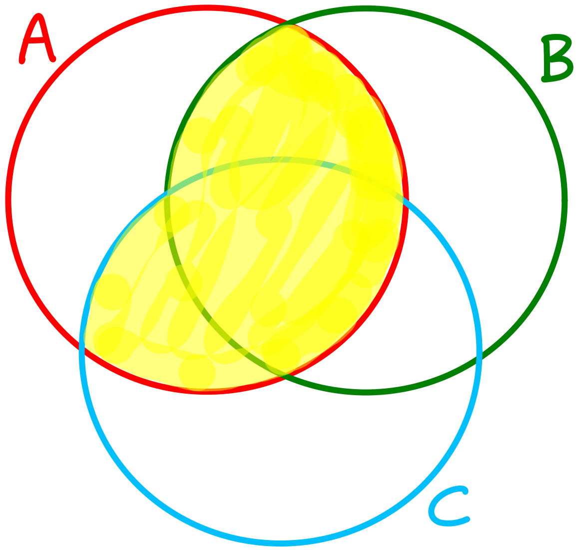

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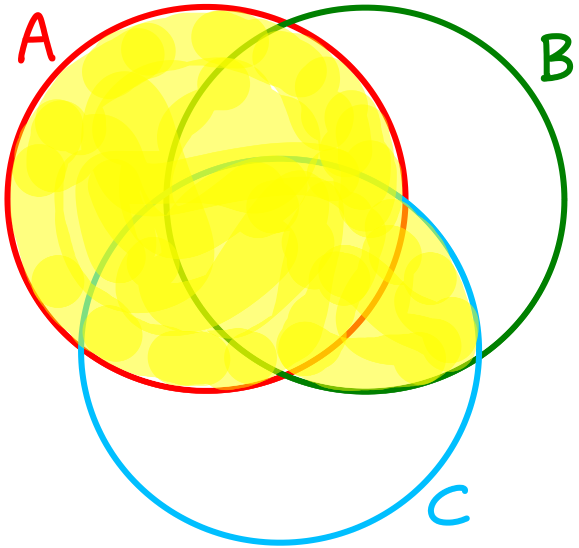

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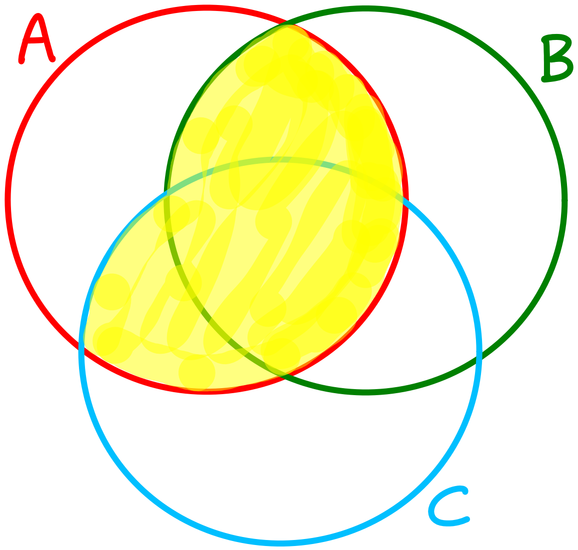

Which seems like a simpler description?

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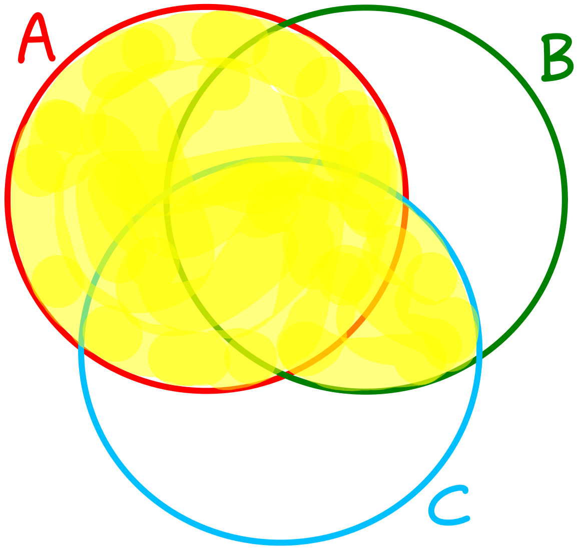
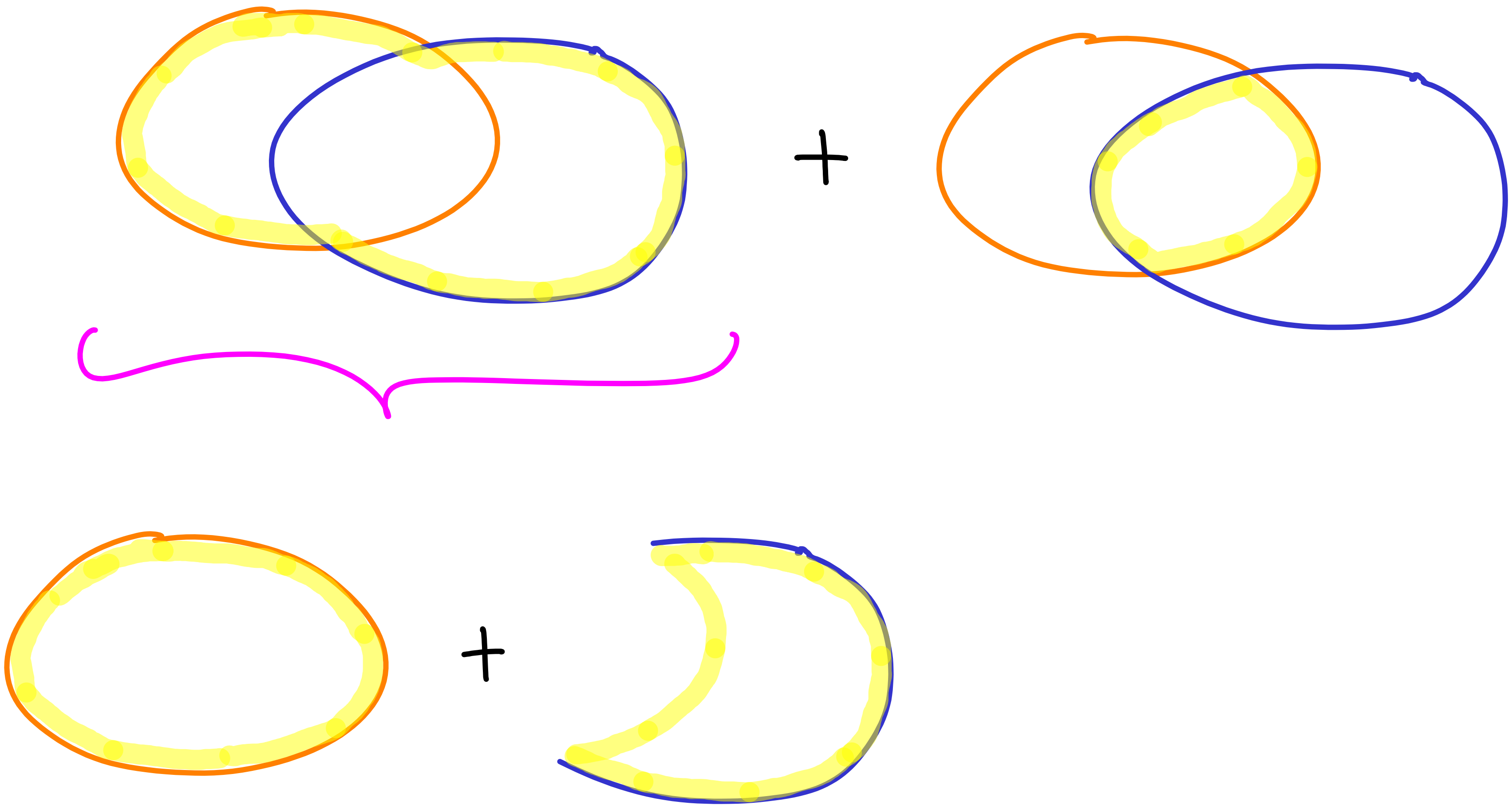


Which seems like a simpler description?

$$|A| + |B| = |A \cup B| + |A \cap B|$$



see **Inclusion-Exclusion Principle**

Set builder notation: use when description of set isn't "basic"

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$$\{5, 14, 19, 23, 28, 32, 37, 41, 46, 50, 55, 64, 69, 73, 78 \ldots\}$$

$$S = ?$$

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$$\{5, 14, 19, 23, 28, 32, 37, 41, 46, 50, 55, 64, 69, 73, 78 \ldots\}$$

$$S = \{x \in \mathbb{N} \mid \text{sum of digits in } x \text{ is a multiple of } 5\}$$

implied: "all x"

"such that" or "for which"

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$$|A \cup B| = ?$$

$$B = \{x \in \mathbb{Z} \mid 1 \leq x \leq 1000 \text{ and } 5 \mid x\}$$

$$|A| = 500$$

$$|B| = 200$$

How many integers in $\{1, \ldots, 1000\}$ are divisible by 2 or 5?

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$x \in A \cap B$ iff $10 \mid x$

$$|A \cap B| = 100$$

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$$|A| + |B| = |A \cup B| + |A \cap B| \rightarrow |A \cup B| = 600$$

De Morgan's Laws

for propositions: $\neg(A \text{ AND } B) \longleftrightarrow \neg A \text{ OR } \neg B$ (see unit on "logic")

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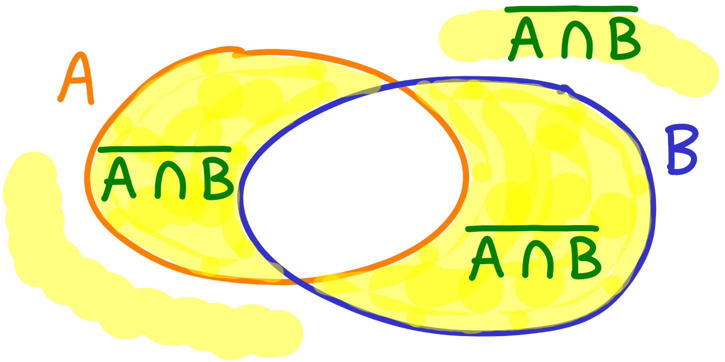

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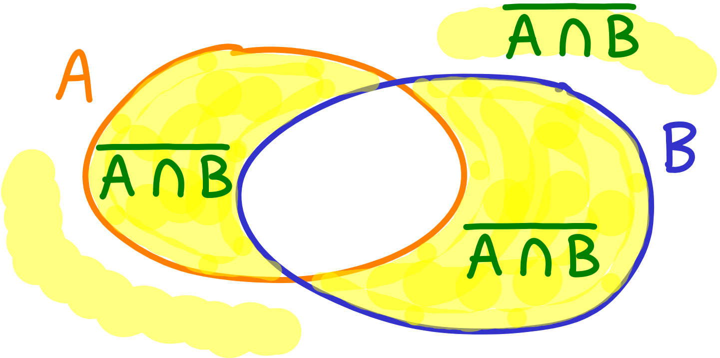

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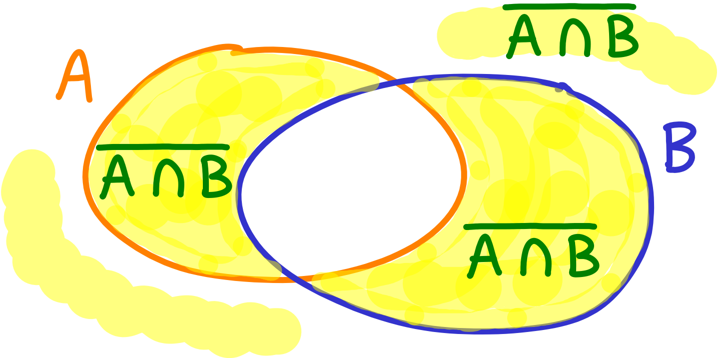

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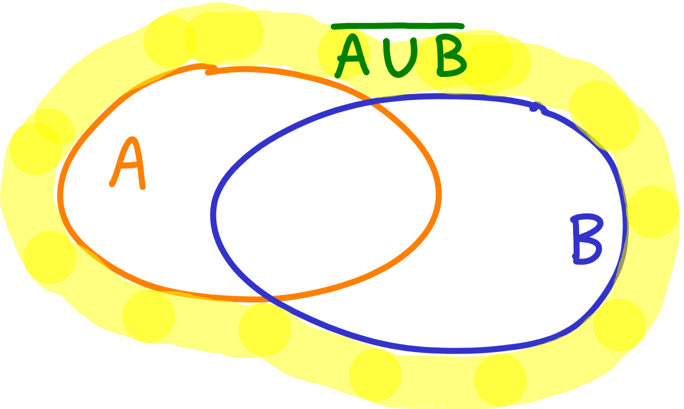

SEQUENCES

Like sets, they are a collection of elements.

2 main differences:

- repeats OK $(a, b, a, b, a, a, b, b, \ldots)$
- order matters $(a, b, c) \neq (c, b, a)$

Cartesian product of sets

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For breakfast you can have one of: $B = \{egg, banana\}$

For lunch you can have one of: $L = \{soup, salad\}$

For dinner you can have one of: $D = \{egg, steak, cake\}$

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$B \times L \times D$

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$B \times L \times D = \{(egg, soup, egg), (egg, soup, steak), (egg, soup, cake), \ldots$

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Cartesian product of sets

produces a set of sequences.

each sequence:

one element per set.

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Cartesian product of sets

produces a set of sequences

If we take the product of n copies of S, we write S^n

e.g., $\{0,1\}^3 = \{(0,0,0),(0,0,1),(0,1,0),(1,0,0),(0,1,1),(1,0,1),(1,1,0),(1,1,1)\}$

set of corners of a cube

Similar to the "useful property" mentioned earlier, about number of subsets.