

What is the probability that at least 2 people
in a group of size n have the same birthday ?

minor assumptions:

- no Feb. 29
- $P(\text{person } i \text{ born on day } j) = \frac{1}{365}$

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$n \leq 365$ otherwise by pigeonhole there must be a match

$P(\geq 2 \text{ of } n \text{ have same birthday})$

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event $m(i,j)$: person i & person j match

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event $m(i,j)$: person i & person j match

$$m(1,2) \cup m(1,3) \cup m(1,4) \cup \dots \cup m(1,n)$$

person 1 matches someone

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$P(\geq 1 \text{ of } \binom{n}{2} \text{ pairs is a birthday match})$

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$$\begin{aligned} P(& m(1,2) \cup m(1,3) \cup m(1,4) \cup \dots \cup m(1,n) \quad \dots \\ & \cup m(2,3) \cup m(2,4) \cup \dots \cup m(2,n) \quad \dots \\ & \vdots \\ & \cup m(n-3,n-1) \cup m(n-2,n-1) \dots \\ & \cup m(n-1,n)) \end{aligned}$$

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$$\rightarrow = P \bigcup_{1 \leq i < j \leq n} m(i,j)$$

$$P \left(\begin{aligned} &m(1,2) \cup m(1,3) \cup m(1,4) \cup \dots \cup m(1,n) \quad \dots \\ &\cup m(2,3) \cup m(2,4) \cup \dots \cup m(2,n) \quad \dots \\ &\vdots \\ &\cup m(n-3,n-1) \cup m(n-2,n-1) \dots \\ &\cup m(n-1,n) \end{aligned} \right)$$

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$$\rightarrow = P \bigcup_{1 \leq i < j \leq n} m(i,j)$$

$$\leq \sum_{1 \leq i < j \leq n} P(m(i,j))$$

$$\begin{aligned} P(& m(1,2) \cup m(1,3) \cup m(1,4) \cup \dots \cup m(1,n) \quad \dots \\ & \cup m(2,3) \cup m(2,4) \cup \dots \cup m(2,n) \quad \dots \\ & \vdots \\ & \cup m(n-3,n-1) \cup m(n-2,n-1) \dots \\ & \cup m(n-1,n)) \end{aligned}$$

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$$\rightarrow = P \bigcup_{1 \leq i < j \leq n} m(i,j)$$

$$P(m(1,2) \cup m(1,3) \cup m(1,4) \cup \dots \cup m(1,n) \dots \\ \cup m(2,3) \cup m(2,4) \cup \dots \cup m(2,n) \dots \\ \vdots \\ \cup m(n-3,n-1) \cup m(n-2,n-1) \dots \\ \cup m(n-1,n))$$

$$\leq \sum_{1 \leq i < j \leq n} P(m(i,j)) = \binom{n}{2} \cdot \frac{1}{365}$$

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$$\leq \sum_{1 \leq i < j \leq n} P(m(i,j)) = \binom{n}{2} \cdot \frac{1}{365}$$

accurate

$$n < 2 : P \leq 0$$

$$n = 2 : P \leq \frac{1}{365}$$

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$$\begin{aligned} &= P \bigcup_{1 \leq i < j \leq n} m(i,j) \\ &\leq \sum_{1 \leq i < j \leq n} P(m(i,j)) = \binom{n}{2} \cdot \frac{1}{365} \\ &\quad \text{not useful,} \\ &\quad \text{even as an upper bound} \end{aligned}$$

$P(m(1,2) \cup m(1,3) \cup m(1,4) \cup \dots \cup m(1,n) \dots$
 $\cup m(2,3) \cup m(2,4) \cup \dots \cup m(2,n) \dots$
 $\vdots$
 $\cup m(n-3, n-1) \cup m(n-2, n-1) \dots$
 $\cup m(n-1, n)$)

accurate
 $n < 2 : P \leq 0$
 $n = 2 : P \leq \frac{1}{365}$

$n = 10 : P \leq 12.4\%$

$n = 28 : P \leq 1.04$

$$P(\geq 2 \text{ of } n \text{ have same birthday}) = 1 - P(\text{all } n \text{ birthdays are different})$$

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event $m(i,j)$: person i & person j match

$\overline{m(i,j)}$: person i & person j don't match

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$\overline{m(i,j)}$: person i & person j don't match

$$1 - P\left(\overline{m(1,2)} \cap \overline{m(1,3)} \cap \overline{m(1,4)} \cap \dots \cap \overline{m(1,n)} \dots \right. \\
\overline{m(2,3)} \cap \overline{m(2,4)} \cap \dots \cap \overline{m(2,n)} \dots \\
\vdots \\
\left. \cap \overline{m(n-3,n-1)} \cap \overline{m(n-2,n-1)} \dots \right. \\
\left. \cap \overline{m(n-1,n)} \right)$$

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Problem: these aren't independent

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- consider people iteratively (in order)

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- event H_i : person i doesn't match previous $i-1$ people

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$$P(H_1) = 1$$

$$P(\geq 2 \text{ of } n \text{ have same birthday}) = 1 - \underbrace{P(\text{all } n \text{ birthdays are different})}_{P_n}$$

- consider people iteratively (in order)
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$$P_1 = P(H_1) = 1$$

$$P(\geq 2 \text{ of } n \text{ have same birthday}) = 1 - \underbrace{P(\text{all } n \text{ birthdays are different})}_{P_n}$$

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$$P_1 = P(H_1) = 1$$

$$P_2 = ?$$

$$P(\geq 2 \text{ of } n \text{ have same birthday}) = 1 - \underbrace{P(\text{all } n \text{ birthdays are different})}_{P_n}$$

- consider people iteratively (in order)
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$$P_1 = P(H_1) = 1$$

$$P_2 = P(H_1 \cap H_2) = ?$$

$$P(\geq 2 \text{ of } n \text{ have same birthday}) = 1 - \underbrace{P(\text{all } n \text{ birthdays are different})}_{P_n}$$

- consider people iteratively (in order)
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-

$$P_1 = P(H_1) = 1$$

$$P_2 = P(H_1 \cap H_2) = P(H_1) \cdot P(H_2 | H_1) = ?$$

$$P(\geq 2 \text{ of } n \text{ have same birthday}) = 1 - \underbrace{P(\text{all } n \text{ birthdays are different})}_{P_n}$$

- consider people iteratively (in order)
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$$P_1 = P(H_1) = 1$$

$$P_2 = P(H_1 \cap H_2) = P(H_1) \cdot \underbrace{P(H_2|H_1)}_{\frac{364}{365}} = 1 \cdot \frac{364}{365} = P(H_2)$$

$$P(\geq 2 \text{ of } n \text{ have same birthday}) = 1 - \underbrace{P(\text{all } n \text{ birthdays are different})}_{P_n}$$

- consider people iteratively (in order)
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$$P_3 = P(H_1 \cap H_2 \cap H_3) = ?$$

$$P(\geq 2 \text{ of } n \text{ have same birthday}) = 1 - \underbrace{P(\text{all } n \text{ birthdays are different})}_{P_n}$$

- consider people iteratively (in order)
 - event H_i : person i doesn't match previous $i-1$ people
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$$P_1 = P(H_1) = 1$$

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$$P_3 = P(H_1 \cap H_2 \cap H_3) = P(H_1) \cdot P(H_2 | H_1) \cdot P(H_3 | (H_2 \cap H_1)) = ?$$

$$P(\geq 2 \text{ of } n \text{ have same birthday}) = 1 - \underbrace{P(\text{all } n \text{ birthdays are different})}_{P_n}$$

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$$P_4 = P(H_1 \cap H_2 \cap H_3 \cap H_4) = ?$$

$$P(\geq 2 \text{ of } n \text{ have same birthday}) = 1 - \underbrace{P(\text{all } n \text{ birthdays are different})}_{P_n}$$

- consider people iteratively (in order)
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$$\begin{aligned} P_4 &= P(H_1 \cap H_2 \cap H_3 \cap H_4) = P(H_1) \cdot P(H_2 | H_1) \cdot P(H_3 | (H_2 \cap H_1)) \cdot P(H_4 | (H_3 \cap H_2 \cap H_1)) \\ &= 1 \cdot \frac{364}{365} \cdot \frac{363}{365} \cdot \frac{362}{365} \end{aligned}$$

$$P(\geq 2 \text{ of } n \text{ have same birthday}) = 1 - \underbrace{P(\text{all } n \text{ birthdays are different})}_{P_n}$$

$$P_4 = P(H_1 \cap H_2 \cap H_3 \cap H_4) = P(H_1) \cdot P(H_2 | H_1) \cdot P(H_3 | (H_2 \cap H_1)) \cdot P(H_4 | (H_3 \cap H_2 \cap H_1))$$

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person: 1 2 3 4 ... n

$$P_n = P(H_1 \cap \dots \cap H_n) = P\left(\bigcap_{i=1}^n H_i\right) = 1 \cdot \frac{364}{365} \cdot \frac{363}{365} \cdot \frac{362}{365} \dots \cdot \frac{365 - (n-1)}{365}$$

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$$= \frac{P(365, n)}{365^n}$$

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$$= \frac{P(365, n)}{365^n} = \frac{\frac{365!}{(365-n)!}}{365^n}$$

$$P(\geq 2 \text{ of } n \text{ have same birthday}) = 1 - \underbrace{P(\text{all } n \text{ birthdays are different})}_{P_n}$$

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$$= \frac{P(365, n)}{365^n} = \frac{\frac{365!}{(365-n)!}}{365^n} \quad \leftarrow \text{sample space (repetition, order matters)}$$

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$$P_n = P(H_1 \cap \dots \cap H_n) = P\left(\bigcap_{i=1}^n H_i\right) = 1 \cdot \frac{364}{365} \cdot \frac{363}{365} \cdot \frac{362}{365} \dots \cdot \frac{365 - (n-1)}{365}$$

$$= \frac{P(365, n)}{365^n} = \frac{\frac{365!}{(365-n)!}}{365^n}$$

$\left\{ \begin{array}{l} \text{ways to choose } n \text{ from } 365 \\ \text{(no repetition, order matters)} \end{array} \right.$
 $\leftarrow$ sample space (repetition, order matters)

$$P(\geq 2 \text{ of } n \text{ have same birthday}) = 1 - \underbrace{P(\text{all } n \text{ birthdays are different})}_{P_n}$$

$$P_4 = P(H_1 \cap H_2 \cap H_3 \cap H_4) = P(H_1) \cdot P(H_2 | H_1) \cdot P(H_3 | (H_2 \cap H_1)) \cdot P(H_4 | (H_3 \cap H_2 \cap H_1))$$

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$$1 - \frac{\frac{365!}{(365-n)!}}{365^n}$$

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$$n = 2 \rightarrow 0.27\% = \frac{1}{365}$$

$P(\geq 2 \text{ of } n \text{ have same birthday}) = 1 - P(\text{all } n \text{ birthdays are different})$

$$1 - \frac{\frac{365!}{(365-n)!}}{365^n}$$

$$n=2 \rightarrow 0.27\% = \frac{1}{365}$$

$$n=4 \rightarrow 1.64\%$$

$$n=10 \rightarrow 11.7\%$$

$$\sum_{1 \leq i < j \leq n} P(m(i,j)) = \binom{n}{2} \cdot \frac{1}{365}$$

$$n=10 : P \leq 12.4\%$$

decent bound

$P(\geq 2 \text{ of } n \text{ have same birthday}) = 1 - P(\text{all } n \text{ birthdays are different})$

$$1 - \frac{\frac{365!}{(365-n)!}}{365^n}$$

$$n=2 \rightarrow 0.27\% = \frac{1}{365}$$

$$n=4 \rightarrow 1.64\%$$

$$n=10 \rightarrow 11.7\%$$

$$n=23 \rightarrow 50.7\%$$

$$\sum_{1 \leq i < j \leq n} P(m(i,j)) = \binom{n}{2} \cdot \frac{1}{365}$$

$$n=23 : P \leq 69.3\%$$

$P(\geq 2 \text{ of } n \text{ have same birthday}) = 1 - P(\text{all } n \text{ birthdays are different})$

$$1 - \frac{\frac{365!}{(365-n)!}}{365^n}$$

$$n=2 \rightarrow 0.27\% = \frac{1}{365}$$

$$n=4 \rightarrow 1.64\%$$

$$n=10 \rightarrow 11.7\%$$

$$n=23 \rightarrow 50.7\%$$

$$n=30 \rightarrow 70.6\%$$

$$n=70 \rightarrow 99.9\%$$

$P(\geq 2 \text{ of } n \text{ have same birthday}) = 1 - P(\text{all } n \text{ birthdays are different})$

$$1 - \frac{\frac{365!}{(365-n)!}}{365^n}$$

$$n \sim 116 \rightarrow 1 - \frac{1}{10^9}$$

$$n = 2 \rightarrow 0.27\% = \frac{1}{365}$$

$$n = 4 \rightarrow 1.64\%$$

$$n = 10 \rightarrow 11.7\%$$

$$n = 23 \rightarrow 50.7\%$$

$$n = 30 \rightarrow 70.6\%$$

$$n = 70 \rightarrow 99.9\%$$

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$$n = 30 \rightarrow 70.6\%$$

$$n = 70 \rightarrow 99.9\%$$

$$n \sim 127 \rightarrow 1 - \frac{1}{10^{11}}$$

{ estimated number of stars
in our galaxy

$P(\geq 2 \text{ of } n \text{ have same birthday}) = 1 - P(\text{all } n \text{ birthdays are different})$

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$$n = 30 \rightarrow 70.6\%$$

$$n = 70 \rightarrow 99.9\%$$

$$n \sim 127 \rightarrow 1 - \frac{1}{10^{11}}$$

{ estimated number of stars
in our galaxy

$$n \sim 300 \rightarrow 1 - \frac{1}{10^{80}}$$

{ estimated number of atoms
in the universe

$$P(\geq 2 \text{ of } n \text{ have same birthday}) = 1 - P(\text{all } n \text{ birthdays are different})$$

$$1 - \frac{\frac{365!}{(365-n)!}}{365^n}$$

$$n \sim 116 \rightarrow 1 - \frac{1}{10^9}$$

$$n = 2 \rightarrow 0.27\% = \frac{1}{365}$$

$$n \sim 127 \rightarrow 1 - \frac{1}{10^{11}}$$

{ estimated number of stars
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$$n = 4 \rightarrow 1.64\%$$

$$n = 10 \rightarrow 11.7\%$$

$$n = 23 \rightarrow 50.7\%$$

$$n \sim 300 \rightarrow 1 - \frac{1}{10^{80}}$$

{ estimated number of atoms
in the universe

$$n = 30 \rightarrow 70.6\%$$

$$n = 70 \rightarrow 99.9\%$$

$$n > 365 \rightarrow 100\%$$

TESTING FOR A DISEASE

TESTING FOR A DISEASE

- suppose 1% of the population has a disease.

TESTING FOR A DISEASE

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D : have disease

$\bar{D}$: don't have disease

Reality :

1%

$P(D)$

99%

$P(\bar{D})$

D : have disease

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Reality :	1% $P(D)$	99% $P(\bar{D})$
Test ☹		
Test ☺		

D : have disease

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Reality :	1% $P(D)$	99% $P(\bar{D})$
Test ☹	98%	4%
Test ☺	2%	96%

D : have disease

$\bar{D}$: don't have disease

Reality :	1% $P(D)$	99% $P(\bar{D})$
Test ☹	98% ?	4% ?
Test ☺	2% ?	96% ?

Using notation?



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$P(D | \text{☹}) = ?$

all people

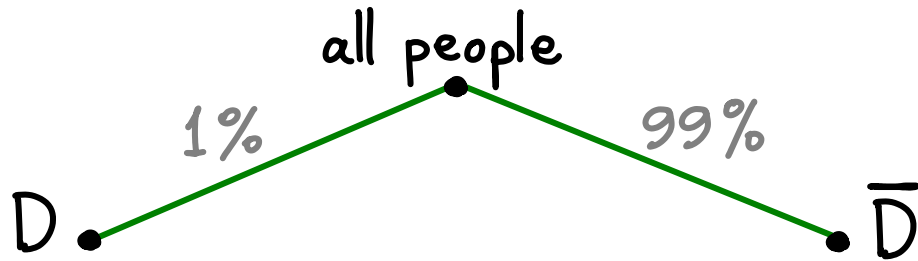
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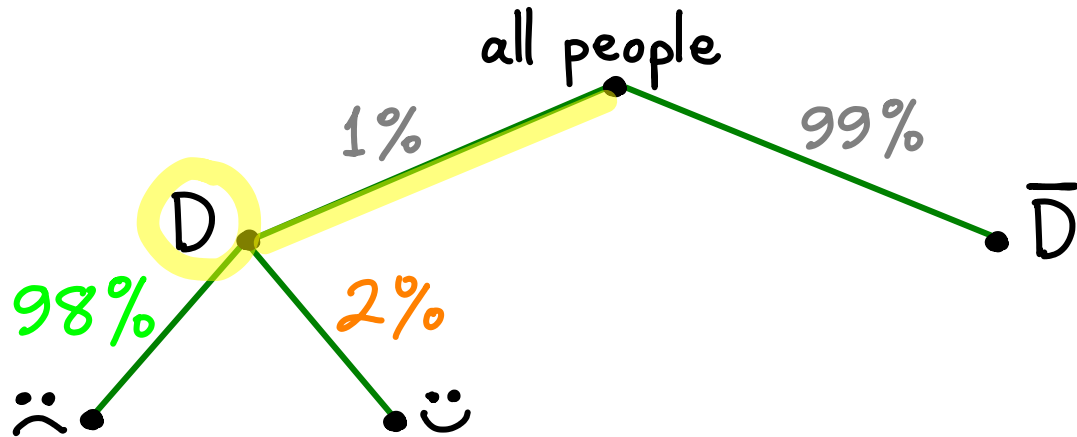
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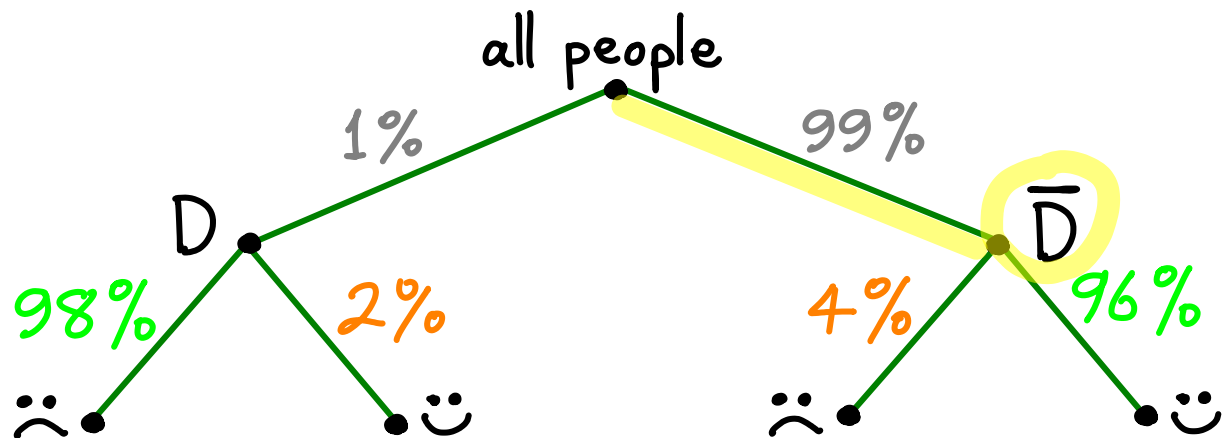
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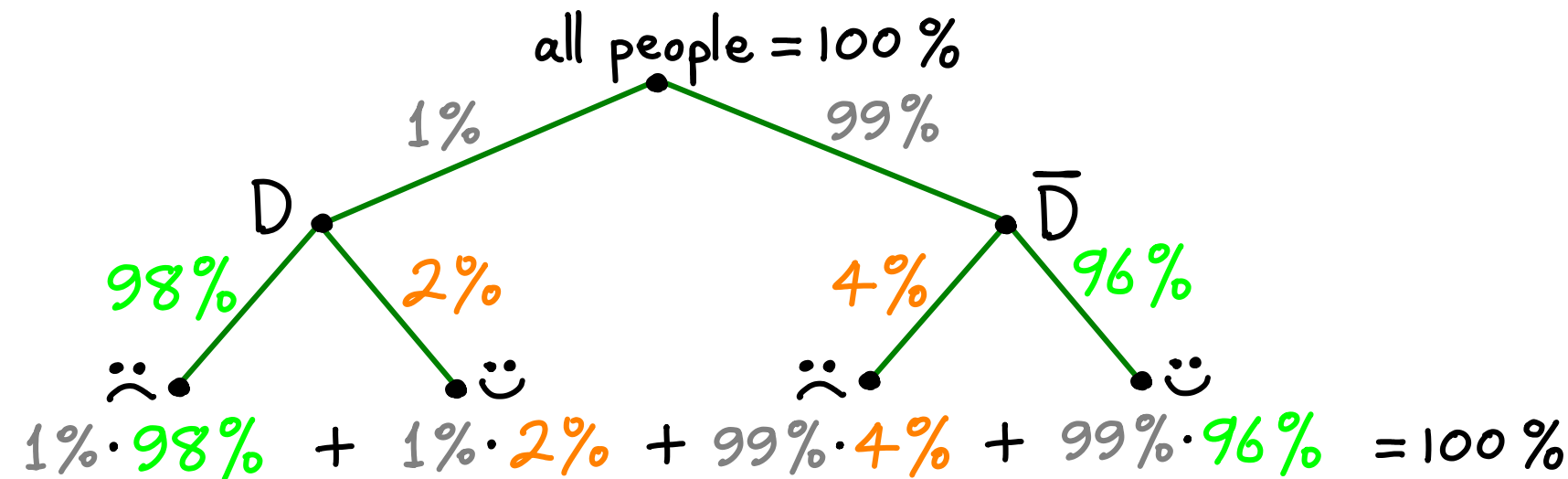
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D : have disease

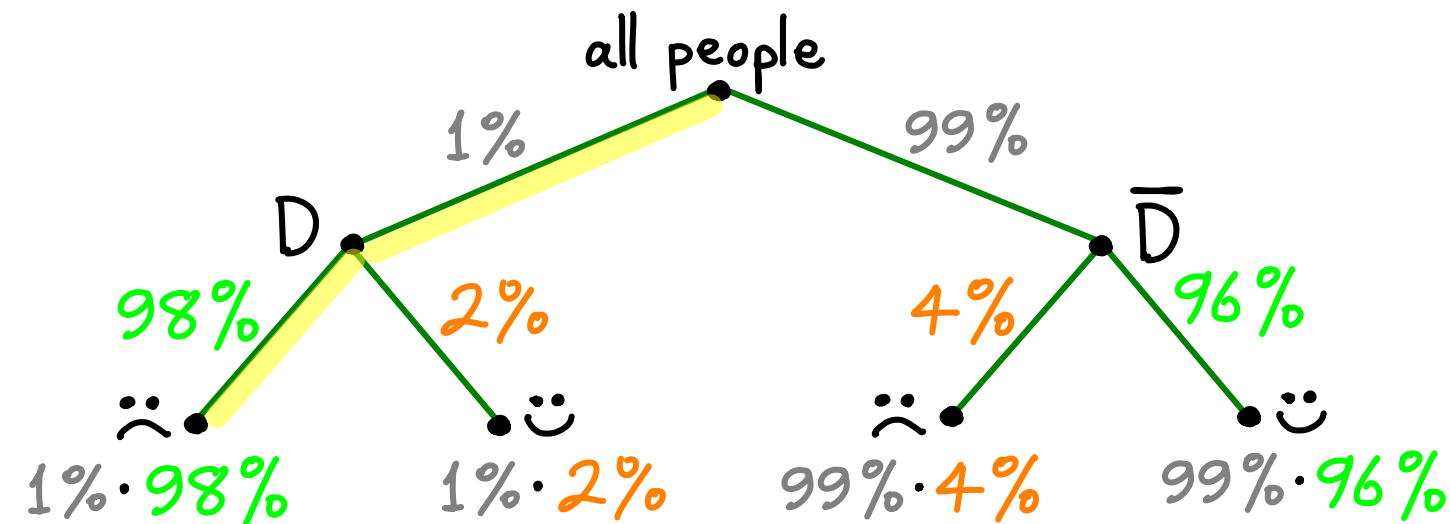
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$P(D | \text{☹}) = ?$

$1\% \cdot 98\%$



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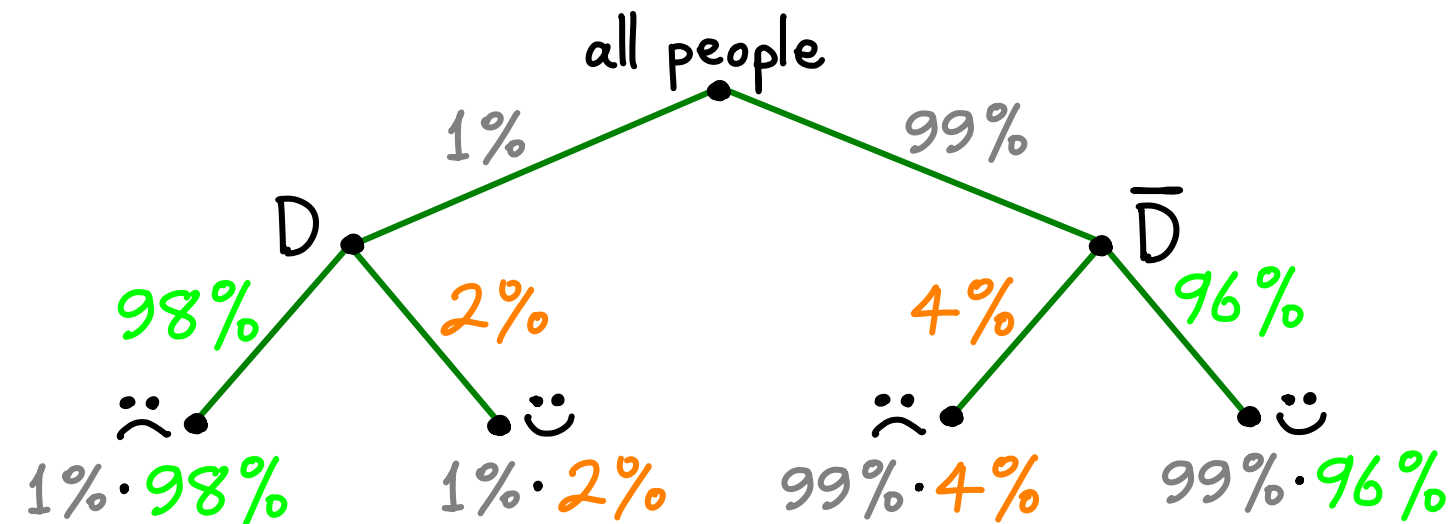
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$P(D | \text{☹}) = ?$

~~$P(D | \text{☹}) = ?$~~

$1\% \cdot 98\% = P(\text{☹} \cap D)$



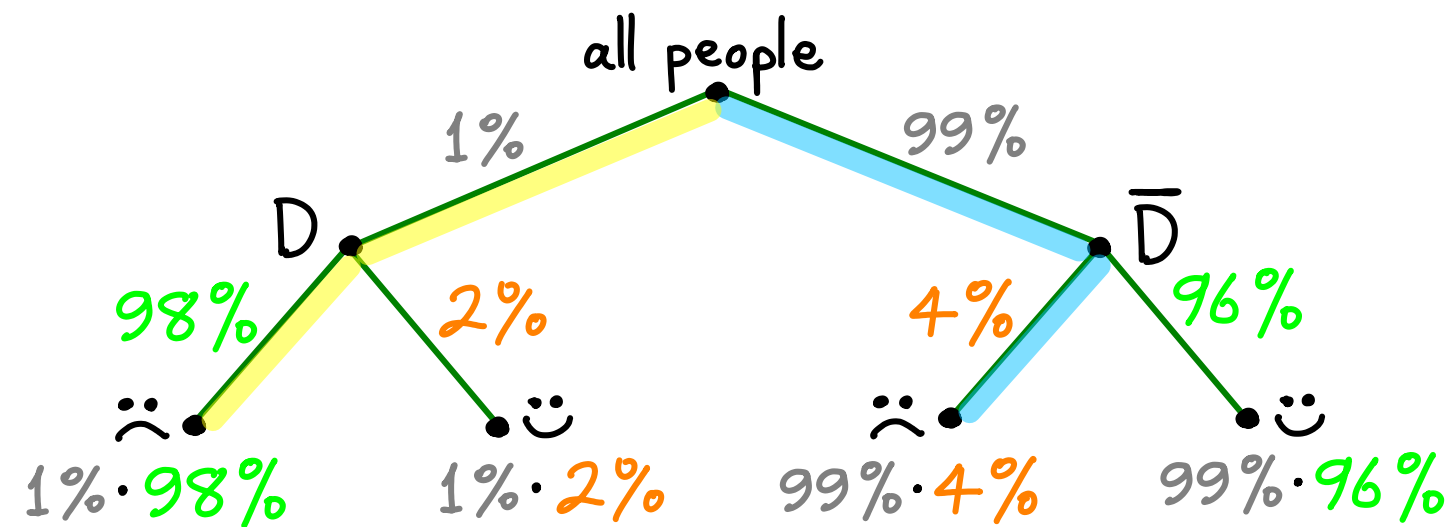
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$$P(D | \text{☹}) = ?$$



● $1\% \cdot 98\% = P(\text{☹} \cap D)$

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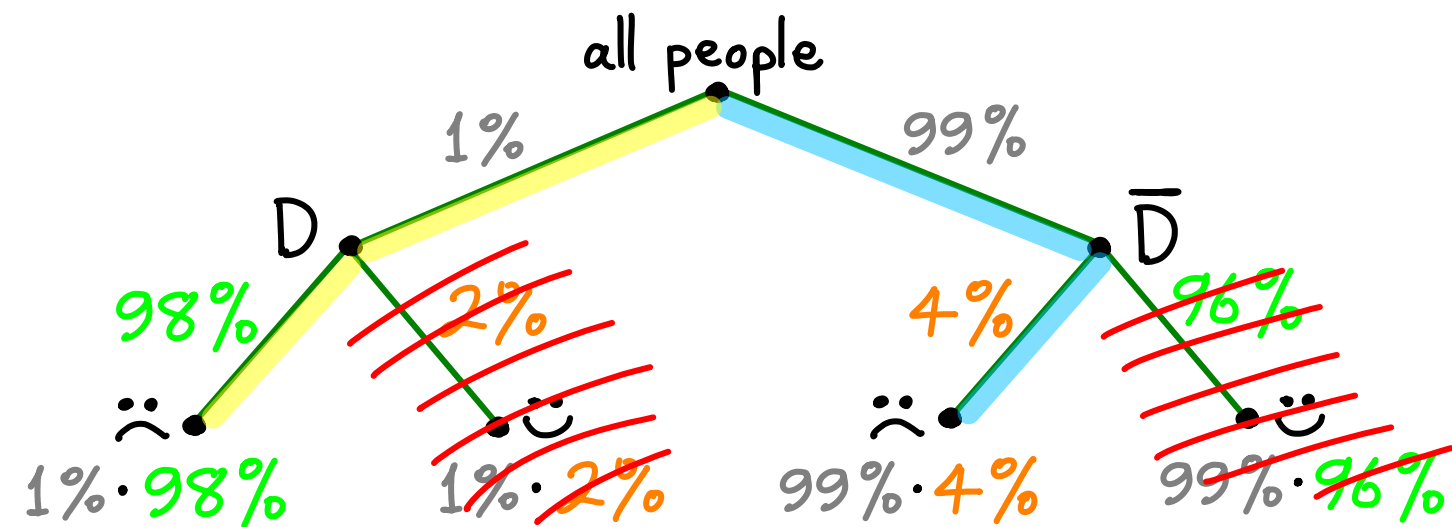
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$P(D | \text{☹}) = ?$
universe shrank



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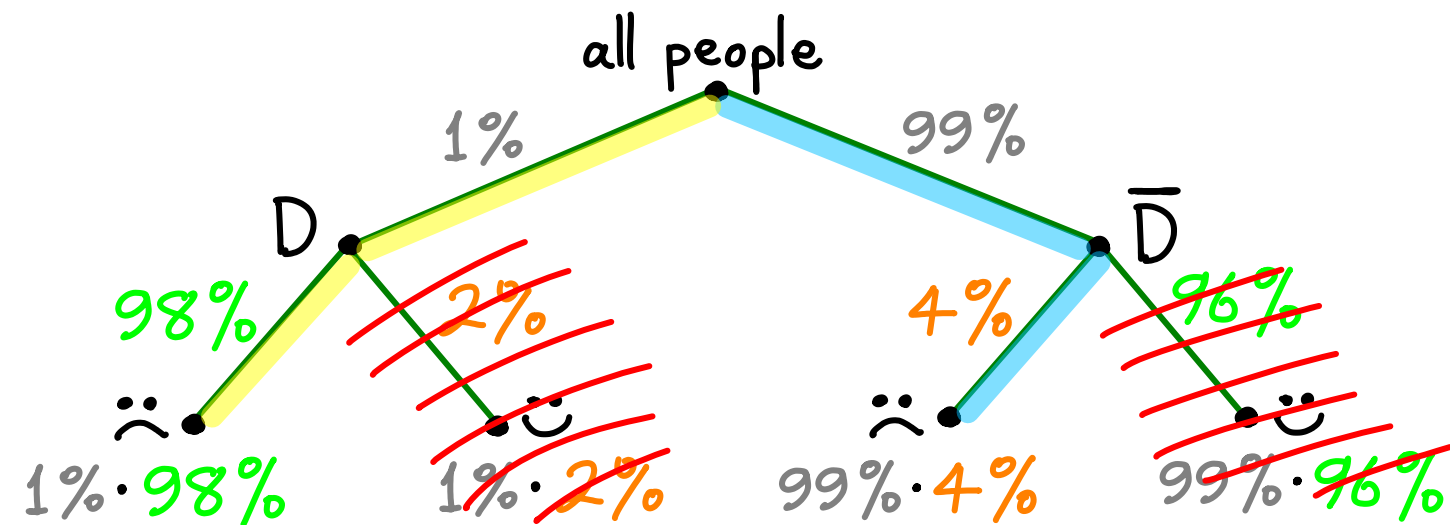
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$P(D | \text{☹}) = ?$
universe shrank



$$1\% \cdot 98\% = P(\text{☹} \cap D)$$
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$$\frac{|\text{new valid outcomes}|}{|\text{new sample space}|}$$

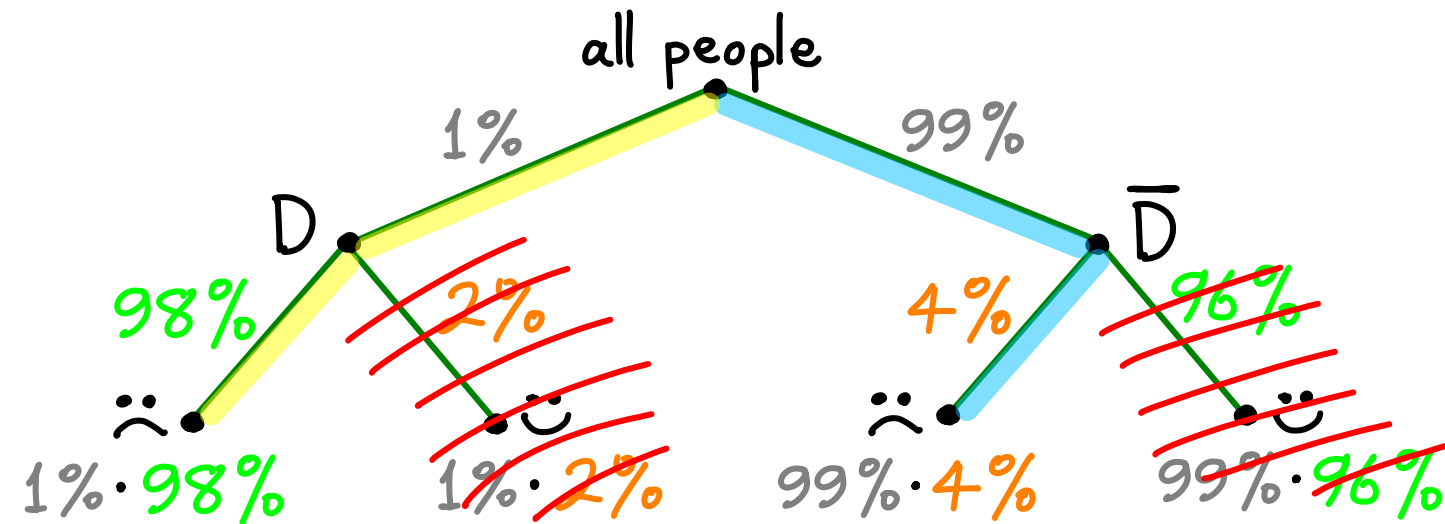
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$P(D | \text{☹}) = ?$
universe shrank



$$1\% \cdot 98\% = P(\text{☹} \cap D)$$
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$\frac{ \text{new valid outcomes} }{1\% \cdot 98\% + 99\% \cdot 4\%}$

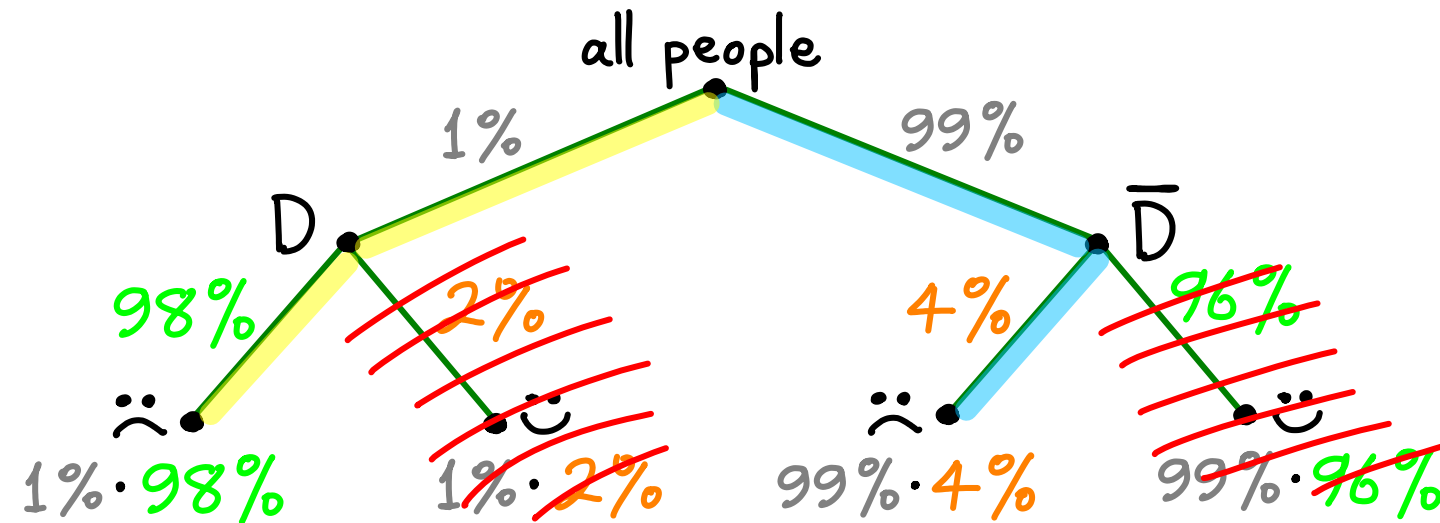
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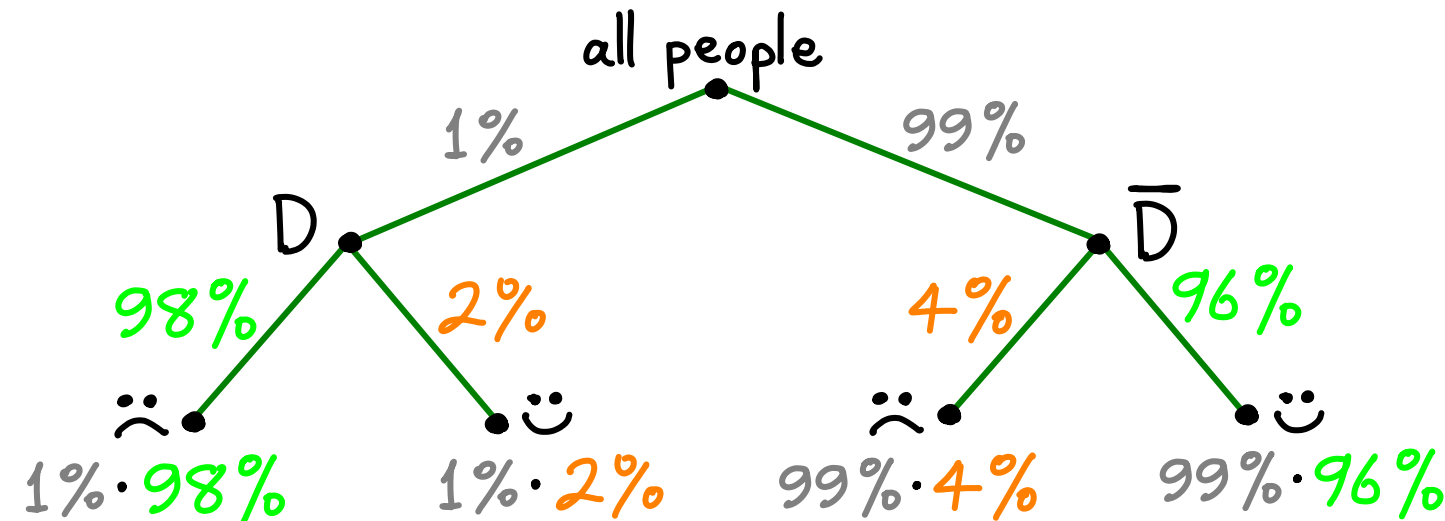
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$$\frac{1\% \cdot 98\%}{1\% \cdot 98\% + 99\% \cdot 4\%}$$

$$\begin{aligned}P(A \cap B) &= P(A|B) \cdot P(B) \\&= P(B|A) \cdot P(A)\end{aligned}$$

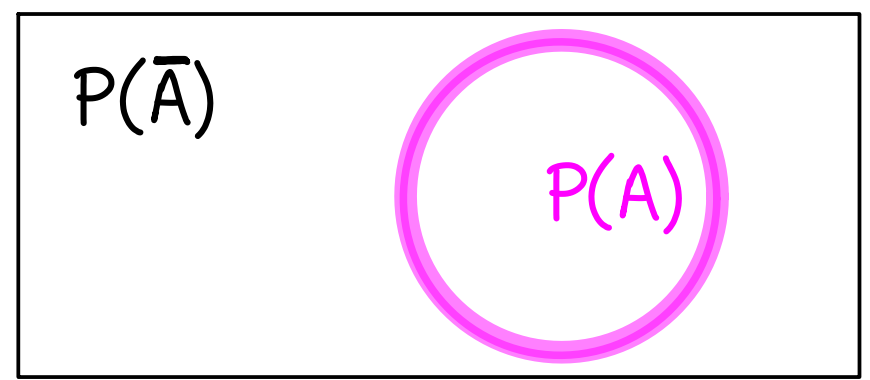
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$$P(A|B) = P(B|A) \cdot \frac{P(A)}{P(B)}$$

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↪

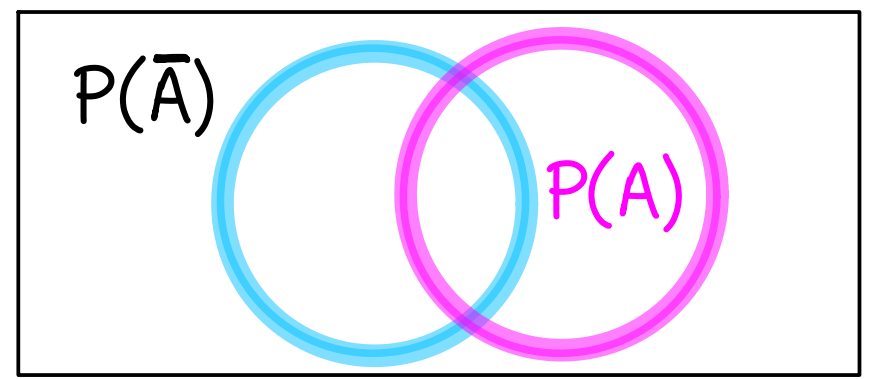
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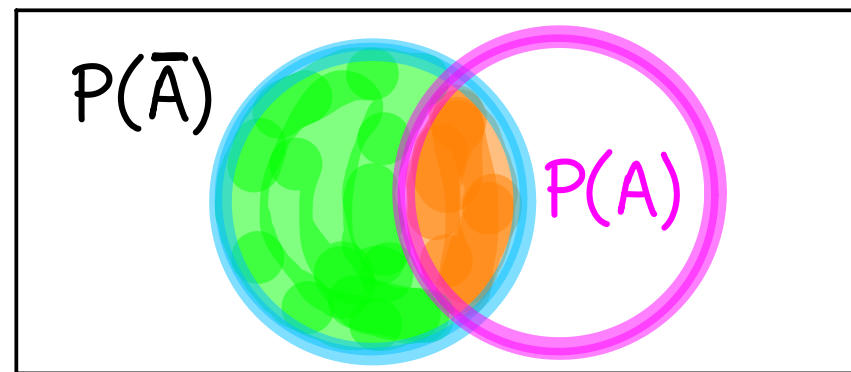


$$P(A \cap B) = P(A|B) \cdot P(B)$$

$$= P(B|A) \cdot P(A)$$

↪

$$P(A|B) = P(B|A) \cdot \frac{P(A)}{P(B)}$$

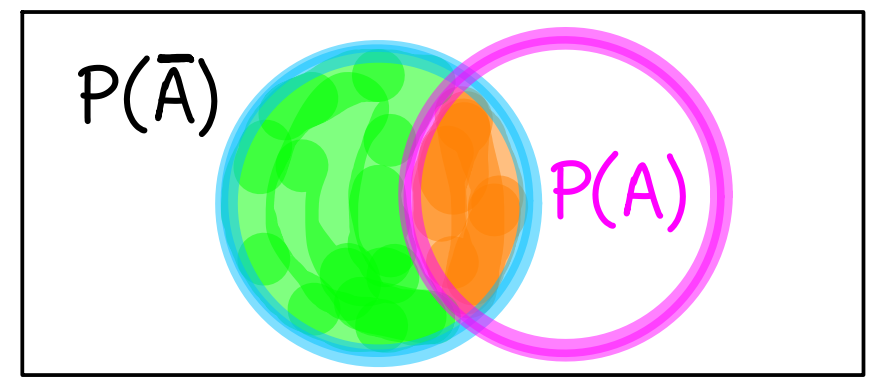


$$\left\{ \begin{array}{l} P(A \cap B) \\ + \\ P(\bar{A} \cap B) \end{array} \right.$$

$$\begin{aligned}\underline{P(A \cap B)} &= P(A|B) \cdot P(B) \\ &= \underline{P(B|A) \cdot P(A)}\end{aligned}$$

↪

$$P(A|B) = P(B|A) \cdot \frac{P(A)}{P(B)}$$



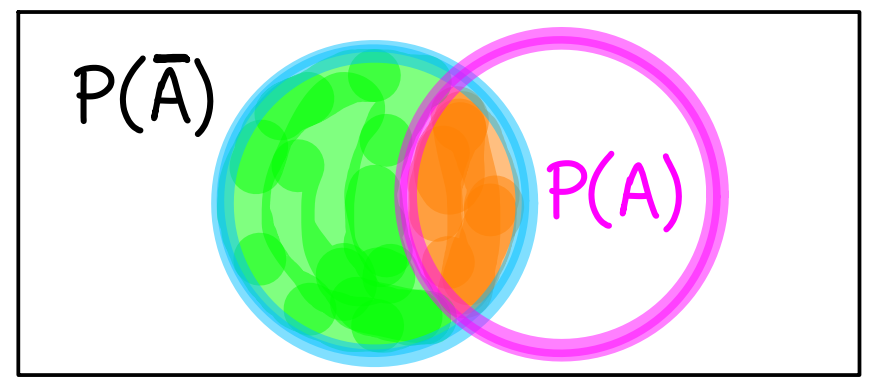
$$P(B) = \frac{P(B|A) \cdot P(A) = \textcolor{brown}{P(A \cap B)}}{+ \textcolor{green}{P(\bar{A} \cap B)}}$$

$$P(A \cap B) = P(A|B) \cdot P(B)$$

$$= P(B|A) \cdot P(A)$$

↪

$$P(A|B) = P(B|A) \cdot \frac{P(A)}{P(B)}$$



$$P(B|A) \cdot P(A) = P(A \cap B)$$

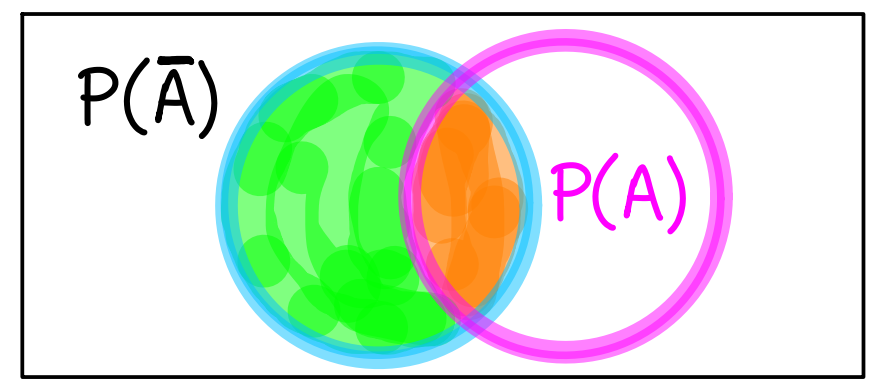
$$+$$

$$P(B|\bar{A}) \cdot P(\bar{A}) = P(\bar{A} \cap B)$$

$$\begin{aligned}P(A \cap B) &= P(A|B) \cdot P(B) \\&= P(B|A) \cdot P(A)\end{aligned}$$

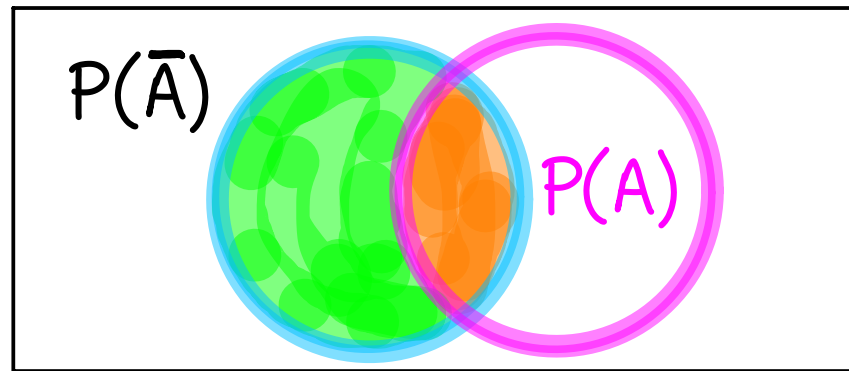


$$P(A|B) = P(B|A) \cdot \frac{P(A)}{P(B)}$$



$$P(B) = \begin{cases} P(B|A) \cdot P(A) = P(A \cap B) \\ + \\ P(B|\bar{A}) \cdot P(\bar{A}) = P(\bar{A} \cap B) \end{cases}$$

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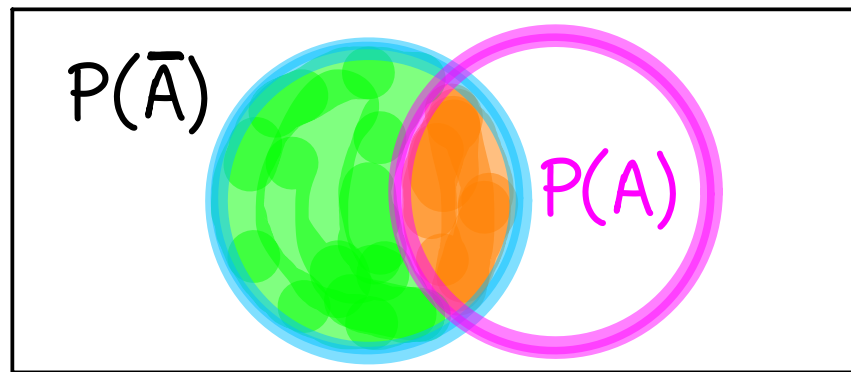
↪

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$$P(A|B) = \frac{P(B|A) \cdot P(A)}{P(B|A) \cdot P(A) + P(B|\bar{A}) \cdot P(\bar{A})}$$

$$P(A \cap B) = P(A|B) \cdot P(B) \\ = P(B|A) \cdot P(A)$$



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BAYES' THEOREM

$$P(A|B) = \frac{P(B|A) \cdot P(A)}{P(B|A) \cdot P(A) + P(B|\bar{A}) \cdot P(\bar{A})}$$

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D: have disease

Reality:	1% $P(D)$	99% $P(\bar{D})$
Test ☹	98% $P(\text{☹} D)$	4% $P(\text{☹} \bar{D})$
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$P(D | \text{☹}) = ?$

$$P(A|B) = \frac{P(B|A) \cdot P(A)}{P(B|A) \cdot P(A) + P(B|\bar{A}) \cdot P(\bar{A})}$$

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D: have disease

Reality:	1% $P(D)$	99% $P(\bar{D})$	} $P(D \text{?}) = ?$
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Test ☺	2% $P(\text{☺} D)$	96% $P(\text{☺} \bar{D})$	

$$P(D|\text{☹}) = \frac{P(\text{☹}|D) \cdot P(D)}{P(\text{☹}|D) \cdot P(D) + P(\text{☹}|\bar{D}) \cdot P(\bar{D})} = \frac{98\% \cdot 1\%}{98\% \cdot 1\% + 4\% \cdot 99\%}$$

$$P(A|B) = \frac{P(B|A) \cdot P(A)}{P(B|A) \cdot P(A) + P(B|\bar{A}) \cdot P(\bar{A})}$$

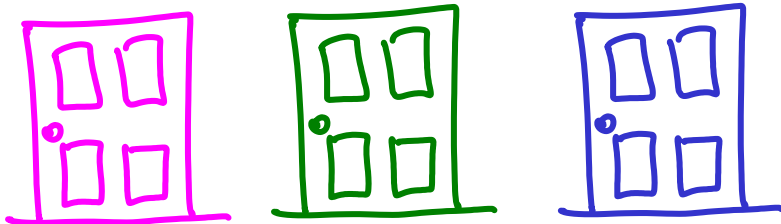
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$$P(D|\text{☹}) = \frac{P(\text{☹}|D) \cdot P(D)}{P(\text{☹}|D) \cdot P(D) + P(\text{☹}|\bar{D}) \cdot P(\bar{D})} = \frac{98\% \cdot 1\%}{98\% \cdot 1\% + 4\% \cdot 99\%} \sim \underline{\underline{19.8\%}}$$

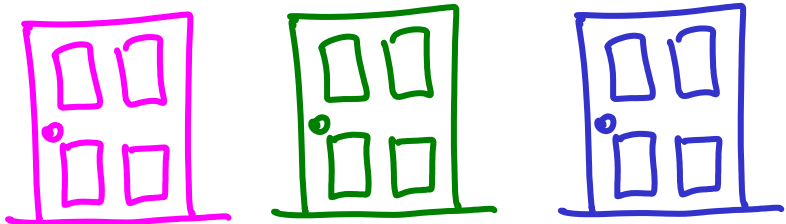
The Monty Hall problem

The Monty Hall problem



{ 1 of these 3 doors hides a car.
The other 2 hide goats.

The Monty Hall problem

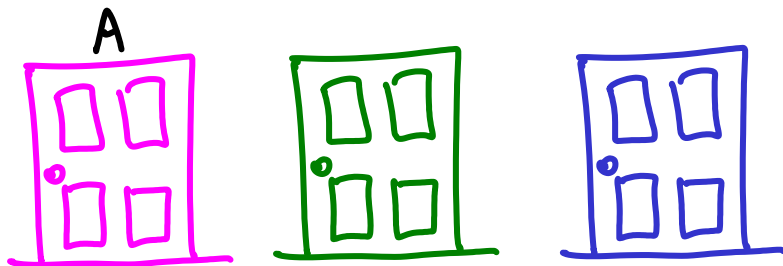


{ 1 of these 3 doors hides a car.
The other 2 hide goats.

You get to choose a door and keep what's behind.



The Monty Hall problem



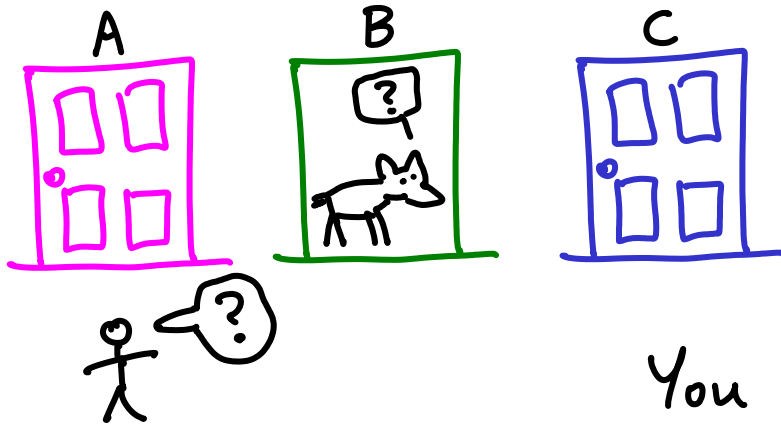
{ 1 of these 3 doors hides a car.
The other 2 hide goats.



You get to choose a door and keep what's behind.

Without loss of generality $\rightarrow$ you choose door A.

The Monty Hall problem

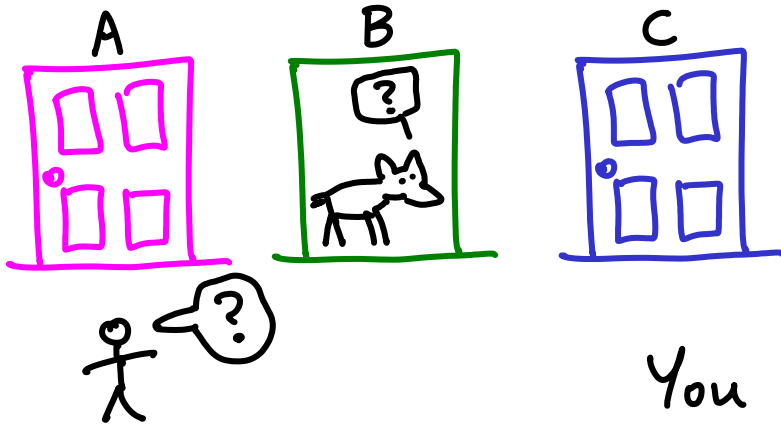


{ 1 of these 3 doors hides a car.
The other 2 hide goats.

You get to choose a door and keep what's behind.

Without loss of generality → 1) you choose door A.
→ 2) door B is opened, revealing a goat.

The Monty Hall problem



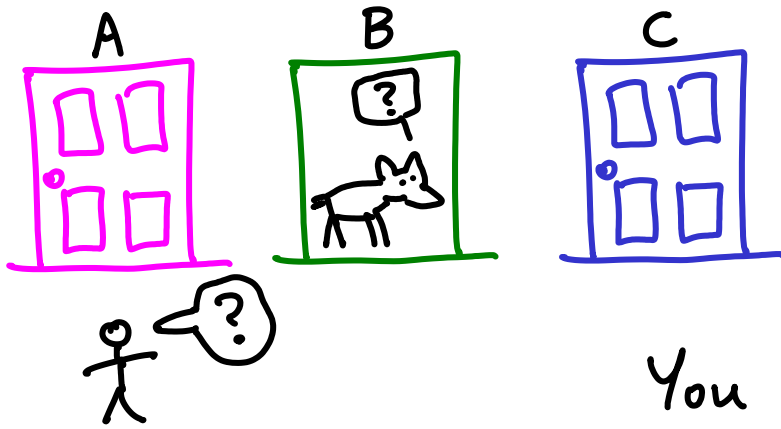
{ 1 of these 3 doors hides a car.
The other 2 hide goats.

You get to choose a door and keep what's behind.

Without loss of generality → 1) you choose door **A**.

just some door with a goat → 2) door **B** is opened, revealing a goat.

The Monty Hall problem



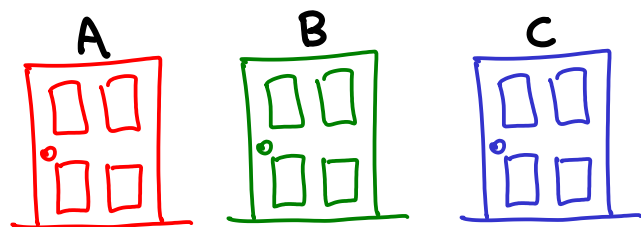
{ 1 of these 3 doors hides a car.
The other 2 hide goats.

You get to choose a door and keep what's behind.

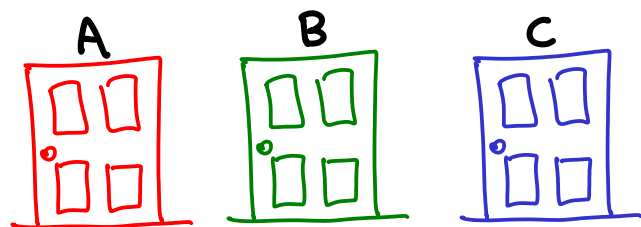
Without loss of generality → 1) you choose door A.

just some door with a goat → 2) door B is opened, revealing a goat.

You're given a choice:
→ keep A
OR
→ switch to C ?



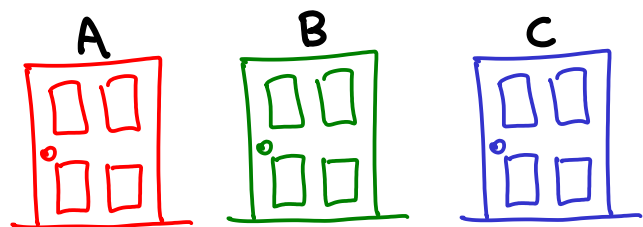
- event i : car is at door i



- event i : car is at door i

$$P(A) = P(B) = P(C) = \frac{1}{3}$$

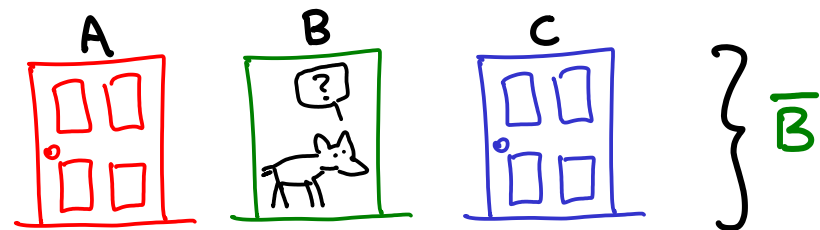
$$P(\bar{A}) = P(\bar{B}) = P(\bar{C}) = \frac{2}{3}$$

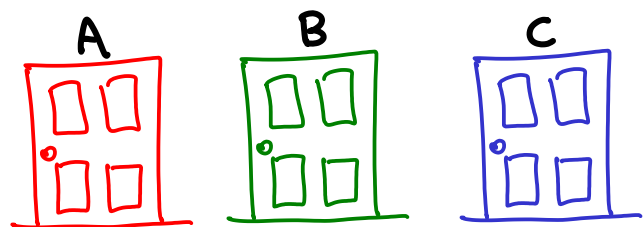


- event i : car is at door i

$$P(A) = P(B) = P(C) = \frac{1}{3}$$

$$P(\bar{A}) = P(\bar{B}) = P(\bar{C}) = \frac{2}{3}$$

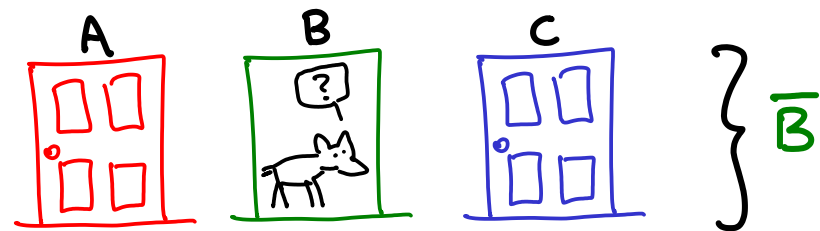




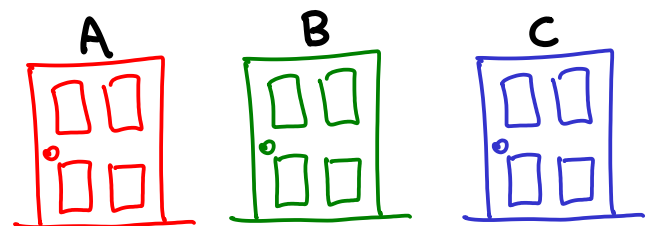
- event i : car is at door i

$$P(A) = P(B) = P(C) = \frac{1}{3}$$

$$P(\bar{A}) = P(\bar{B}) = P(\bar{C}) = \frac{2}{3}$$



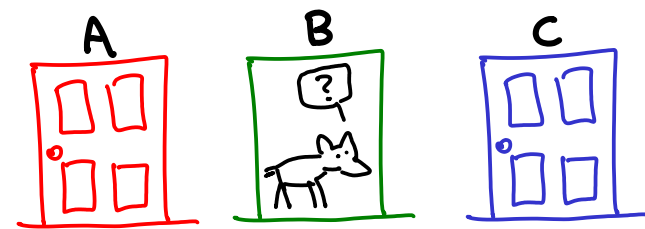
$$P(A|\bar{B}) = ?$$



- event i : car is at door i

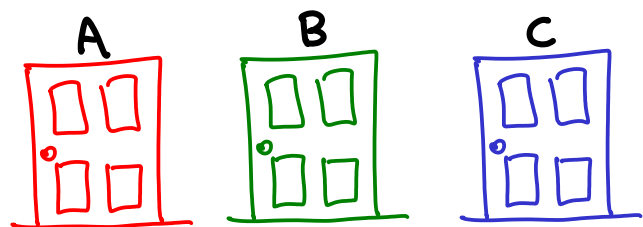
$$P(A) = P(B) = P(C) = \frac{1}{3}$$

$$P(\bar{A}) = P(\bar{B}) = P(\bar{C}) = \frac{2}{3}$$



} $\bar{B}$

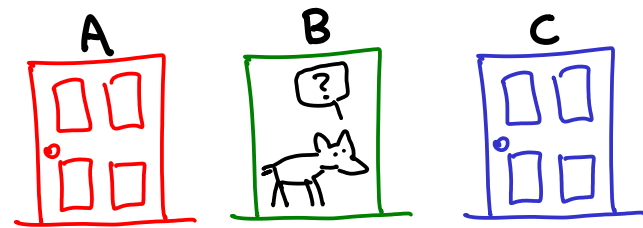
Bayes' theorem $\rightarrow P(A|\bar{B}) = ?$



- event i : car is at door i

$$P(A) = P(B) = P(C) = \frac{1}{3}$$

$$P(\bar{A}) = P(\bar{B}) = P(\bar{C}) = \frac{2}{3}$$

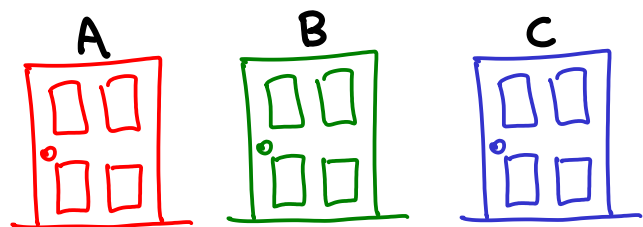


} $\bar{B}$

Bayes' theorem $\rightarrow P(A|\bar{B}) = ?$

$$P(A|\bar{B}) = \frac{P(A \cap \bar{B})}{P(\bar{B})}$$

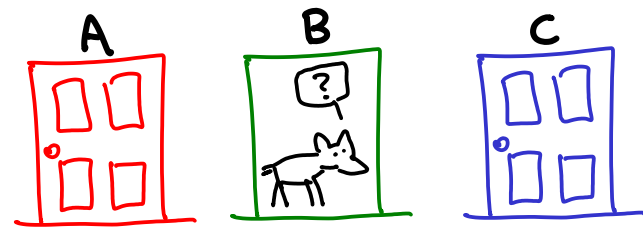
$$P(\bar{B}|A) = \frac{P(\bar{B} \cap A)}{P(A)}$$



• event i : car is at door i

$$P(A) = P(B) = P(C) = \frac{1}{3}$$

$$P(\bar{A}) = P(\bar{B}) = P(\bar{C}) = \frac{2}{3}$$



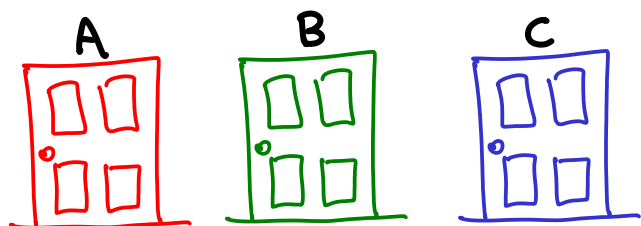
} $\bar{B}$

Bayes' theorem $\rightarrow P(A|\bar{B}) = \frac{P(\bar{B}|A) \cdot P(A)}{P(\bar{B})}$

$$= \frac{? \cdot ?}{?}$$

$$P(A|\bar{B}) = \frac{P(A \cap \bar{B})}{P(\bar{B})}$$

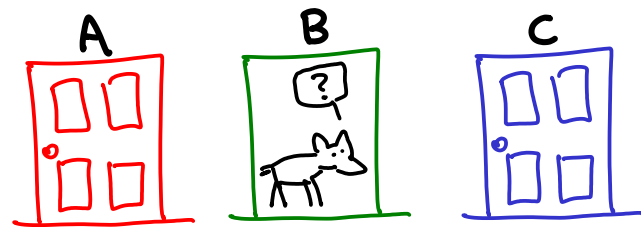
$$P(\bar{B}|A) = \frac{P(\bar{B} \cap A)}{P(A)}$$



- event i : car is at door i

$$P(A) = P(B) = P(C) = \frac{1}{3}$$

$$P(\bar{A}) = P(\bar{B}) = P(\bar{C}) = \frac{2}{3}$$



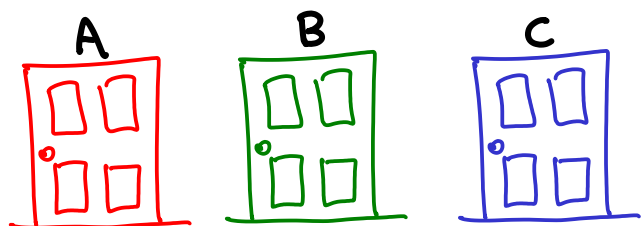
} $\bar{B}$

Bayes' theorem $\rightarrow P(A|\bar{B}) = \frac{P(\bar{B}|A) \cdot P(A)}{P(\bar{B})}$

$$= \frac{1 \cdot \frac{1}{3}}{\frac{2}{3}}$$

$$P(A|\bar{B}) = \frac{P(A \cap \bar{B})}{P(\bar{B})}$$

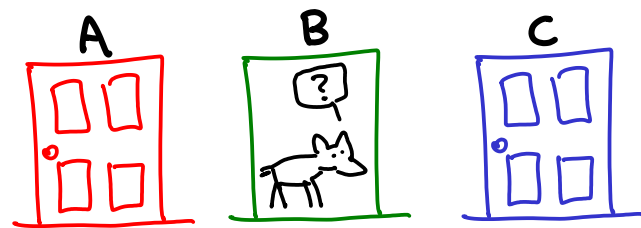
$$P(\bar{B}|A) = \frac{P(\bar{B} \cap A)}{P(A)}$$



- event i : car is at door i

$$P(A) = P(B) = P(C) = \frac{1}{3}$$

$$P(\bar{A}) = P(\bar{B}) = P(\bar{C}) = \frac{2}{3}$$



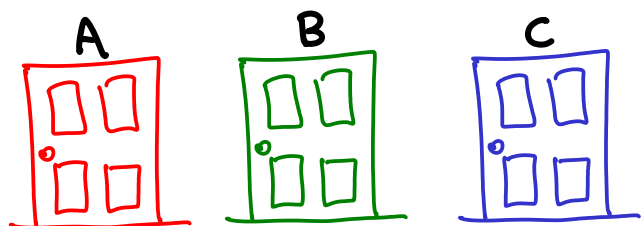
} $\bar{B}$

Bayes' theorem $\rightarrow P(A|\bar{B}) = \frac{P(\bar{B}|A) \cdot P(A)}{P(\bar{B})}$

$$\frac{1 \cdot \frac{1}{3}}{\frac{2}{3}} = \frac{1}{2}$$

$$P(A|\bar{B}) = \frac{P(A \cap \bar{B})}{P(\bar{B})}$$

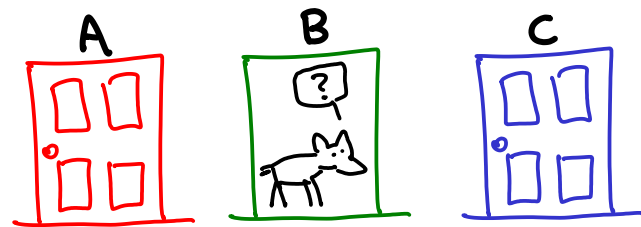
$$P(\bar{B}|A) = \frac{P(\bar{B} \cap A)}{P(A)}$$



• event i : car is at door i

$$P(A) = P(B) = P(C) = \frac{1}{3}$$

$$P(\bar{A}) = P(\bar{B}) = P(\bar{C}) = \frac{2}{3}$$



$\left. \begin{array}{c} A \\ B \\ C \end{array} \right\} \bar{B}$

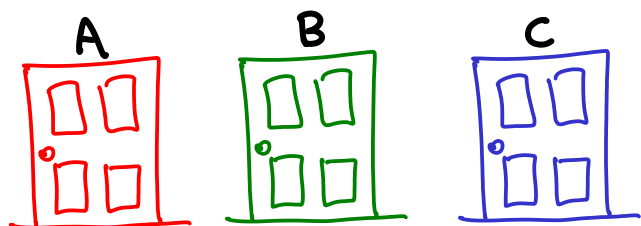
Bayes' theorem $\rightarrow P(A|\bar{B}) = \frac{P(\bar{B}|A) \cdot P(A)}{P(\bar{B})}$

$P(C|\bar{B}) = \frac{P(\bar{B}|C) \cdot P(C)}{P(\bar{B})}$

$$\frac{1 \cdot \frac{1}{3}}{\frac{2}{3}} = \frac{1}{2}$$

$$P(A|\bar{B}) = \frac{P(A \cap \bar{B})}{P(\bar{B})}$$

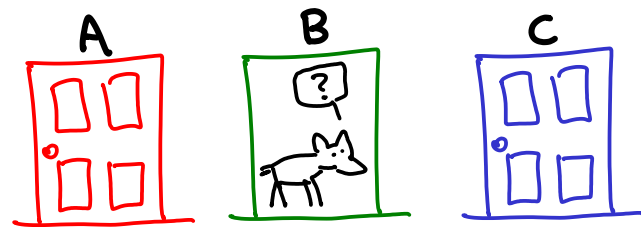
$$P(\bar{B}|A) = \frac{P(\bar{B} \cap A)}{P(A)}$$



• event i : car is at door i

$$P(A) = P(B) = P(C) = \frac{1}{3}$$

$$P(\bar{A}) = P(\bar{B}) = P(\bar{C}) = \frac{2}{3}$$



$\left. \begin{array}{c} A \\ B \\ C \end{array} \right\} \bar{B}$

Bayes' theorem $\rightarrow P(A|\bar{B}) = \frac{P(\bar{B}|A) \cdot P(A)}{P(\bar{B})}$

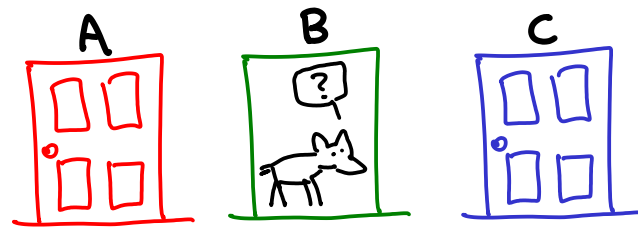
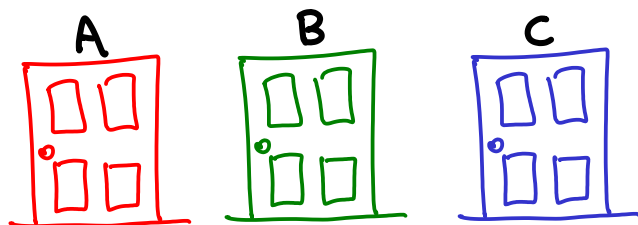
$$P(C|\bar{B}) = \frac{P(\bar{B}|C) \cdot P(C)}{P(\bar{B})} = \frac{1 \cdot \frac{1}{3}}{\frac{2}{3}} = \frac{1}{2}$$

$$P(A|\bar{B}) = \frac{P(A \cap \bar{B})}{P(\bar{B})}$$

$$P(\bar{B}|A) = \frac{P(\bar{B} \cap A)}{P(A)}$$

$\hookrightarrow$ conclusion: it makes no difference

An incorrect analysis:



} $\bar{B}$

- event i : car is at door i

$$P(A) = P(B) = P(C) = \frac{1}{3}$$

$$P(\bar{A}) = P(\bar{B}) = P(\bar{C}) = \frac{2}{3}$$

Bayes' theorem $\rightarrow P(A|\bar{B}) = \frac{P(\bar{B}|A) \cdot P(A)}{P(\bar{B})}$

$$P(C|\bar{B}) = \frac{P(\bar{B}|C) \cdot P(C)}{P(\bar{B})} = \frac{1 \cdot \frac{1}{3}}{\frac{2}{3}} = \frac{1}{2}$$

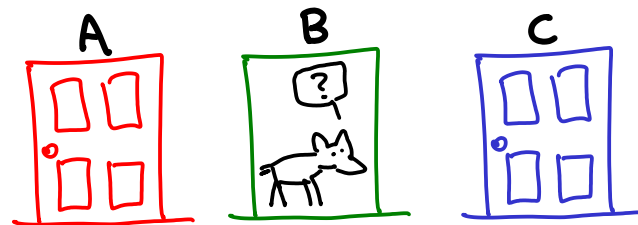
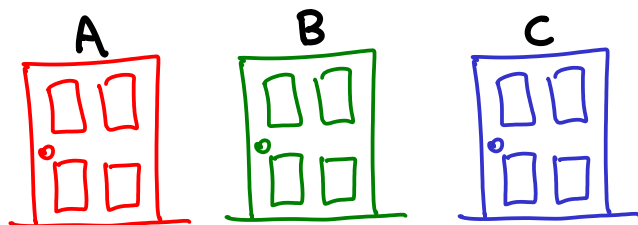
$$P(A|\bar{B}) = \frac{P(A \cap \bar{B})}{P(\bar{B})}$$

$$P(\bar{B}|A) = \frac{P(\bar{B} \cap A)}{P(A)}$$

↪ conclusion: it makes no difference

• What is wrong?

An incorrect analysis:



- event i : car is at door i

$$P(A) = P(B) = P(C) = \frac{1}{3}$$

$$P(\bar{A}) = P(\bar{B}) = P(\bar{C}) = \frac{2}{3}$$

$\bar{B}$

Bayes' theorem $\rightarrow P(A|\bar{B}) = \frac{P(\bar{B}|A) \cdot P(A)}{P(\bar{B})}$

$$P(C|\bar{B}) = \frac{P(\bar{B}|C) \cdot P(C)}{P(\bar{B})} = \frac{1 \cdot \frac{1}{3}}{\frac{2}{3}} = \frac{1}{2}$$

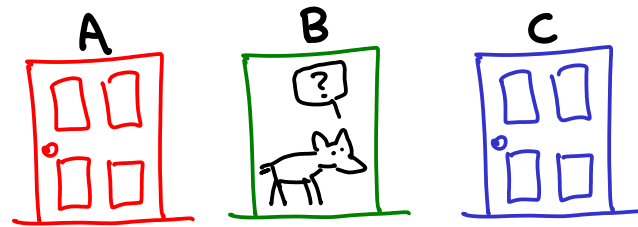
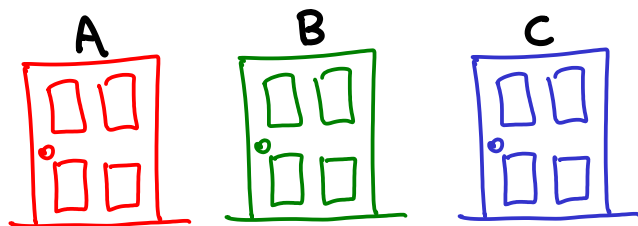
↪ conclusion: it makes no difference

- What is wrong? $\rightarrow$ You are not in the picture.

$$P(A|\bar{B}) = \frac{P(A \cap \bar{B})}{P(\bar{B})}$$

$$P(\bar{B}|A) = \frac{P(\bar{B} \cap A)}{P(A)}$$

An incorrect analysis:



- event i : car is at door i

$$P(A) = P(B) = P(C) = \frac{1}{3}$$

$$P(\bar{A}) = P(\bar{B}) = P(\bar{C}) = \frac{2}{3}$$

Bayes' theorem $\rightarrow P(A|\bar{B}) = \frac{P(\bar{B}|A) \cdot P(A)}{P(\bar{B})}$

$\downarrow$

$$P(C|\bar{B}) = \frac{P(\bar{B}|C) \cdot P(C)}{P(\bar{B})} = \frac{1 \cdot \frac{1}{3}}{\frac{2}{3}} = \frac{1}{2}$$

$$P(A|\bar{B}) = \frac{P(A \cap \bar{B})}{P(\bar{B})}$$

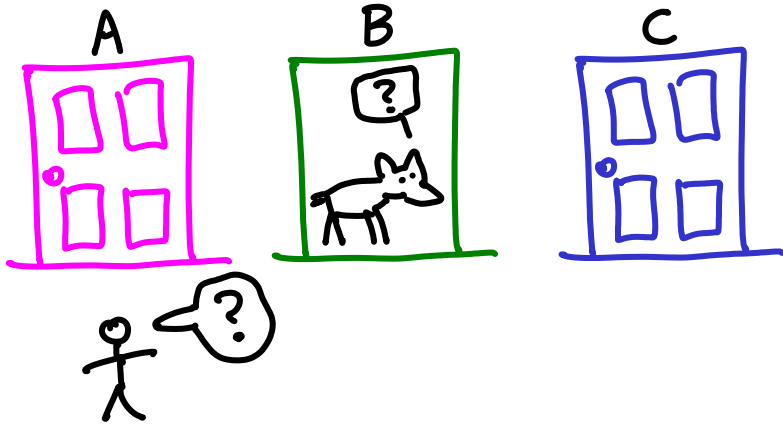
$$P(\bar{B}|A) = \frac{P(\bar{B} \cap A)}{P(A)}$$

↪ conclusion: it makes no difference

- What is wrong? $\rightarrow$ You are not in the picture.
- if you hadn't selected A, this analysis would be correct.

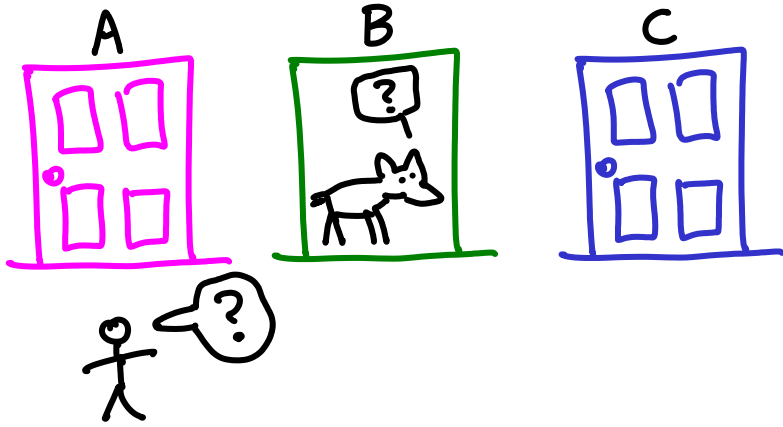
Without loss of generality → 1) you choose door **A**.

just some door with a goat → 2) door **B** is opened, revealing a goat.



Without loss of generality → 1) you choose door **A**.

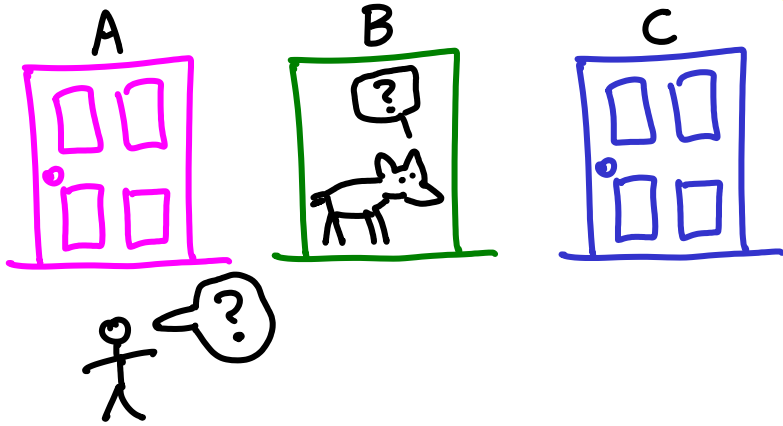
just some door with a goat → 2) door **B** is opened, revealing a goat.



→ if the car had been at B,
then **C** would have opened.
(doors could have been renamed)

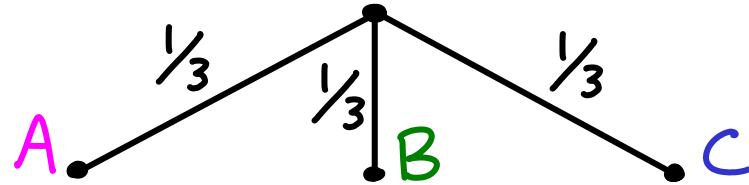
Without loss of generality → 1) you choose door **A**.

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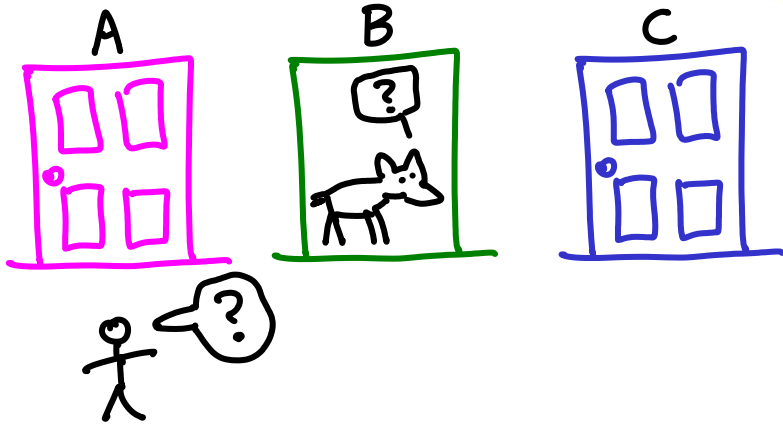
→ if the car had been at B,
then **C** would have opened.
(doors could have been renamed)

car is at:



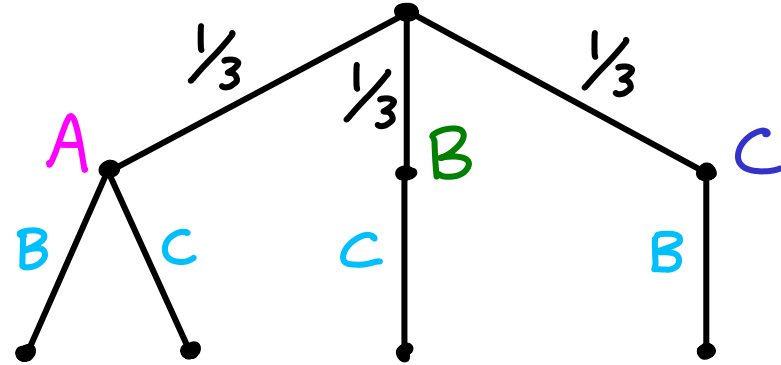
Without loss of generality → 1) you choose door **A**.

just some door with a goat → 2) door **B** is opened, revealing a goat.



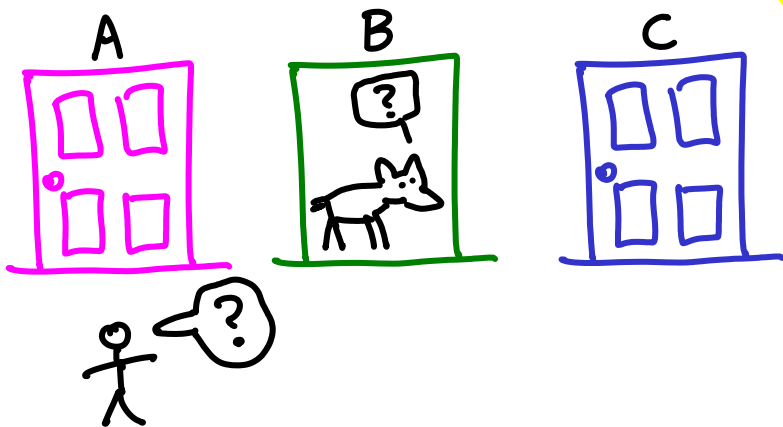
if the car had been at B,
then **C** would have opened.
(doors could have been renamed)

car is at:
open:



Without loss of generality → 1) you choose door **A**.

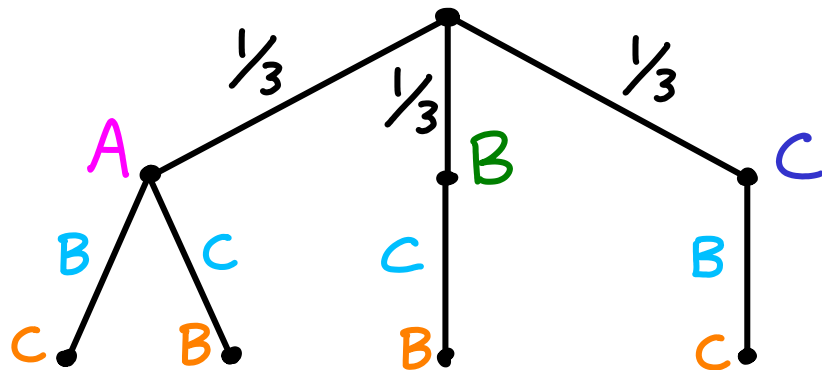
just some door with a goat → 2) door **B** is opened, revealing a goat.



→ if the car had been at B,
then **C** would have opened.
(doors could have been renamed)

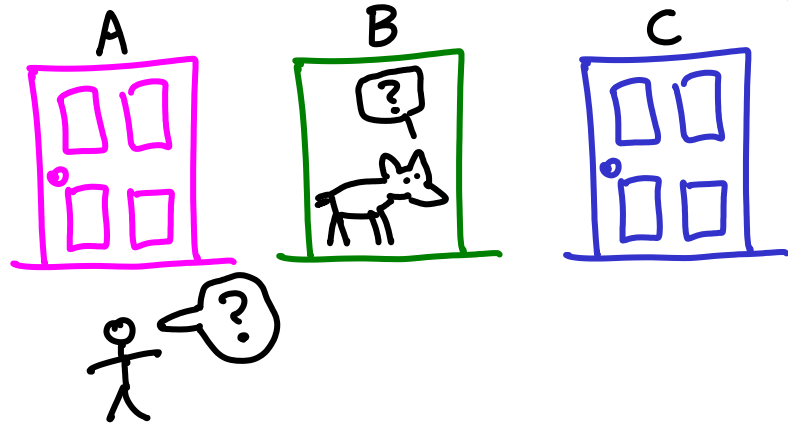
strategy:
switch

car is at:
open:
switch to:



Without loss of generality → 1) you choose door **A**.

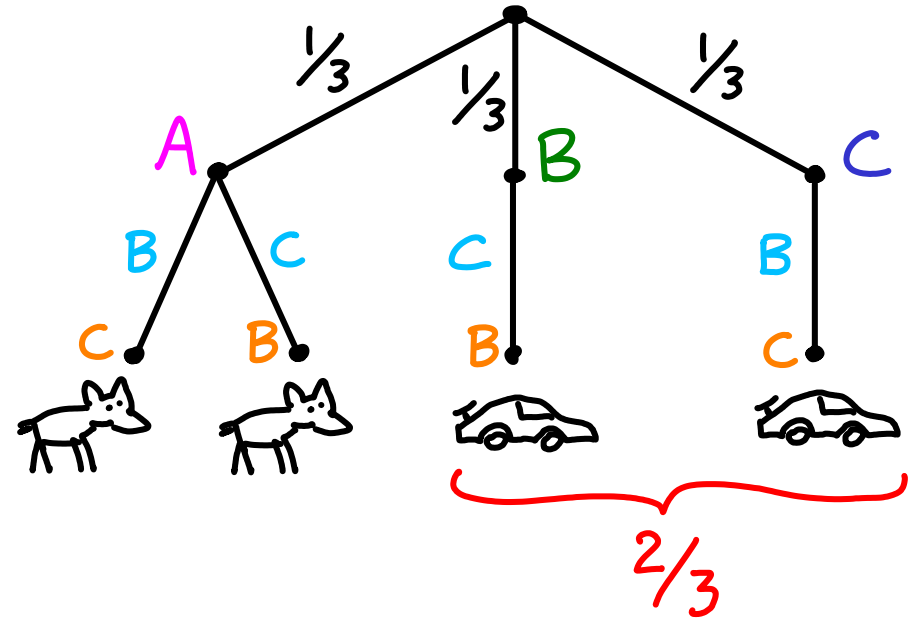
just some door with a goat → 2) door **B** is opened, revealing a goat.



→ if the car had been at B,
then **C** would have opened.
(doors could have been renamed)

strategy:
switch

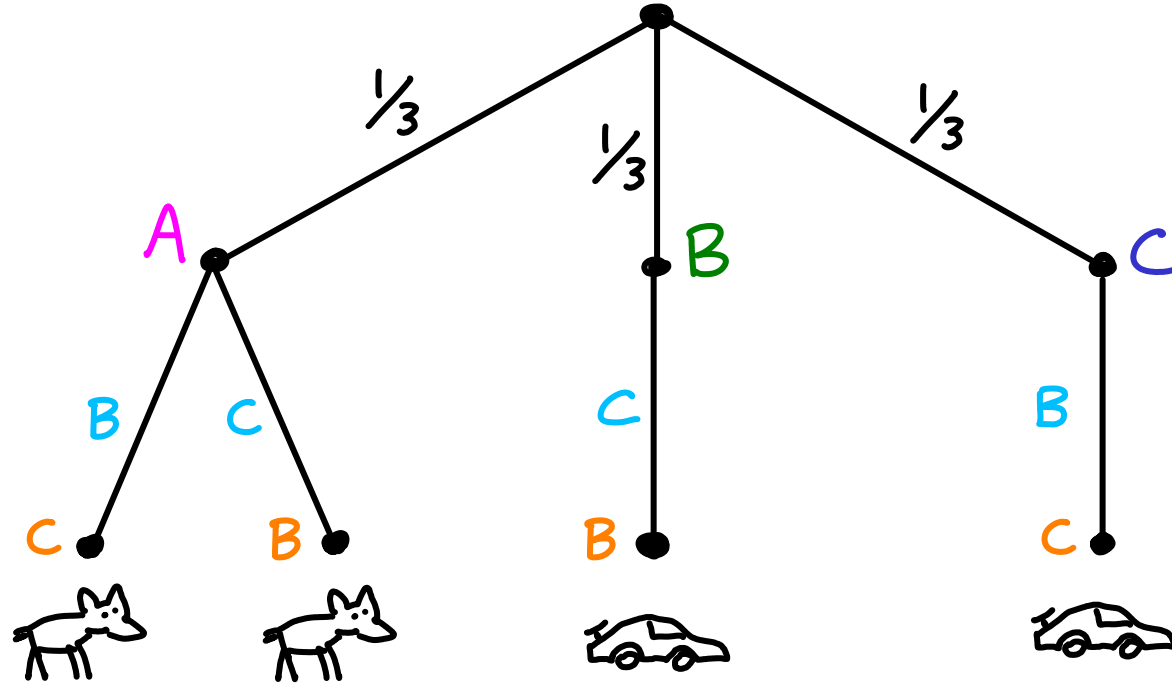
car is at:
open:
switch to:



car is at:

open:

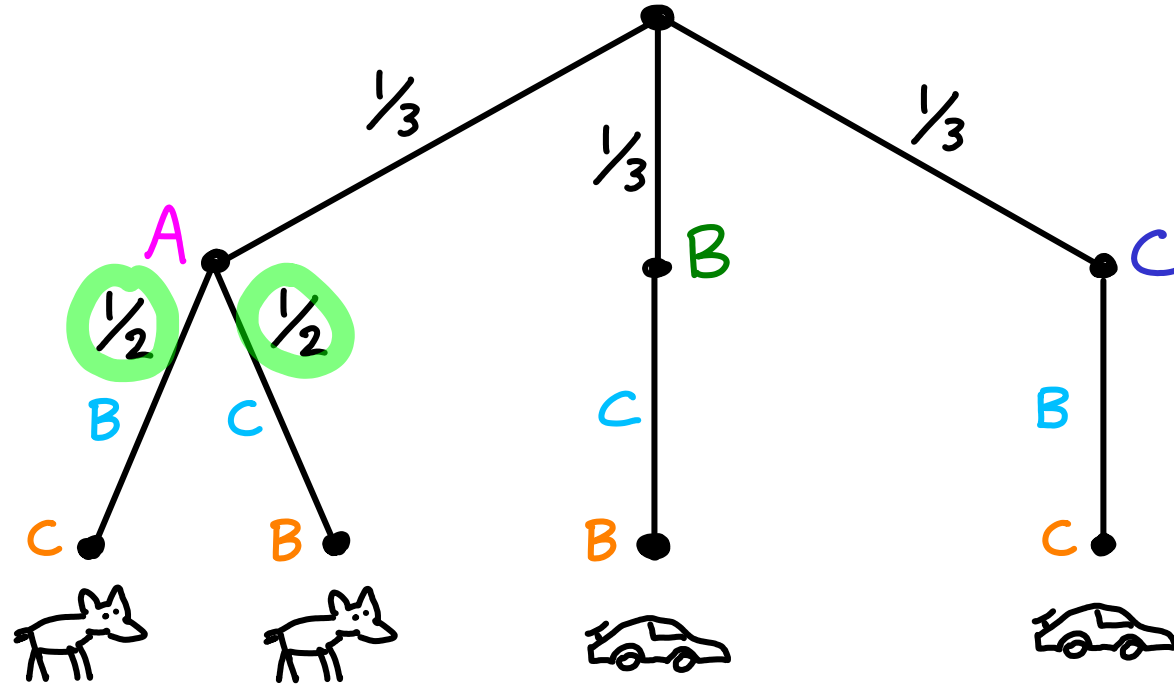
switch to:



car is at:

open:

switch to:

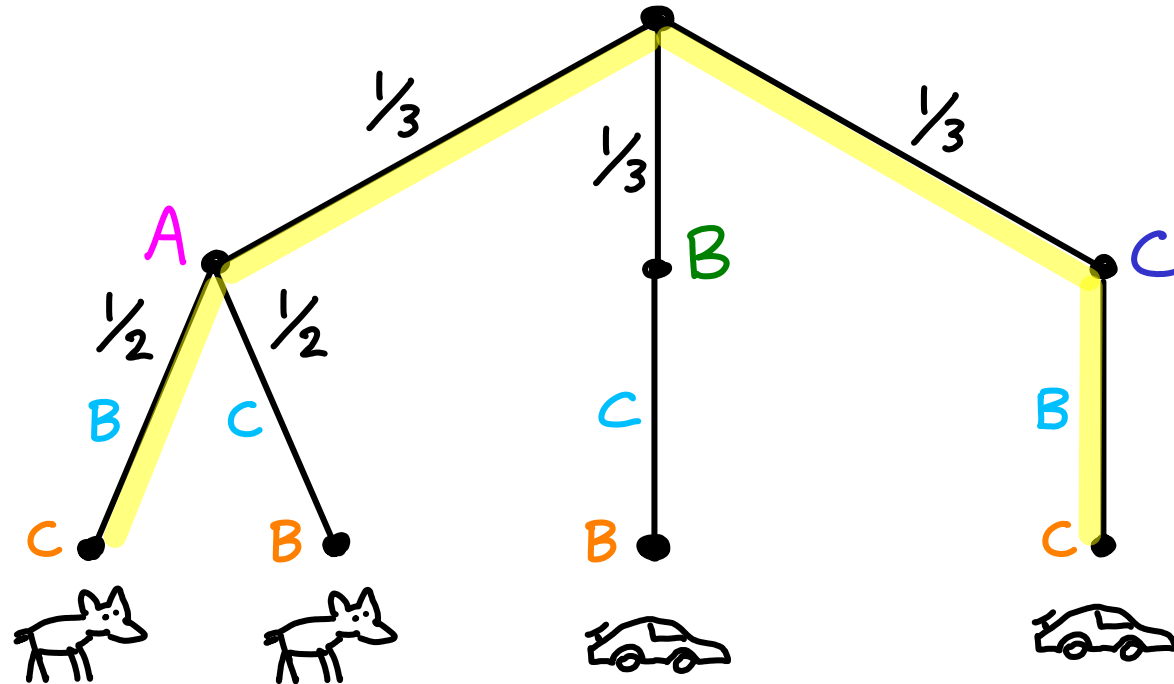


2 scenarios consistent with B opened

car is at:

open:

switch to:

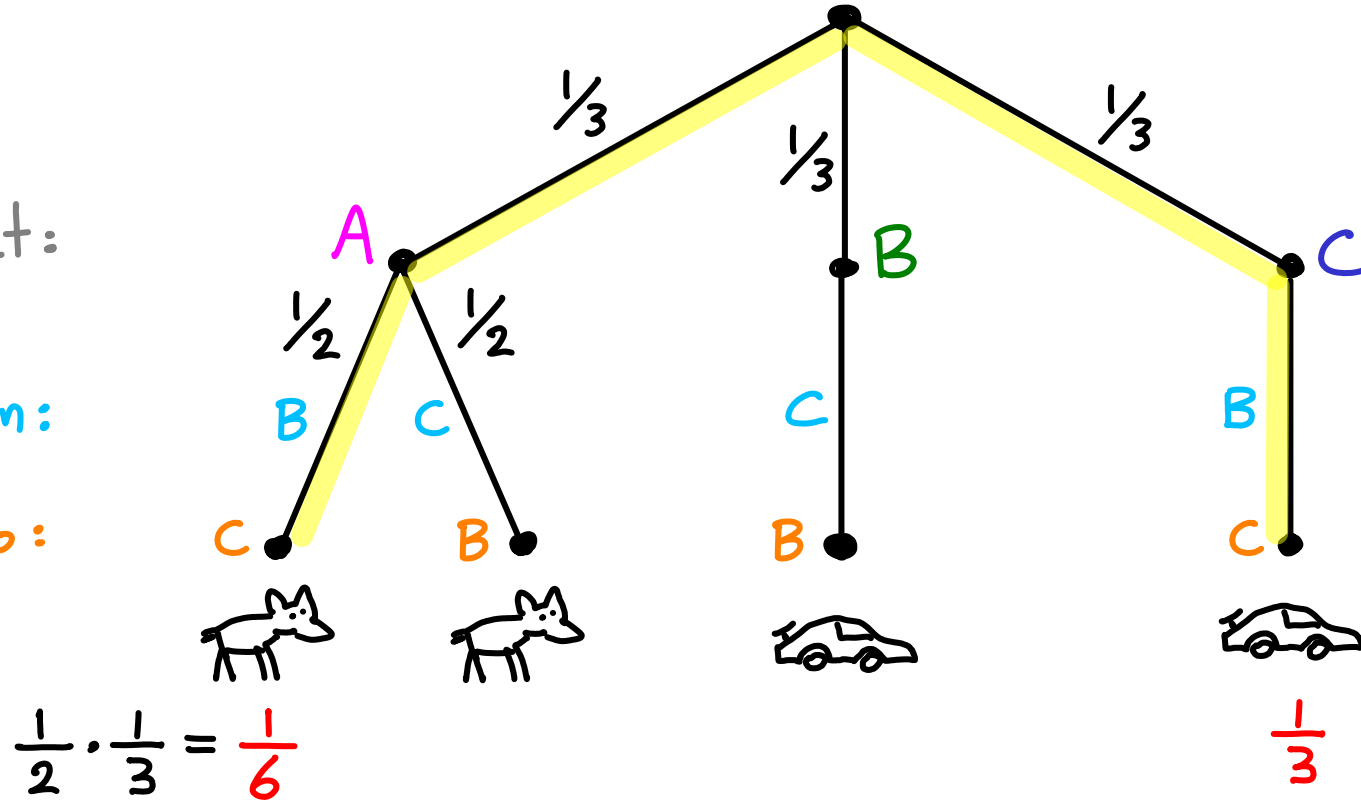


2 scenarios consistent with B opened

car is at:

open:

switch to:

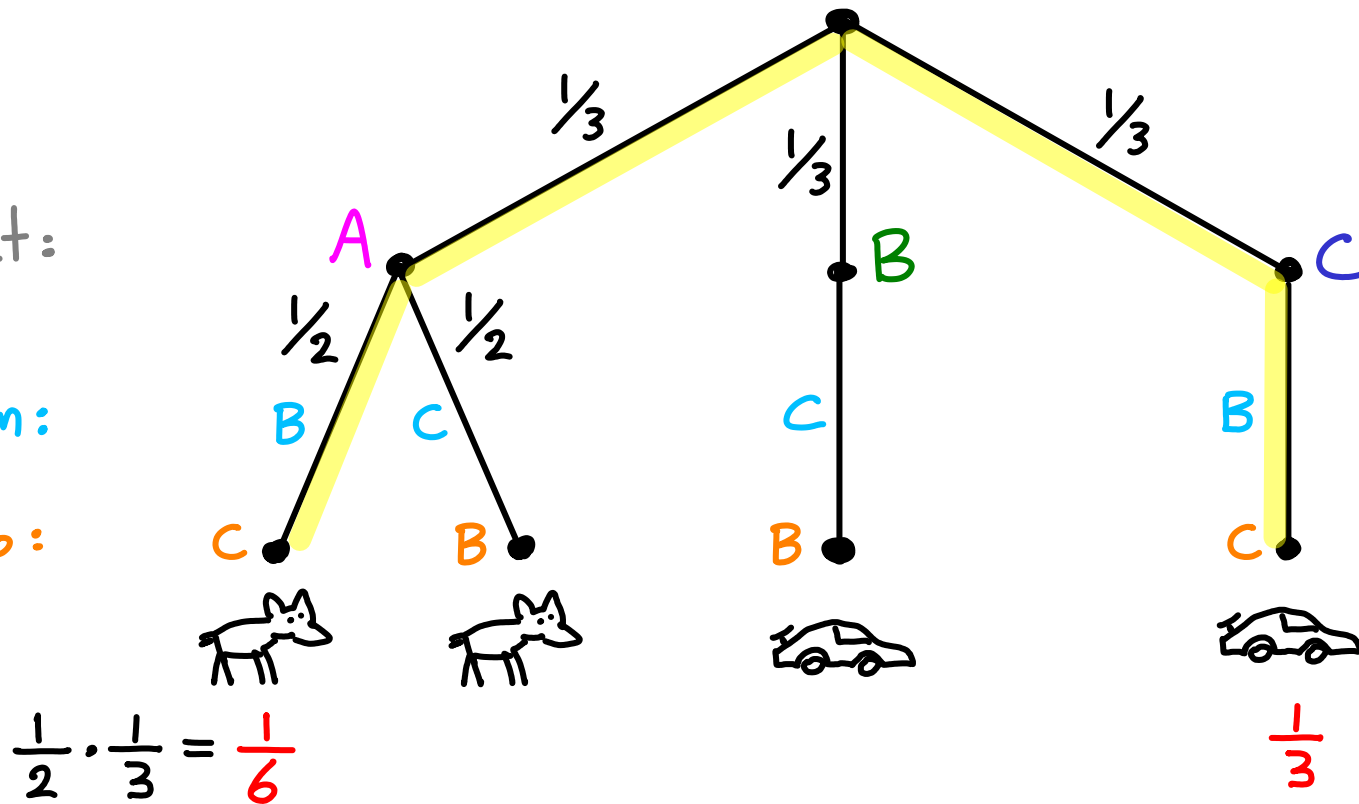


2 scenarios consistent with B opened

car is at:

open:

switch to:



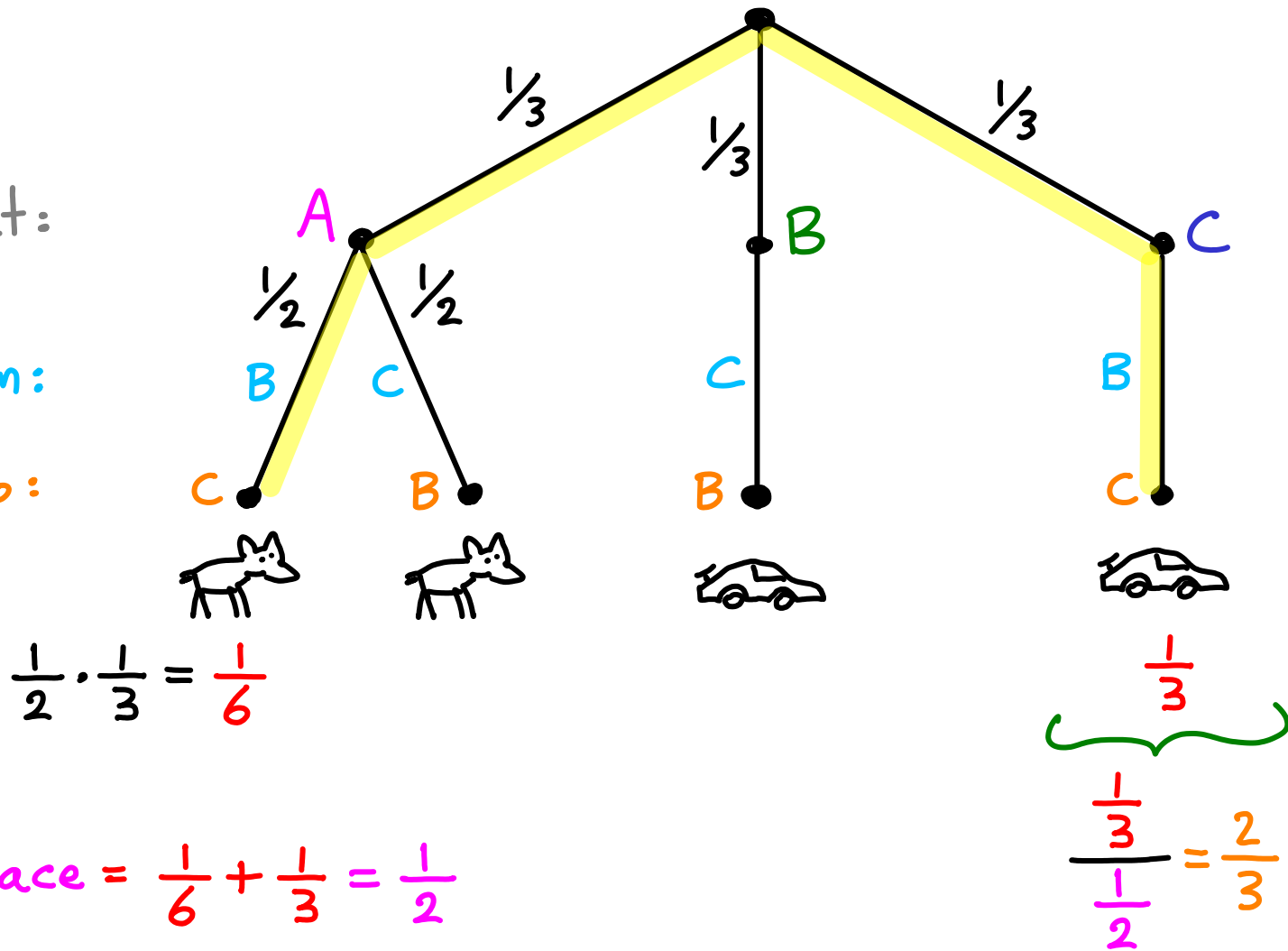
new sample space = $\frac{1}{6} + \frac{1}{3} = \frac{1}{2}$

2 scenarios consistent with B opened

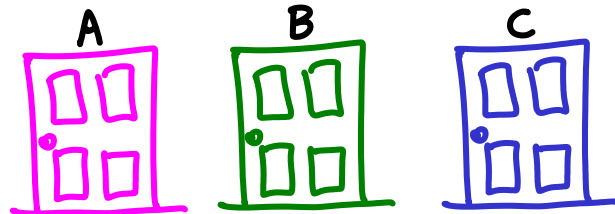
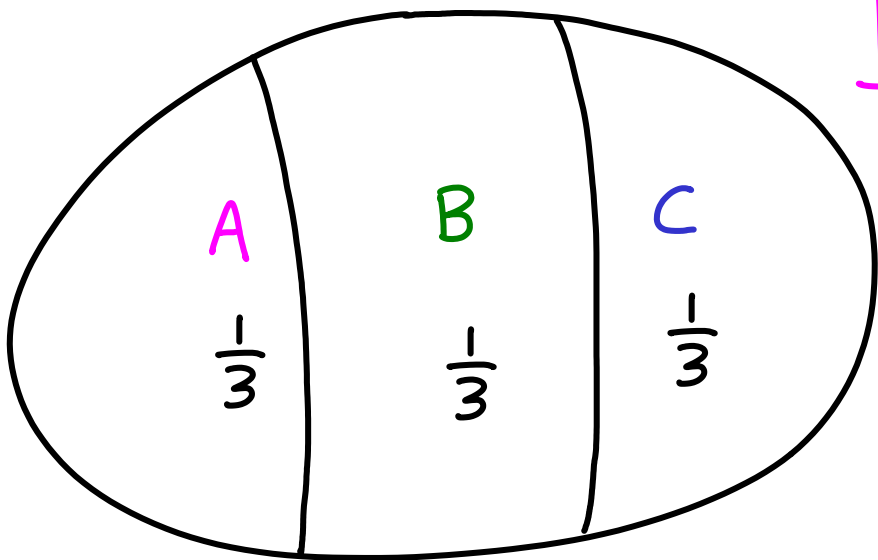
car is at:

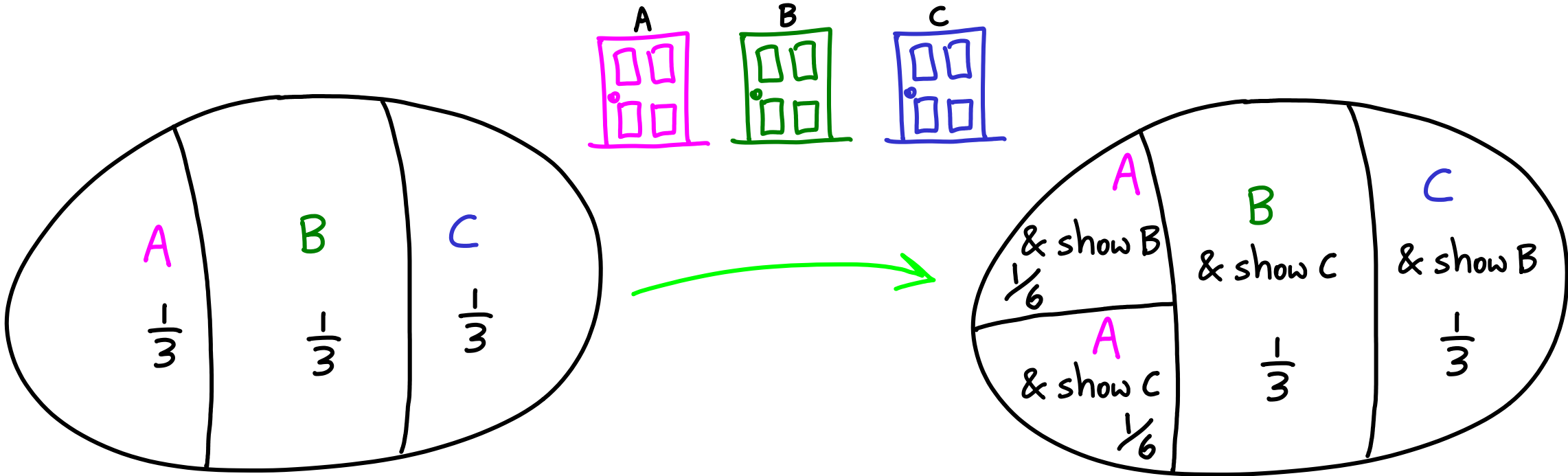
open:

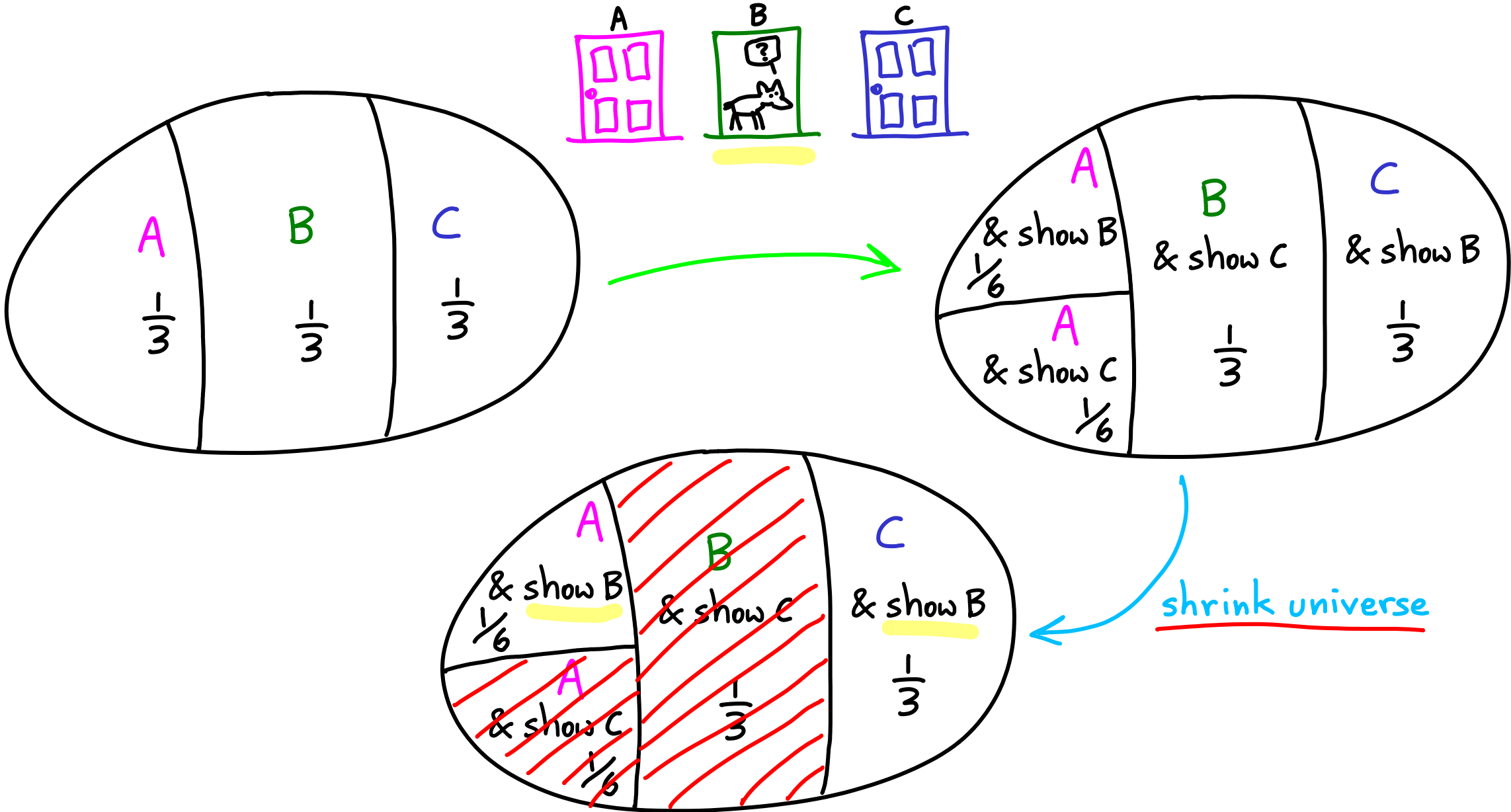
switch to:

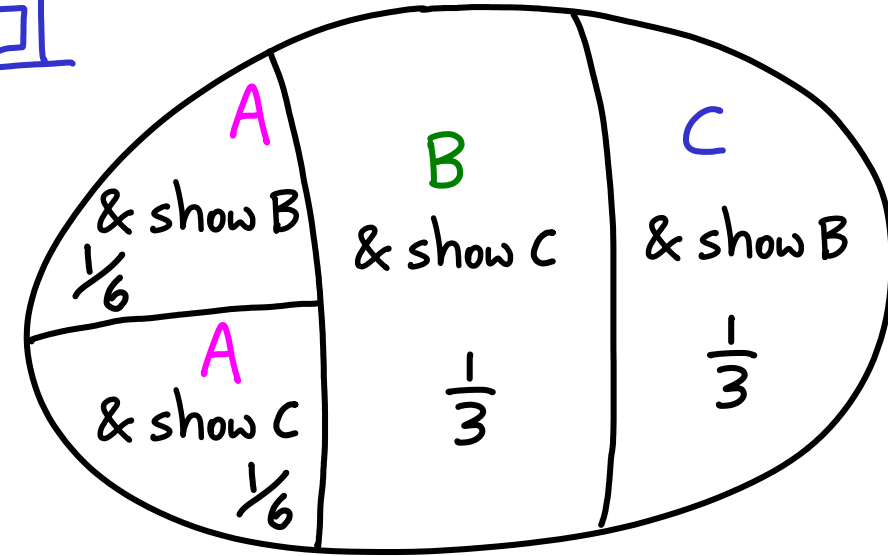
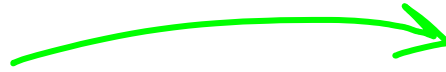
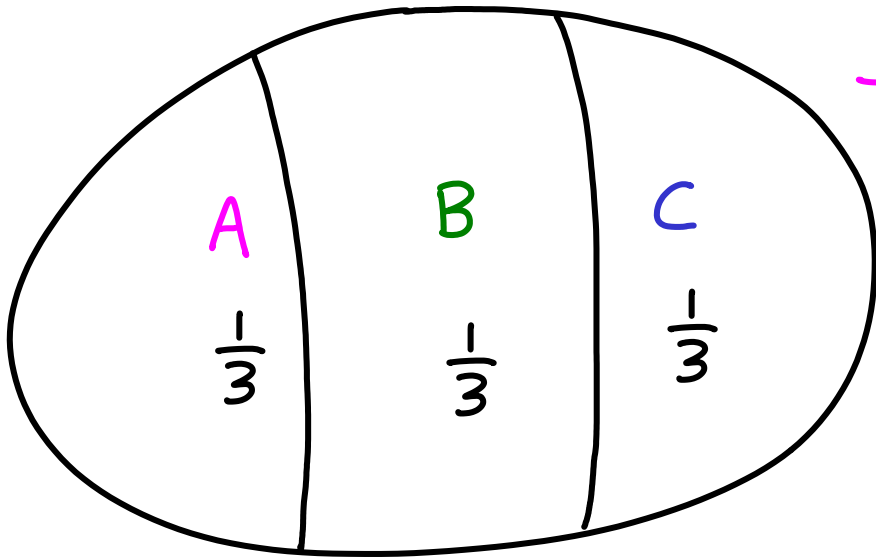
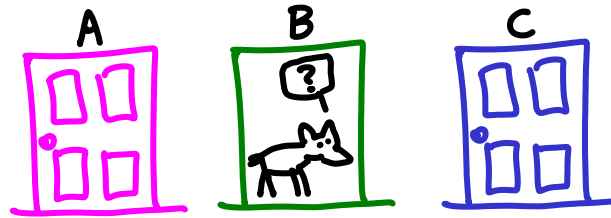


new sample space = $\frac{1}{6} + \frac{1}{3} = \frac{1}{2}$









$$P(C \cap \text{show B})$$

$$= \frac{\frac{1}{3}}{\frac{1}{6} + \frac{1}{3}} = \frac{2}{3}$$

