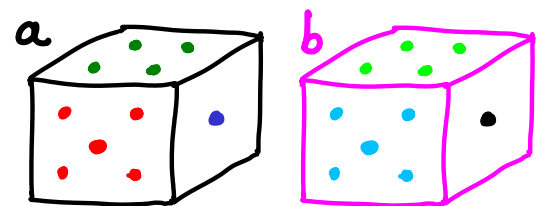


CONDITIONAL PROBABILITY

Roll 2 dice...



$$P(A) = P(\text{sum}=8) = \frac{5}{36}$$

$$P(B) = P(\text{both even}) = \frac{1}{4}$$

- what is the probability that the sum is 8, given that both are even?

$P(A \text{ given } B)$

$\downarrow$

$$P(A|B) = \frac{3}{9}$$

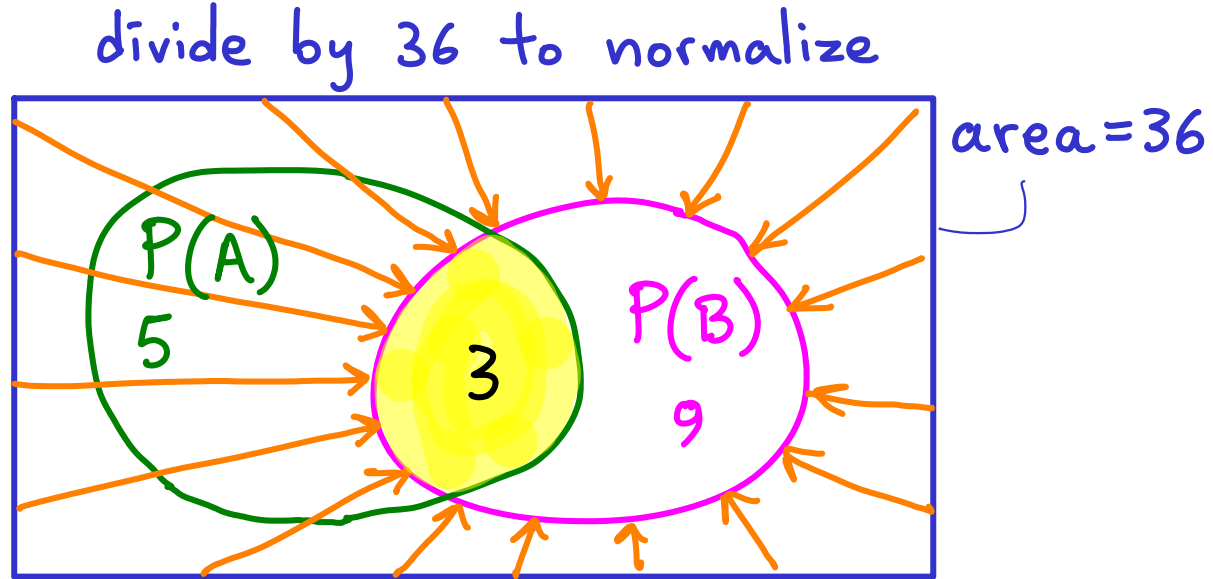
$$A = \{(2,6), (3,5), (4,4), (5,3), (6,2)\}$$

$$B = \{(2,2), (2,4), (2,6), (4,2), (4,4), (4,6), (6,2), (6,4), (6,6)\}$$

$\neq P(A)$
in this example

$$P(A) = P(\text{sum}=8) = \frac{5}{36}$$

$$P(B) = P(\text{both even}) = \frac{9}{36}$$



• When we establish B , the "universe" shrinks. sample space $\rightarrow |B|$

• probability that A holds $\rightarrow$ valid remaining subset of A
new universe

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{3}{9}$$

Flip a coin 5 times $|\text{sample space}| = 2^5$ $P(F) = P(\text{1st flip} = T) = \frac{1}{2}$

What if you are told that 3 of the 5 flips were H? "3H"

$$P(F|3H) = ?$$

quick intuition:

if 3 of 5 were H,

then for any flip the probability of H is $\frac{3}{5}$

so for T it's $\frac{2}{5}$

Flip a coin 5 times $|\text{sample space}| = 2^5$ $P(F) = P(\text{1st flip} = T) = \frac{1}{2}$

What if you are told that 3 of the 5 flips were H? "3H"

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$P(F|3H) = \frac{P(F \cap 3H)}{P(3H)} = \frac{2}{5}$$

T	H	H	H	T
T	H	H	T	H
T	H	T	H	H
T	T	H	H	H

$$P(F \cap 3H) = \frac{\binom{4}{3}}{2^5} = \frac{4}{32}$$

$$P(3H) = \frac{\binom{5}{3}}{2^5} = \frac{\frac{5!}{3!2!}}{32} = \frac{10}{32}$$

$$P(F) = \frac{1}{2}$$

unconditional probability

$$P(F|3H) = \frac{2}{5}$$

conditional probability
given extra information

$$P(F) \neq P(F|3H)$$

The probability of F happening depends on $3H$

flip a coin 3 times

|sample space| = 2^3

$$P(c=T) = \frac{1}{2}$$

$$P(a=H) = \frac{1}{2}$$

abc

HHH
HHT
HTH
HTT
THH
THT
TTH
TTT

$$P((c=T) \mid (a=H)) = \frac{P((c=T) \cap (a=H))}{P(a=H)} = \frac{\frac{2}{8}}{\frac{1}{2}} = \frac{1}{2}$$

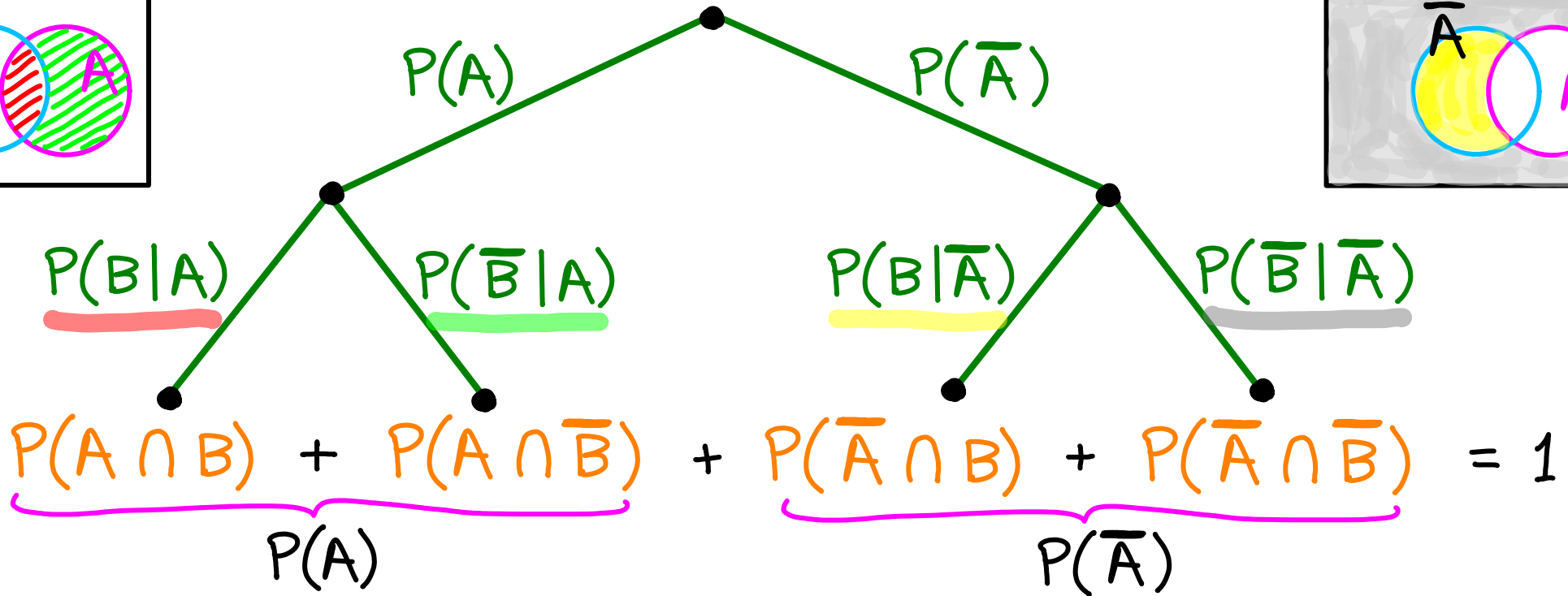
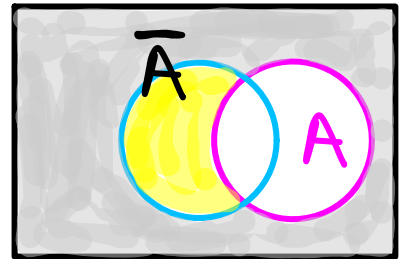
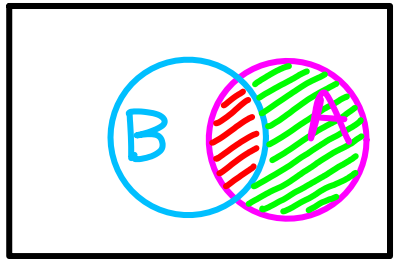
- knowledge of $a=H$ had no effect on c
- the outcome for c is independent of the outcome for a
- events c and a are independent

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

← identical →

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

$$P(B) \cdot P(A|B) = P(A \cap B) = P(A) \cdot P(B|A)$$



Roll one die : $P(\text{prime AND even})?$

$$= P(\text{prime}) \cdot P(\text{even} \mid \text{prime})$$

$$= \frac{3}{6} \cdot \frac{1}{3} = \frac{1}{6}$$

$$\overbrace{2, 3, 5} \\ P(\text{prime}) = \frac{3}{6}$$

$$\overbrace{2, 4, 6} \\ P(\text{even}) = \frac{3}{6}$$

$$\overbrace{2} \\ P(\text{even} \mid \text{prime}) = \frac{1}{3}$$

$$P(A \cap B) = P(A) \cdot P(B|A)$$

$$P(A \cap B \cap C) = P((A \cap B) \cap C)$$

$$= P(A \cap B) \cdot P(C|(A \cap B))$$

$$= P(A) \cdot P(B|A) \cdot P(C|(A \cap B))$$

$$P(A \cap B \cap C \cap D) = P((A \cap B \cap C) \cap D)$$

$$= P(A \cap B \cap C) \cdot P(D|(A \cap B \cap C))$$

$$= P(A) \cdot P(B|A) \cdot P(C|(A \cap B)) \cdot P(D|(A \cap B \cap C))$$

$$P(e_1 \cap \dots \cap e_n) = P(e_1) \cdot \prod_{i=2}^n P(e_i | e_1 \cap \dots \cap e_{i-1})$$

$$P\left(\bigcap_{i=1}^n e_i\right) = P(e_1) \cdot \prod_{i=2}^n P(e_i | \bigcap_{j=1}^{i-1} e_j)$$

$$= \prod_{i=1}^n P(e_i | \bigcap_{j=1}^{i-1} e_j)$$

INDEPENDENCE

definition: A & B are independent $\leftrightarrow$ $\underbrace{P(A \cap B) = P(A) \cdot P(B)}_{\text{symmetry}}$

we already know: $P(B|A) = \frac{P(A \cap B)}{P(A)}$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

if independent: $P(B|A) = \frac{P(A) \cdot P(B)}{P(A)}$

$$P(A|B) = \frac{P(A) \cdot P(B)}{P(B)}$$

$* P(A) \neq 0$

$$P(B|A) = P(B)$$

$$P(A|B) = P(A)$$

$* P(B) \neq 0$

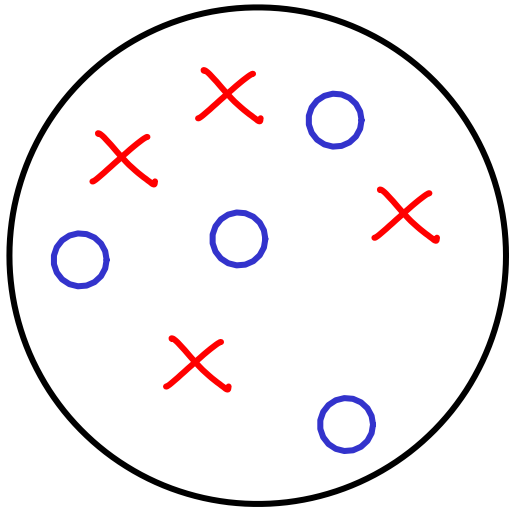
INDEPENDENCE

A, B, C are independent $\leftrightarrow$ $\underbrace{P(A \cap B \cap C) = P(A) \cdot P(B) \cdot P(C)}_{\text{symmetry}}$

$$P(e_1 \cap \dots \cap e_n) = P\left(\bigcap_{i=1}^n e_i\right) = \prod_{i=1}^n P(e_i)$$

Select 2 objects from this group (order matters, no replacement)

$$|\text{sample space}| = P(8, 2) = 8 \cdot 7 = 56$$



event A: 1st choice is X
event B: 2nd choice is X } dependent

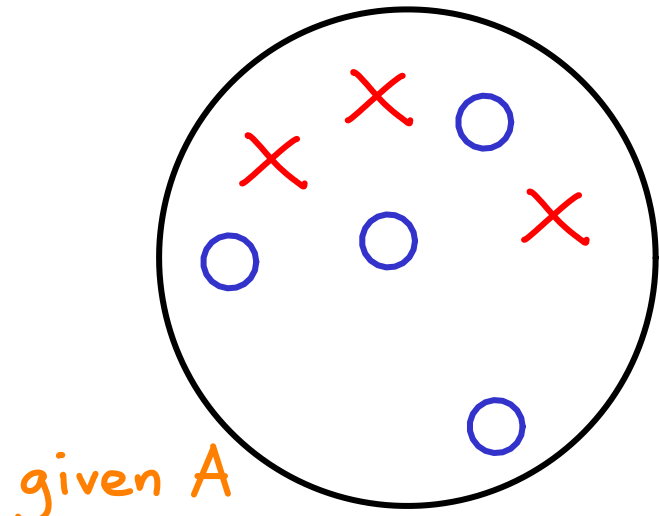
$$P(A) = \frac{4}{8}$$

$$P(B) = \frac{4}{8}$$

symmetric
... or examine
sample space

$\neq$

$$P(B|A) = \frac{3}{7}$$



given A