CONDITIONAL PROBABILITY

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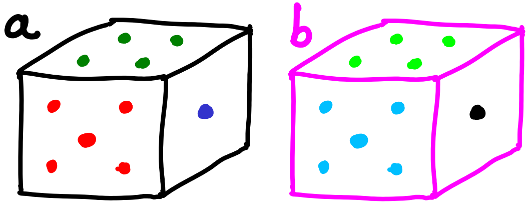
Roll 2 dice...



$$P(\text{sum}=8) = \frac{5}{36}$$

$$\{(2,6),(3,5),(4,4),(5,3),(6,2)\}$$

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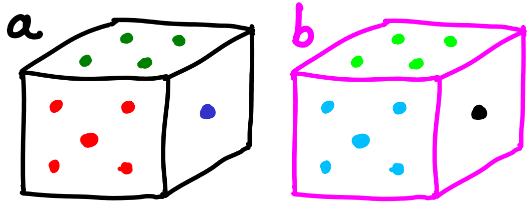
Roll 2 dice...



$$P(\text{sum}=8) = \frac{5}{36}$$

$$P(\text{both even}) = \frac{1}{4}$$

$$\{(2,6), (3,5), (4,4), (5,3), (6,2)\}$$

$$\{(2,2), (2,4), (2,6), (4,2), (4,4), (4,6), (6,2), (6,4), (6,6)\}$$

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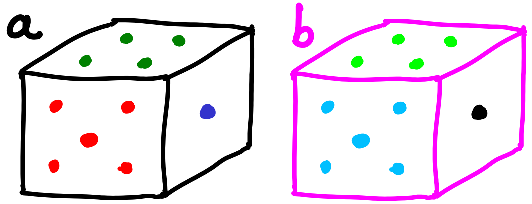


$P(\text{sum}=8) = \frac{5}{36}$

$P(\text{both even}) = \frac{1}{4}$

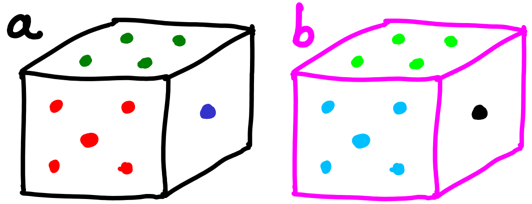
- what is the probability that the sum is 8, given that both are even?

$A = \{(2,6), (3,5), (4,4), (5,3), (6,2)\}$

$B = \{(2,2), (2,4), (2,6), (4,2), (4,4), (4,6), (6,2), (6,4), (6,6)\}$

CONDITIONAL PROBABILITY

Roll 2 dice...



$P(A) = P(\text{sum}=8) = \frac{5}{36}$

$P(B) = P(\text{both even}) = \frac{1}{4}$

- what is the probability that the sum is 8, given that both are even? } $P(A \text{ given } B)$

$A = \{(2,6), (3,5), (4,4), (5,3), (6,2)\}$

$B = \{(2,2), (2,4), (2,6), (4,2), (4,4), (4,6), (6,2), (6,4), (6,6)\}$

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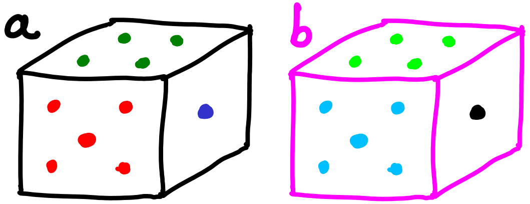
Roll 2 dice...



$P(A) = P(\text{sum}=8) = \frac{5}{36}$

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- what is the probability that the sum is 8, given that both are even?

P(A given B)

$\downarrow$

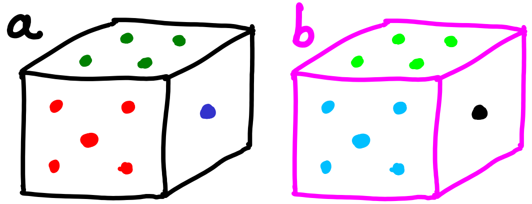
$P(A|B)$

$A = \{(2,6), (3,5), (4,4), (5,3), (6,2)\}$

$B = \{(2,2), (2,4), (2,6), (4,2), (4,4), (4,6), (6,2), (6,4), (6,6)\}$

CONDITIONAL PROBABILITY

Roll 2 dice...



$$P(A) = P(\text{sum}=8) = \frac{5}{36}$$

$$P(B) = P(\text{both even}) = \frac{1}{4}$$

- what is the probability that the sum is 8, given that both are even?

$\Bigg\}$ P(A given B)

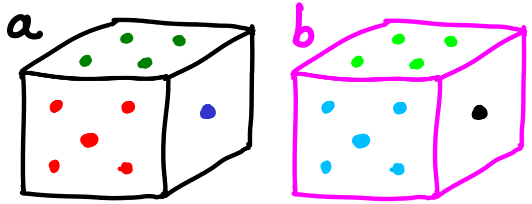
$$P(A|B) = \frac{3}{9}$$

$$A = \{(2,6), (3,5), (4,4), (5,3), (6,2)\}$$

$$B = \{(2,2), (2,4), (2,6), (4,2), (4,4), (4,6), (6,2), (6,4), (6,6)\}$$

CONDITIONAL PROBABILITY

Roll 2 dice...



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$P(A \text{ given } B)$

$$P(A|B) = \frac{3}{9}$$

$$A = \{(2,6), (3,5), (4,4), (5,3), (6,2)\}$$

$$B = \{(2,2), (2,4), (2,6), (4,2), (4,4), (4,6), (6,2), (6,4), (6,6)\}$$

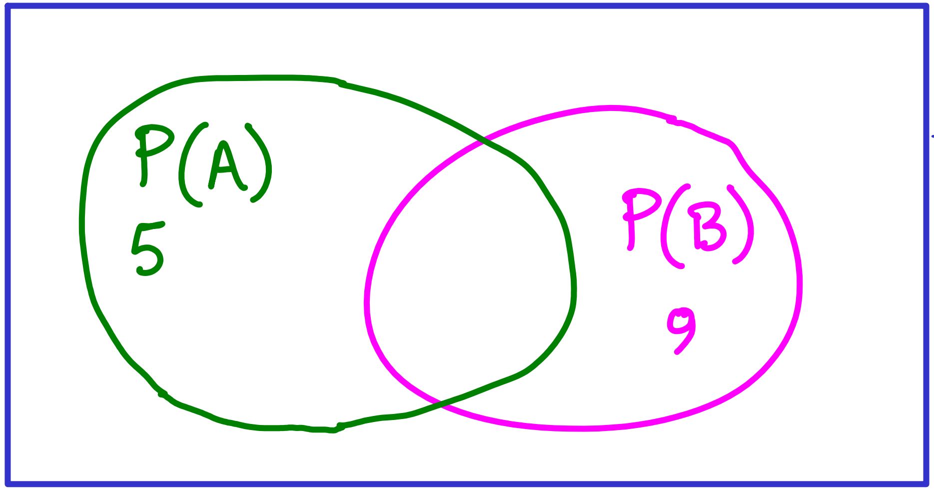
$\neq P(A)$

in this example

$P(A) = P(\text{sum}=8) = \frac{5}{36}$

$P(B) = P(\text{both even}) = \frac{9}{36}$

divide by 36 to normalize

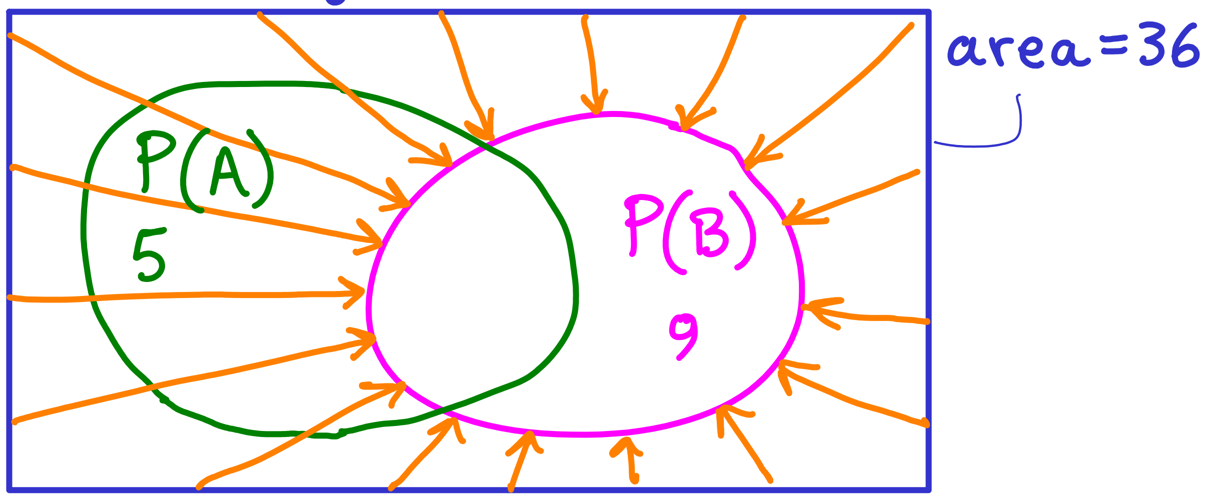


area=36

$P(A) = P(\text{sum}=8) = \frac{5}{36}$

$P(B) = P(\text{both even}) = \frac{9}{36}$

divide by 36 to normalize



- When we establish B, the "universe" shrinks. sample space $\rightarrow |B|$

$P(A) = P(\text{sum}=8) = \frac{5}{36}$

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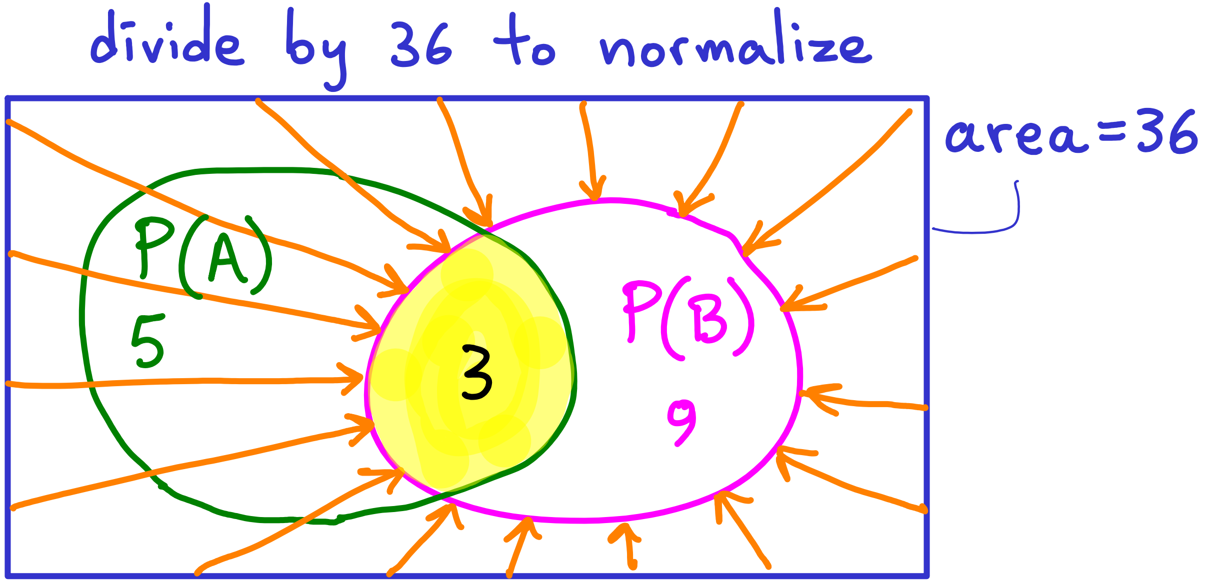

- When we establish B, the "universe" shrinks. sample space → |B|

- probability that A holds → $\dfrac{\text{valid remaining subset of A}}{\text{new universe}}$

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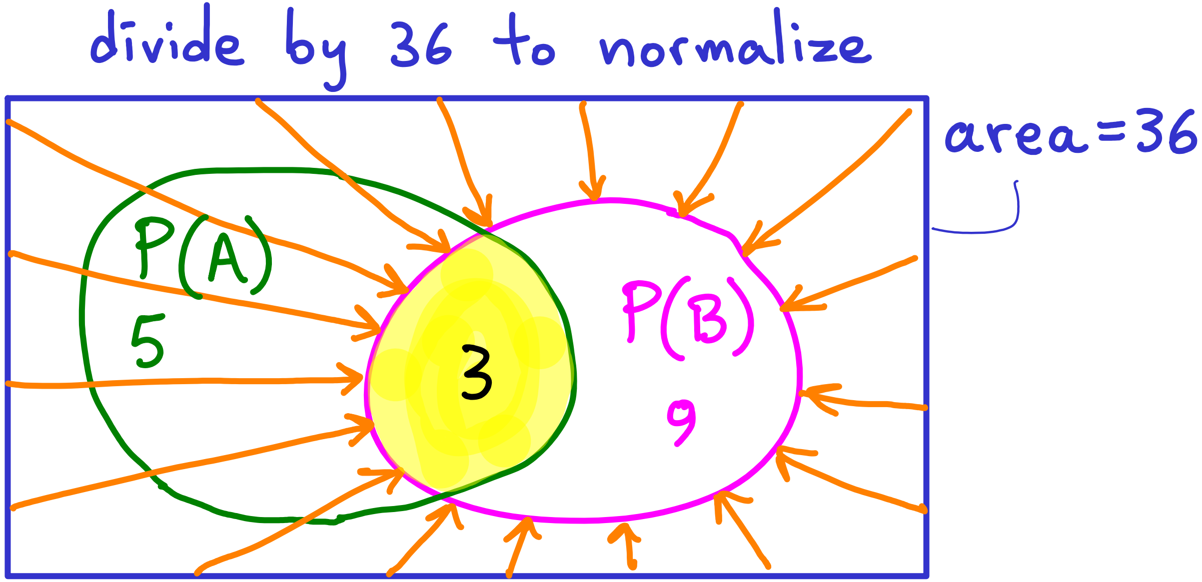

- When we establish B, the "universe" shrinks. sample space → |B|

- probability that A holds → $\dfrac{\text{valid remaining subset of A}}{\text{new universe}}$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{3}{9}$$

Flip a coin 5 times $\quad |\text{sample space}| = 2^5$

Flip a coin 5 times $\quad |\text{sample space}| = 2^5 \quad P(\text{1st flip} = T) = \frac{1}{2}$

Flip a coin 5 times $\quad |\text{sample space}| = 2^5 \quad P(F) = P(\text{1st flip} = T) = \frac{1}{2}$

Flip a coin 5 times $\quad |\text{sample space}| = 2^5 \quad P(F) = P(\underline{\text{1st flip} = T}) = \frac{1}{2}$

What if you are told that 3 of the 5 flips were H?

Flip a coin 5 times $|\text{sample space}| = 2^5$ $P(F) = P(\underline{\text{1st flip} = T}) = \frac{1}{2}$

What if you are told that $\underbrace{\text{3 of the 5 flips were H}}$? "3H"

$$P(F \mid 3H) = ?$$

Flip a coin 5 times $\quad |\text{sample space}| = 2^5 \quad P(F) = P(\underline{\text{1st flip} = T}) = \frac{1}{2}$

What if you are told that 3 of the 5 flips were H? "3H"

$$P(F \mid 3H) = ?$$

quick intuition:

if 3 of 5 were H,

then for any flip the probability of H is $\frac{3}{5}$

so for T it's $\frac{2}{5}$

Flip a coin 5 times $\quad |\text{sample space}| = 2^5 \quad P(F) = P(\underline{\text{1st flip} = T}) = \frac{1}{2}$

What if you are told that $\underbrace{\text{3 of the 5 flips were H}}$? "3H"

$$\boxed{P(A|B) = \frac{P(A \cap B)}{P(B)}}$$

$$P(F|3H) = \frac{P(F \cap 3H)}{P(3H)}$$

Flip a coin 5 times |sample space| $= 2^5$ $P(F) = P(\text{1st flip} = T) = \frac{1}{2}$

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Flip a coin 5 times |sample space| $= 2^5$ $P(F) = P(\underline{\text{1st flip} = T}) = \frac{1}{2}$

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$$\boxed{P(A|B) = \frac{P(A \cap B)}{P(B)}}$$

$$P(F|3H) = \frac{P(F \cap 3H)}{P(3H)}$$

$$\boxed{\begin{array}{ll} T & HHHT \\ T & HHTH \\ T & HTHH \\ T & THHH \end{array}}$$

$P(F \cap 3H) = \ldots$

Flip a coin 5 times $\quad |\text{sample space}| = 2^5 \quad P(F) = P(\underline{\text{1st flip} = T}) = \frac{1}{2}$

What if you are told that $\underline{\text{3 of the 5 flips were H}}$? "3H"

$$\boxed{P(A|B) = \frac{P(A \cap B)}{P(B)}}$$

$$P(F|3H) = \frac{P(F \cap 3H)}{P(3H)}$$

$$\boxed{\begin{matrix} T & HHHT \\ T & HHTH \\ T & HTHH \\ T & THHH \end{matrix}} \quad P(F \cap 3H) = \frac{\binom{4}{3}}{2^5}$$

Flip a coin 5 times |sample space| $= 2^5$ $P(F) = P(\underline{\text{1st flip} = T}) = \frac{1}{2}$

What if you are told that $\underbrace{\text{3 of the 5 flips were H}}$? "3H"

$$\boxed{P(A|B) = \frac{P(A \cap B)}{P(B)}}$$

$$P(F|3H) = \frac{P(F \cap 3H)}{P(3H)}$$

T	HHHT
T	HHTH
T	HTHH
T	THHH

$$P(F \cap 3H) = \frac{\binom{4}{3}}{2^5} = \frac{4}{32}$$

Flip a coin 5 times $\quad |\text{sample space}| = 2^5 \quad P(F) = P(\underline{1\text{st flip} = T}) = \frac{1}{2}$

What if you are told that 3 of the 5 flips were H? "3H"

$$\boxed{P(A|B) = \frac{P(A \cap B)}{P(B)}}$$

$$P(F|3H) = \frac{P(F \cap 3H)}{P(3H)}$$

T	HHHT
T	HHTH
T	HTHH
T	THHH

$$P(F \cap 3H) = \frac{\binom{4}{3}}{2^5} = \frac{4}{32}$$

$$P(3H) = ?$$

Flip a coin 5 times $\quad$ $|\text{sample space}| = 2^5$ $\quad$ $P(F) = P(\underline{\text{1st flip} = T}) = \frac{1}{2}$

What if you are told that $\underbrace{\text{3 of the 5 flips were H}}$? "3H"

$$\boxed{P(A|B) = \frac{P(A \cap B)}{P(B)}}$$

$$P(F|3H) = \frac{P(F \cap 3H)}{P(3H)}$$

T	HHHT
T	HHTH
T	HTHH
T	THHH

$$P(F \cap 3H) = \frac{\binom{4}{3}}{2^5} = \frac{4}{32}$$

$$P(3H) = \frac{\binom{5}{3}}{2^5}$$

Flip a coin 5 times $\quad |\text{sample space}| = 2^5 \quad P(F) = P(\underbrace{\text{1st flip} = T}_{}) = \frac{1}{2}$

What if you are told that 3 of the 5 flips were H? "3H"

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$$\boxed{\begin{matrix} T & HHHT \\ T & HHTH \\ T & HTHH \\ T & THHH \end{matrix}}$$

$$P(F \cap 3H) = \frac{\binom{4}{3}}{2^5} = \frac{4}{32}$$

$$P(3H) = \frac{\binom{5}{3}}{2^5} = \frac{\frac{5!}{3!2!}}{32} = \frac{10}{32}$$

Flip a coin 5 times $\quad |\text{sample space}| = 2^5 \quad P(F) = P(\underline{\text{1st flip} = T}) = \frac{1}{2}$

What if you are told that 3 of the 5 flips were H? "3H"

$$\boxed{P(A|B) = \frac{P(A \cap B)}{P(B)}}$$

$$P(F|3H) = \frac{P(F \cap 3H)}{P(3H)} = \frac{2}{5}$$

$$\boxed{\begin{matrix} T & HHHT \\ T & HHTH \\ T & HTHH \\ T & THHH \end{matrix}} \quad P(F \cap 3H) = \frac{\binom{4}{3}}{2^5} = \frac{4}{32}$$

$$P(3H) = \frac{\binom{5}{3}}{2^5} = \frac{\frac{5!}{3!2!}}{32} = \frac{10}{32}$$

$$P(F) = \frac{1}{2}$$

unconditional probability

$$P(F \mid 3H) = \frac{2}{5}$$

conditional probability

given extra information

$$P(F) = \frac{1}{2}$$

unconditional probability

$$P(F|3H) = \frac{2}{5}$$

conditional probability

given extra information

$$P(F) \neq P(F|3H)$$

The probability of F happening <u>depends</u> on 3H

flip a coin 3 times

$|\text{sample space}| = 2^3$

a	b	c
H	H	H
H	H	T
H	T	H
H	T	T
T	H	H
T	H	T
T	T	H
T	T	T

flip a coin 3 times

|sample space| $= 2^3$

$P(c = T) = \frac{1}{2}$

a b c

HHH
HHT
HTH
HTT
THH
THT
TTH
TTT

flip a coin 3 times

$|\text{sample space}| = 2^3$

$P(c = T) = \frac{1}{2}$

a	b	c
H	H	H
H	H	T
H	T	H
H	T	T
T	H	H
T	H	T
T	T	H
T	T	T

$$P\big((c = T) \mid (a = H)\big) = ?$$

flip a coin 3 times

$|\text{sample space}| = 2^3$

$P(c=T) = \frac{1}{2}$

a	b	c
H	H	H
H	H	T
H	T	H
H	T	T
T	H	H
T	H	T
T	T	H
T	T	T

$$P\big((c=T) \mid (a=H)\big) = \frac{P\big((c=T) \cap (a=H)\big)}{P(a=H)} = \frac{?}{?}$$

flip a coin 3 times

$|\text{sample space}| = 2^3$

$P(c=T) = \frac{1}{2}$

$P(a=H) = \frac{1}{2}$

a	b	c
H	H	H
H	H	T
H	T	H
H	T	T
T	H	H
T	H	T
T	T	H
T	T	T

$$P\big((c=T) \mid (a=H)\big) = \frac{P\big((c=T) \cap (a=H)\big)}{P(a=H)} = \frac{?}{1/2}$$

flip a coin 3 times

$|\text{sample space}| = 2^3$

$P(c = T) = \frac{1}{2}$

$P(a = H) = \frac{1}{2}$

a	b	c
H	H	H
H	H	T
H	T	H
H	T	T
T	H	H
T	H	T
T	T	H
T	T	T

$$P\big((c = T) \mid (a = H)\big) = \frac{P\big((c = T) \cap (a = H)\big)}{P(a = H)} = \frac{2/8}{1/2}$$

flip a coin 3 times

$|\text{sample space}| = 2^3$

$P(c=T) = \frac{1}{2}$

$P(a=H) = \frac{1}{2}$

a	b	c
H	H	H
H	H	T
H	T	H
H	T	T
T	H	H
T	H	T
T	T	H
T	T	T

$$P\big((c=T) \mid (a=H)\big) = \frac{P\big((c=T) \cap (a=H)\big)}{P(a=H)} = \frac{2/8}{1/2} = \frac{1}{2}$$

flip a coin 3 times

$|\text{sample space}| = 2^3$

$P(c = T) = \frac{1}{2}$

$P(a = H) = \frac{1}{2}$

a	b	c
H	H	H
H	H	T
H	T	H
H	T	T
T	H	H
T	H	T
T	T	H
T	T	T

$$P\big((c = T) \mid (a = H)\big) = \frac{P\big((c = T) \cap (a = H)\big)}{P(a = H)} = \frac{2/8}{1/2} = \frac{1}{2}$$

- knowledge of a=H had no effect on c

flip a coin 3 times

$|\text{sample space}| = 2^3$

$P(c=T) = \frac{1}{2}$

$P(a=H) = \frac{1}{2}$

a	b	c
H	H	H
H	H	T
H	T	H
H	T	T
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$$P\big((c=T) \mid (a=H)\big) = \frac{P\big((c=T) \cap (a=H)\big)}{P(a=H)} = \frac{2/8}{1/2} = \frac{1}{2}$$

- knowledge of $a=H$ had no effect on c
- the outcome for c is independent of the outcome for a

flip a coin 3 times

$|\text{sample space}| = 2^3$

$P(c = T) = \frac{1}{2}$

$P(a = H) = \frac{1}{2}$

a	b	c
H	H	H
H	H	T
H	T	H
H	T	T
T	H	H
T	H	T
T	T	H
T	T	T

$$P\big((c = T) \mid (a = H)\big) = \frac{P\big((c = T) \cap (a = H)\big)}{P(a = H)} = \frac{2/8}{1/2} = \frac{1}{2}$$

- knowledge of $a = H$ had no effect on c
- the outcome for c is independent of the outcome for a
- events c and a are independent

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} \quad \leftarrow \text{identical} \rightarrow \quad P(B|A) = \frac{P(A \cap B)}{P(A)}$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} \quad \longleftarrow \text{identical} \longrightarrow \quad P(B|A) = \frac{P(A \cap B)}{P(A)}$$

$$P(A \cap B) = P(A) \cdot P(B|A)$$

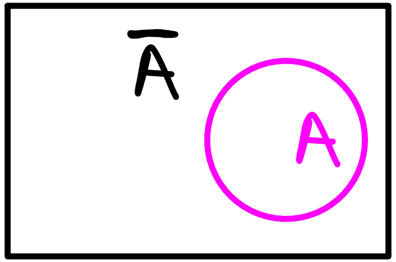
$$P(A|B) = \frac{P(A \cap B)}{P(B)} \quad \leftarrow \text{identical} \rightarrow \quad P(B|A) = \frac{P(A \cap B)}{P(A)}$$

$$P(B) \cdot P(A|B) = P(A \cap B) = P(A) \cdot P(B|A)$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} \quad \leftarrow \text{identical} \rightarrow \quad P(B|A) = \frac{P(A \cap B)}{P(A)}$$

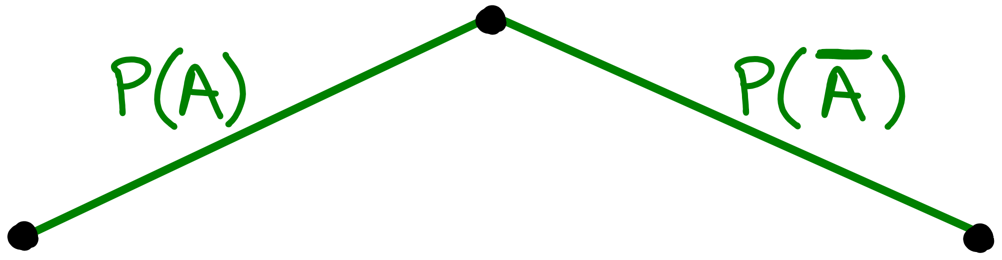
$$P(B) \cdot P(A|B) = P(A \cap B) = P(A) \cdot P(B|A)$$

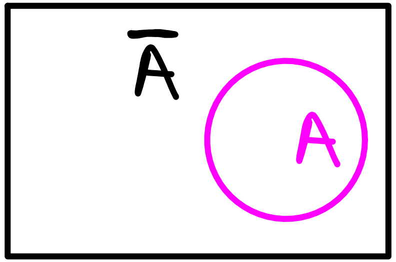
$$P(A) \qquad + \qquad P(\bar{A}) = 1$$



$$P(A|B) = \frac{P(A \cap B)}{P(B)} \quad \leftarrow \text{identical} \rightarrow \quad P(B|A) = \frac{P(A \cap B)}{P(A)}$$

$$P(B) \cdot P(A|B) = P(A \cap B) = P(A) \cdot P(B|A)$$

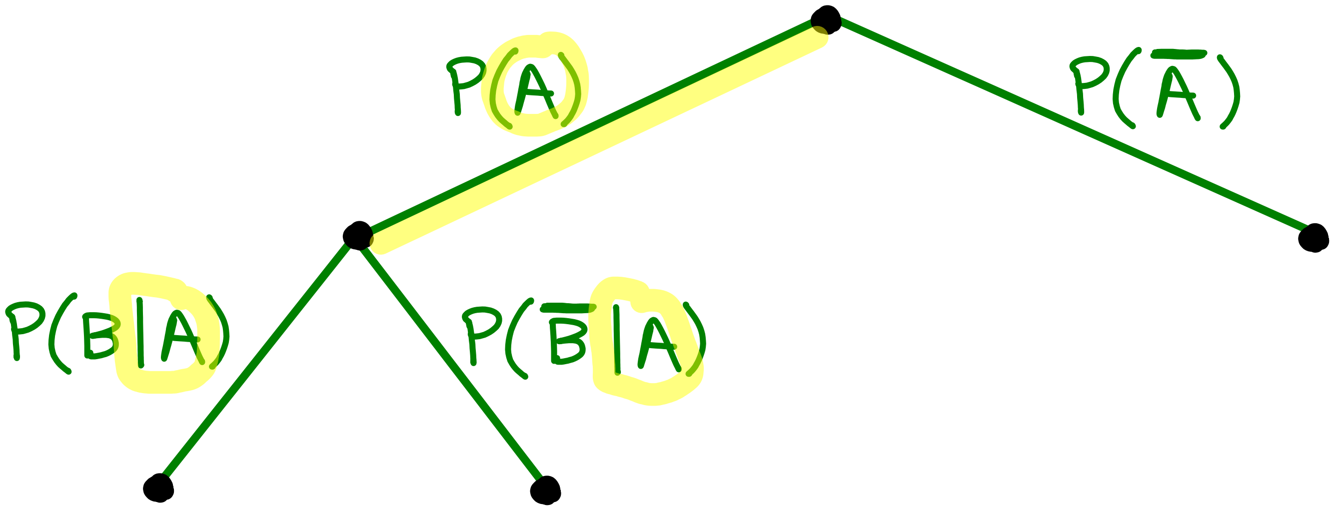


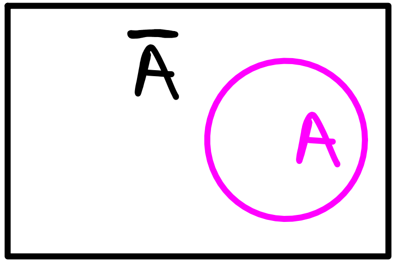


$$P(A) \quad + \quad P(\bar{A}) = 1$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} \quad \leftarrow \text{identical} \rightarrow \quad P(B|A) = \frac{P(A \cap B)}{P(A)}$$

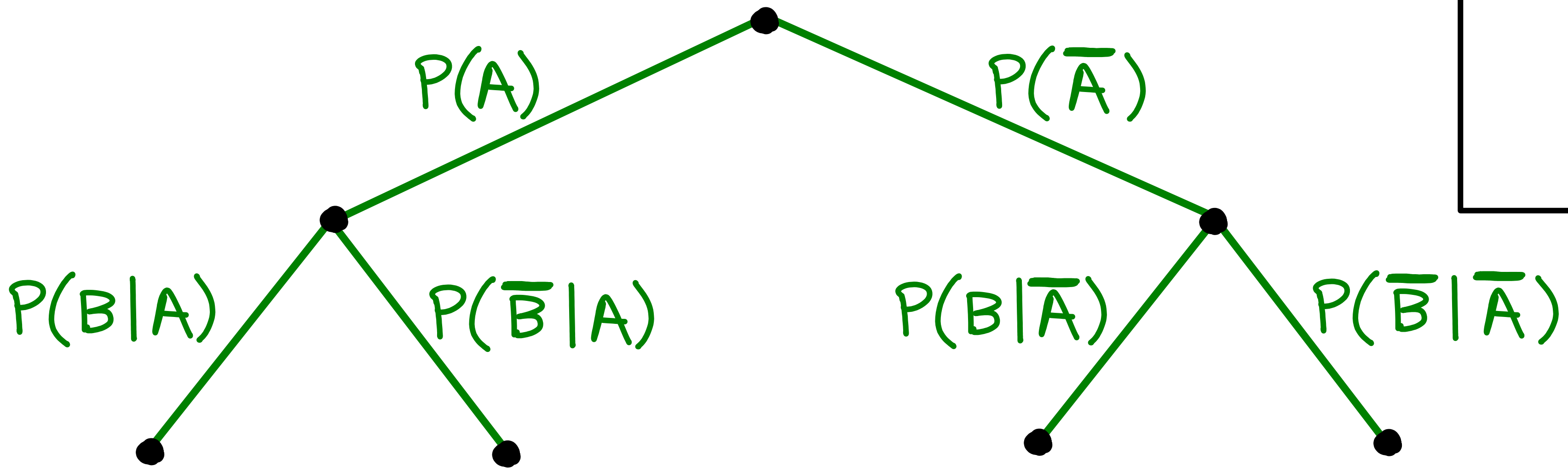
$$P(B) \cdot P(A|B) = P(A \cap B) = P(A) \cdot P(B|A)$$

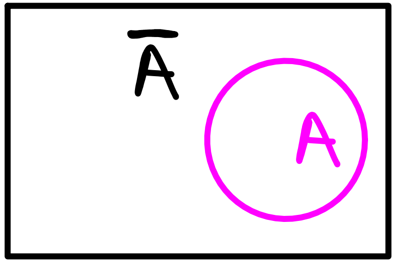




$$P(A|B) = \frac{P(A \cap B)}{P(B)} \quad \leftarrow \text{identical} \rightarrow \quad P(B|A) = \frac{P(A \cap B)}{P(A)}$$

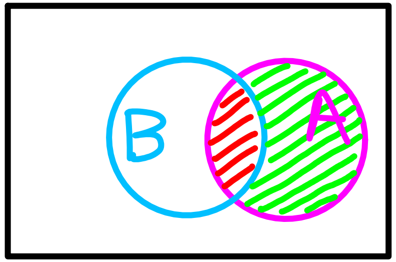
$$P(B) \cdot P(A|B) = P(A \cap B) = P(A) \cdot P(B|A)$$

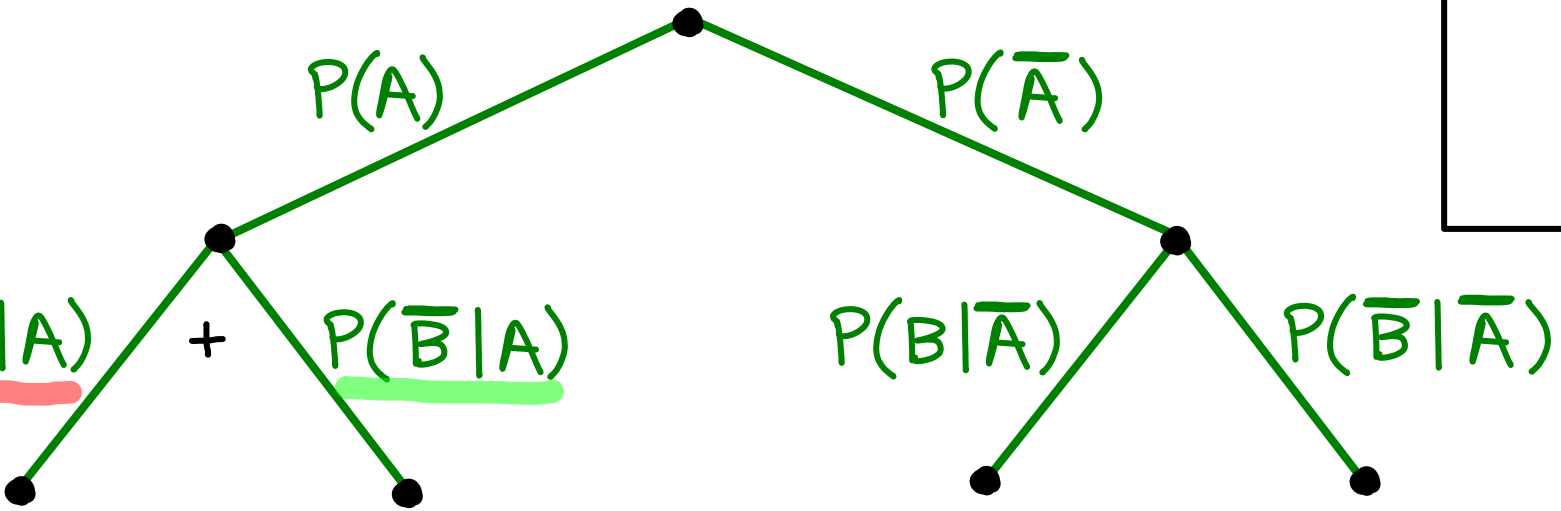


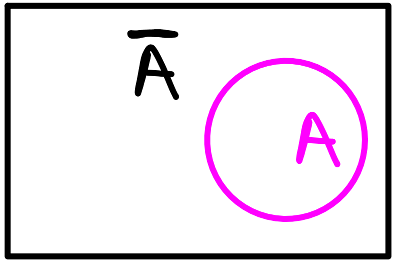


$$P(A|B) = \frac{P(A \cap B)}{P(B)} \quad \xleftarrow{\text{identical}} \!\!\!\! \xrightarrow{} \quad P(B|A) = \frac{P(A \cap B)}{P(A)}$$

$$P(B) \cdot P(A|B) = P(A \cap B) = P(A) \cdot P(B|A)$$



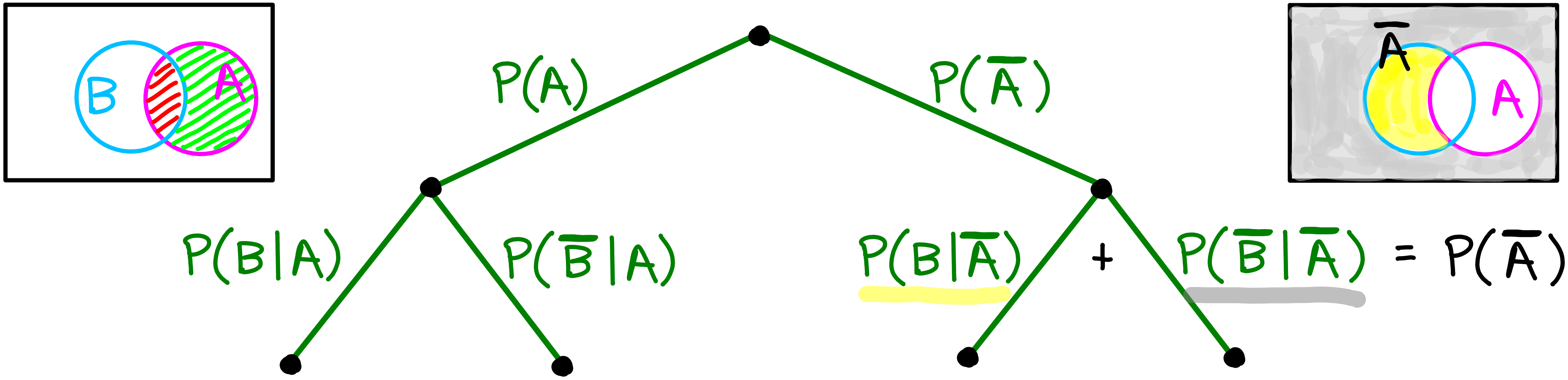




$$P(A) = P(B|A) + P(\bar{B}|A)$$

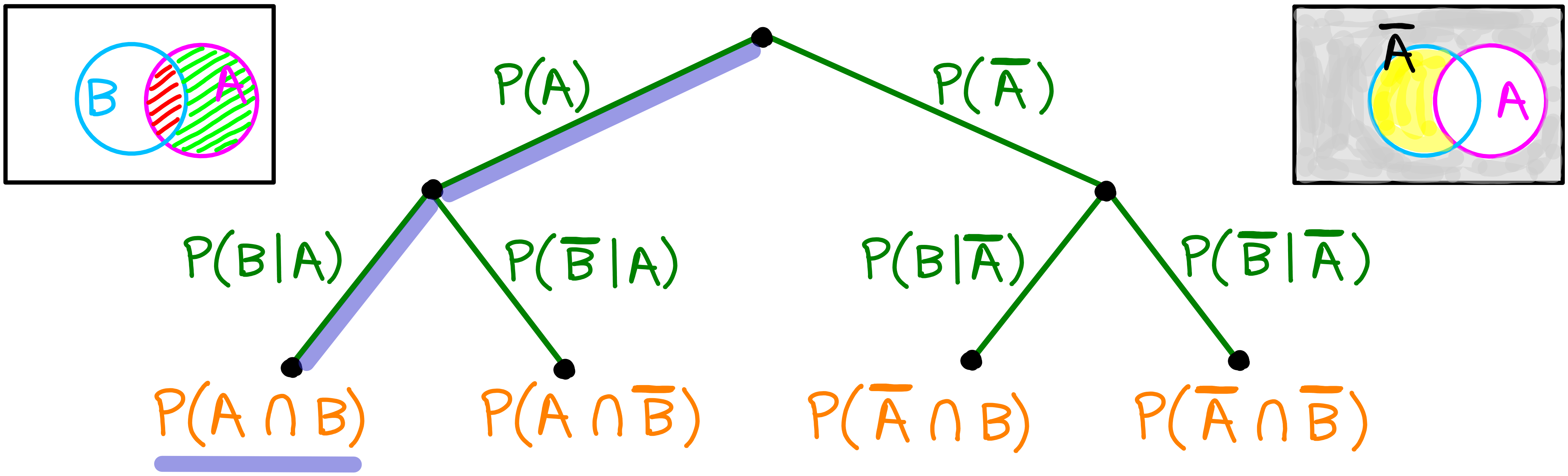
$$P(A|B) = \frac{P(A \cap B)}{P(B)} \quad \leftarrow \text{identical} \rightarrow \quad P(B|A) = \frac{P(A \cap B)}{P(A)}$$

$$P(B) \cdot P(A|B) = P(A \cap B) = P(A) \cdot P(B|A)$$



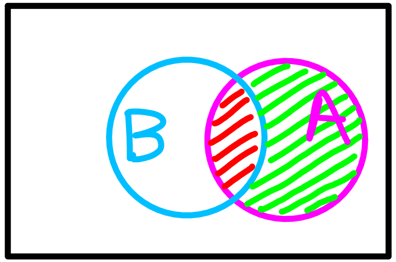
$$P(A|B) = \frac{P(A \cap B)}{P(B)} \quad \leftarrow \text{identical} \rightarrow \quad P(B|A) = \frac{P(A \cap B)}{P(A)}$$

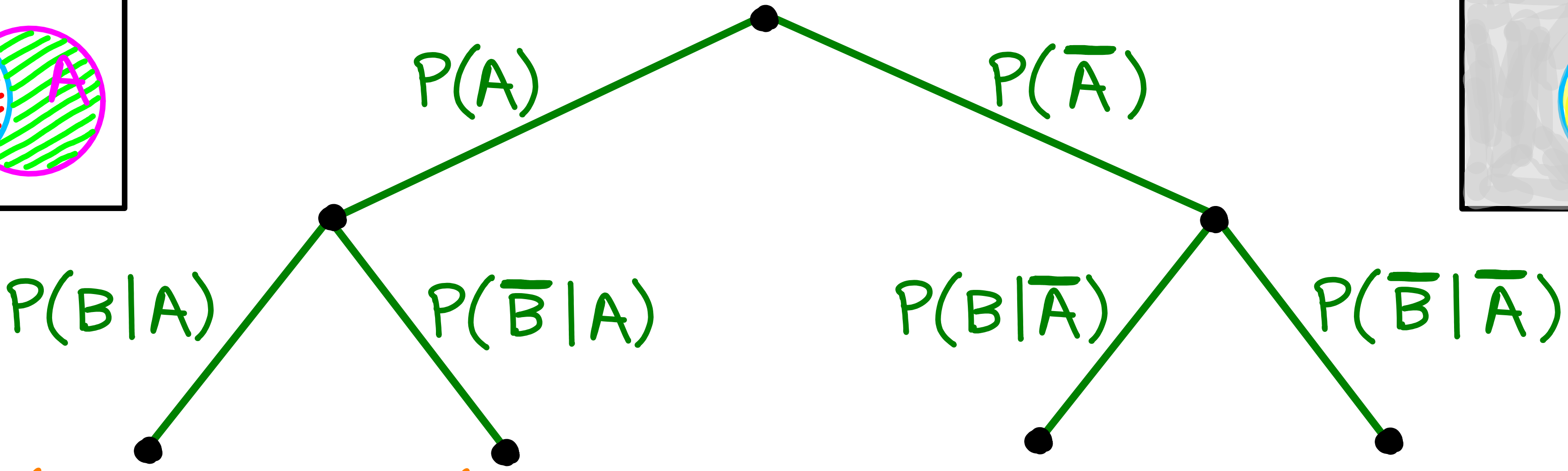
$$P(B) \cdot P(A|B) = \underline{P(A \cap B)} = P(A) \cdot P(B|A)$$

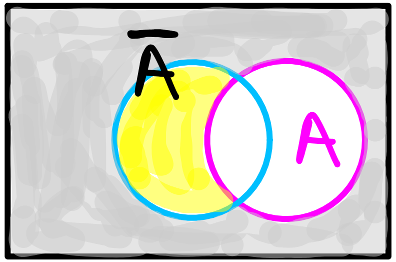


$$P(A|B) = \frac{P(A \cap B)}{P(B)} \quad \leftarrow \text{identical} \rightarrow \quad P(B|A) = \frac{P(A \cap B)}{P(A)}$$

$$P(B) \cdot P(A|B) = P(A \cap B) = P(A) \cdot P(B|A)$$



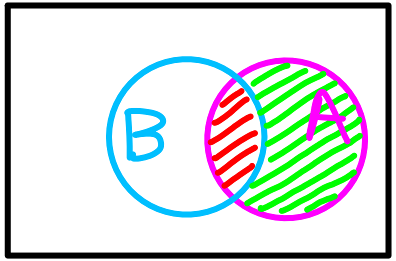


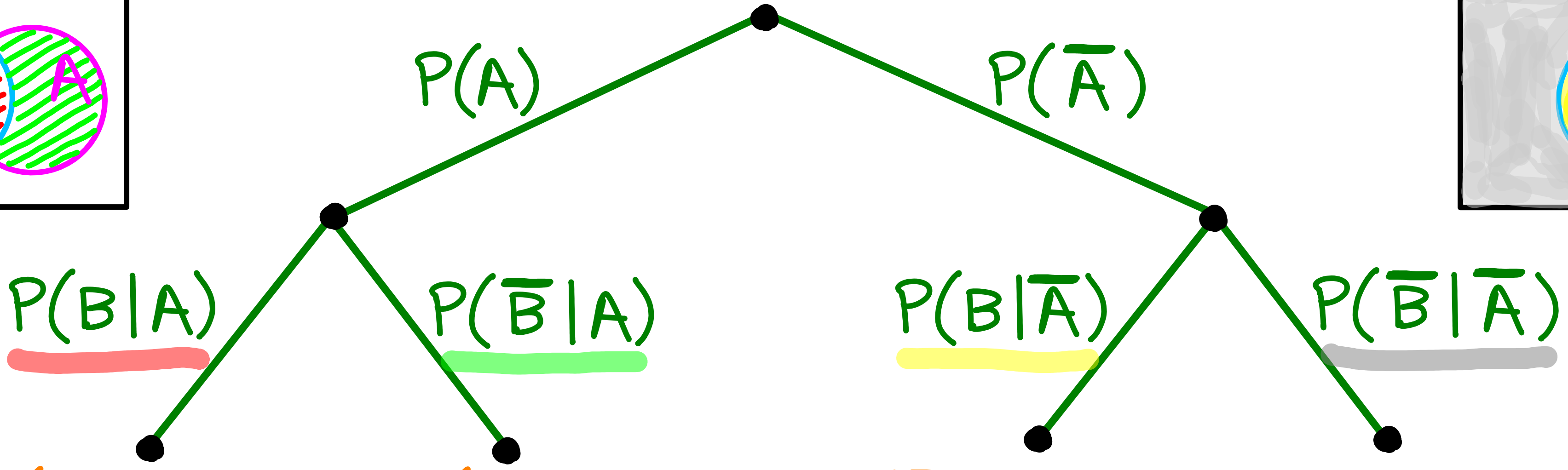


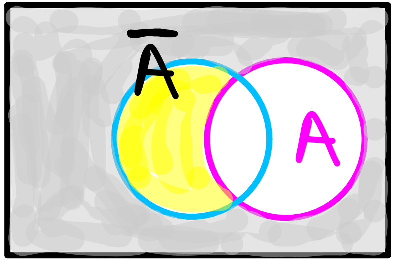
$$\underbrace{P(A \cap B) + P(A \cap \overline{B})}_{P(A)} + \underbrace{P(\overline{A} \cap B) + P(\overline{A} \cap \overline{B})}_{P(\overline{A})} = 1$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} \quad \leftarrow \text{identical} \rightarrow \quad P(B|A) = \frac{P(A \cap B)}{P(A)}$$

$$P(B) \cdot P(A|B) = P(A \cap B) = P(A) \cdot P(B|A)$$







$$\underbrace{P(A \cap B) + P(A \cap \overline{B})}_{P(A)} + \underbrace{P(\overline{A} \cap B) + P(\overline{A} \cap \overline{B})}_{P(\overline{A})} = 1$$

Roll one die : $P(\text{prime AND even})$?

Roll one die : $P(\text{prime AND even})$?

$$P(\text{prime}) = \underbrace{\frac{3}{6}}_{}$$

$$\overbrace{2, 3, 5}$$

$$P(\text{even}) = \frac{3}{6}$$

$$\overbrace{2, 4, 6}$$

Roll one die : $P(\text{prime AND even})$?

$$= P(\text{prime}) \cdot P(\text{even} \mid \text{prime})$$

$$\underbrace{P(\text{prime})}_{}{} \quad \overbrace{2,3,5}$$

$$\overbrace{2,3,5}\qquad \overbrace{2,4,6}$$

$$P(\text{prime}) = \frac{3}{6} \qquad P(\text{even}) = \frac{3}{6}$$

Roll one die : $P(\text{prime AND even})$?

$$= P(\text{prime}) \cdot P(\text{even} \mid \text{prime})$$

$$\underbrace{2, 3, 5}\qquad \underbrace{2, 4, 6}\qquad \underbrace{2}$$

$$P(\text{prime}) = \frac{3}{6} \qquad P(\text{even}) = \frac{3}{6} \qquad P(\text{even} \mid \text{prime}) = \frac{1}{3}$$

Roll one die: $P(\text{prime AND even})$?

$$= P(\text{prime}) \cdot P(\text{even} \mid \text{prime})$$

$$= \frac{3}{6} \cdot \frac{1}{3} = \frac{1}{6}$$

$$P(\text{prime}) = \frac{\overbrace{2,3,5}}{6} = \frac{3}{6}$$

$$P(\text{even}) = \frac{\overbrace{2,4,6}}{6} = \frac{3}{6}$$

$$P(\text{even} \mid \text{prime}) = \frac{\overbrace{2}}{3} = \frac{1}{3}$$

$$P(A \cap B) = P(A) \cdot P(B|A)$$

$$P(A \cap B) = P(A) \cdot P(B|A)$$

$$P(A \cap B \cap C) = ?$$

$$P(A \cap B) = P(A) \cdot P(B|A)$$

$$P(A \cap B \cap C) = P((A \cap B) \cap C)$$

$$= ?$$

$$P(A \cap B) = P(A) \cdot P(B \mid A)$$

$$P(A \cap B \cap C) = P((A \cap B) \cap C)$$

$$= P(A \cap B) \cdot P(C \mid (A \cap B))$$

$$= ?$$

$$P(A \cap B) = P(A) \cdot P(B|A)$$

$$P(A \cap B \cap C) = P((A \cap B) \cap C)$$

$$= P(A \cap B) \cdot P(C|(A \cap B))$$

$$= P(A) \cdot P(B|A) \cdot P(C|(A \cap B))$$

$$P(A \cap B) = P(A) \cdot P(B|A)$$

$$P(A \cap B \cap C) = P((A \cap B) \cap C)$$

$$= P(A \cap B) \cdot P(C|(A \cap B))$$

$$= P(A) \cdot P(B|A) \cdot P(C|(A \cap B))$$

$$P(A \cap B \cap C \cap D) = ?$$

$$P(A \cap B) = P(A) \cdot P(B|A)$$

$$P(A \cap B \cap C) = P((A \cap B) \cap C)$$

$$= P(A \cap B) \cdot P(C|(A \cap B))$$

$$= P(A) \cdot P(B|A) \cdot P(C|(A \cap B))$$

$$P(A \cap B \cap C \cap D) = P((A \cap B \cap C) \cap D)$$

$$= ?$$

$$P(A \cap B) = P(A) \cdot P(B|A)$$

$$P(A \cap B \cap C) = P((A \cap B) \cap C)$$

$$= P(A \cap B) \cdot P(C|(A \cap B))$$

$$= P(A) \cdot P(B|A) \cdot P(C|(A \cap B))$$

$$P(A \cap B \cap C \cap D) = P((A \cap B \cap C) \cap D)$$

$$= P(A \cap B \cap C) \cdot P(D|(A \cap B \cap C))$$

$$= ?$$

$$P(A \cap B) = P(A) \cdot P(B|A)$$

$$P(A \cap B \cap C) = P((A \cap B) \cap C)$$

$$= P(A \cap B) \cdot P(C|(A \cap B))$$

$$= P(A) \cdot P(B|A) \cdot P(C|(A \cap B))$$

$$P(A \cap B \cap C \cap D) = P((A \cap B \cap C) \cap D)$$

$$= P(A \cap B \cap C) \cdot P(D|(A \cap B \cap C))$$

$$= P(A) \cdot P(B|A) \cdot P(C|(A \cap B)) \cdot P(D|(A \cap B \cap C))$$

$$P(e_1 \cap \ldots \cap e_n) = ?$$

$$P(A \cap B) = P(A) \cdot P(B|A)$$

$$P(A \cap B \cap C) = P((A \cap B) \cap C)$$

$$= P(A \cap B) \cdot P(C|(A \cap B))$$

$$= P(A) \cdot P(B|A) \cdot P(C|(A \cap B))$$

$$P(A \cap B \cap C \cap D) = P((A \cap B \cap C) \cap D)$$

$$= P(A \cap B \cap C) \cdot P(D|(A \cap B \cap C))$$

$$= P(A) \cdot P(B|A) \cdot P(C|(A \cap B)) \cdot P(D|(A \cap B \cap C))$$

$$P(e_1 \cap \ldots \cap e_n) = P(e_1) \cdot \,?$$

$$P(A \cap B) = P(A) \cdot P(B|A)$$

$$P(A \cap B \cap C) = P((A \cap B) \cap C)$$

$$= P(A \cap B) \cdot P(C|(A \cap B))$$

$$= P(A) \cdot P(B|A) \cdot P(C|(A \cap B))$$

$$P(A \cap B \cap C \cap D) = P((A \cap B \cap C) \cap D)$$

$$= P(A \cap B \cap C) \cdot P(D|(A \cap B \cap C))$$

$$= P(A) \cdot P(B|A) \cdot P(C|(A \cap B)) \cdot P(D|(A \cap B \cap C))$$

$$P(e_1 \cap \ldots \cap e_n) = P(e_1) \cdot \prod_{i=2}^{n} ?$$

$$P(A \cap B) = P(A) \cdot P(B|A)$$

$$P(A \cap B \cap C) = P((A \cap B) \cap C)$$
$$= P(A \cap B) \cdot P(C|(A \cap B))$$
$$= P(A) \cdot P(B|A) \cdot P(C|(A \cap B))$$

$$P(A \cap B \cap C \cap D) = P((A \cap B \cap C) \cap D)$$
$$= P(A \cap B \cap C) \cdot P(D|(A \cap B \cap C))$$
$$= P(A) \cdot P(B|A) \cdot P(C|(A \cap B)) \cdot P(D|(A \cap B \cap C))$$

$$P(e_1 \cap \ldots \cap e_n) = P(e_1) \cdot \prod_{i=2}^{n} P(e_i \mid e_1 \cap \ldots \cap e_{i-1})$$

$$P(A \cap B) = P(A) \cdot P(B \mid A)$$

$$\begin{aligned} P(A \cap B \cap C) &= P((A \cap B) \cap C) \\ &= P(A \cap B) \cdot P(C \mid (A \cap B)) \\ &= P(A) \cdot P(B \mid A) \cdot P(C \mid (A \cap B)) \end{aligned}$$

$$\begin{aligned} P(A \cap B \cap C \cap D) &= P((A \cap B \cap C) \cap D) \\ &= P(A \cap B \cap C) \cdot P(D \mid (A \cap B \cap C)) \\ &= P(A) \cdot P(B \mid A) \cdot P(C \mid (A \cap B)) \cdot P(D \mid (A \cap B \cap C)) \end{aligned}$$

$$P(A \cap B) = P(A) \cdot P(B|A)$$

$$P(e_1 \cap \ldots \cap e_n) = P(e_1) \cdot \prod_{i=2}^{n} P(e_i | e_1 \cap \ldots \cap e_{i-1})$$

$$P\left(\bigcap_{i=1}^{n} e_i\right) = P(e_1) \cdot \prod_{i=2}^{n} P\left(e_i \middle| \bigcap_{j=1}^{i-1} e_j\right)$$

$$P(A \cap B \cap C) = P((A \cap B) \cap C)$$

$$= P(A \cap B) \cdot P(C|(A \cap B))$$

$$= P(A) \cdot P(B|A) \cdot P(C|(A \cap B))$$

$$P(A \cap B \cap C \cap D) = P((A \cap B \cap C) \cap D)$$

$$= P(A \cap B \cap C) \cdot P(D|(A \cap B \cap C))$$

$$= P(A) \cdot P(B|A) \cdot P(C|(A \cap B)) \cdot P(D|(A \cap B \cap C))$$

$$P(A \cap B) = P(A) \cdot P(B|A)$$

$$P(A \cap B \cap C) = P((A \cap B) \cap C)$$

$$= P(A \cap B) \cdot P(C|(A \cap B))$$

$$= P(A) \cdot P(B|A) \cdot P(C|(A \cap B))$$

$$P(A \cap B \cap C \cap D) = P((A \cap B \cap C) \cap D)$$

$$= P(A \cap B \cap C) \cdot P(D|(A \cap B \cap C))$$

$$= P(A) \cdot P(B|A) \cdot P(C|(A \cap B)) \cdot P(D|(A \cap B \cap C))$$

$$P(e_1 \cap \ldots \cap e_n) = P(e_1) \cdot \prod_{i=2}^{n} P(e_i | e_1 \cap \ldots \cap e_{i-1})$$

$$P\left(\bigcap_{i=1}^{n} e_i\right) = P(e_1) \cdot \prod_{i=2}^{n} P\left(e_i \middle| \bigcap_{j=1}^{i-1} e_j\right)$$

$$= \prod_{i=1}^{n} P\left(e_i \middle| \bigcap_{j=1}^{i-1} e_j\right)$$

INDEPENDENCE

INDEPENDENCE

definition: A & B are independent $\longleftrightarrow$ $P(A \cap B) = P(A) \cdot P(B)$

INDEPENDENCE

definition: A & B are independent $\longleftrightarrow$ $\underbrace{P(A \cap B) = P(A) \cdot P(B)}_{\text{symmetry}}$

INDEPENDENCE

definition: A & B are independent $\longleftrightarrow$ $\underbrace{P(A \cap B) = P(A) \cdot P(B)}_{\text{symmetry}}$

we already know: $P(B|A) = \dfrac{P(A \cap B)}{P(A)}$

$P(A|B) = \dfrac{P(A \cap B)}{P(B)}$

INDEPENDENCE

definition: A & B are independent $\longleftrightarrow$ $\underbrace{P(A \cap B) = P(A) \cdot P(B)}_{\text{symmetry}}$

we already know: $P(B|A) = \dfrac{P(A \cap B)}{P(A)}$ $\qquad$ $P(A|B) = \dfrac{P(A \cap B)}{P(B)}$

if independent: $P(B|A) = \dfrac{P(A) \cdot P(B)}{P(A)}$ $\qquad$ $P(A|B) = \dfrac{P(A) \cdot P(B)}{P(B)}$

INDEPENDENCE

definition: A & B are independent $\longleftrightarrow$ $\underbrace{P(A \cap B) = P(A) \cdot P(B)}_{\text{symmetry}}$

we already know: $P(B|A) = \dfrac{P(A \cap B)}{P(A)}$

$P(A|B) = \dfrac{P(A \cap B)}{P(B)}$

if independent: $P(B|A) = \dfrac{P(A) \cdot P(B)}{P(A)}$

$P(A|B) = \dfrac{P(A) \cdot P(B)}{P(B)}$

$P(B|A) = P(B)$

$P(A|B) = P(A)$

INDEPENDENCE

definition: A & B are independent $\longleftrightarrow$ $\underbrace{P(A \cap B) = P(A) \cdot P(B)}_{\text{symmetry}}$

we already know: $P(B|A) = \dfrac{P(A \cap B)}{P(A)}$ $\quad$ $P(A|B) = \dfrac{P(A \cap B)}{P(B)}$

if independent: $P(B|A) = \dfrac{P(A) \cdot P(B)}{P(A)}$ $\quad$ $P(A|B) = \dfrac{P(A) \cdot P(B)}{P(B)}$

*$P(A) \neq 0$ $\quad$ $P(B|A) = P(B)$ $\quad$ $P(A|B) = P(A)$ $\quad$ *$P(B) \neq 0$

INDEPENDENCE

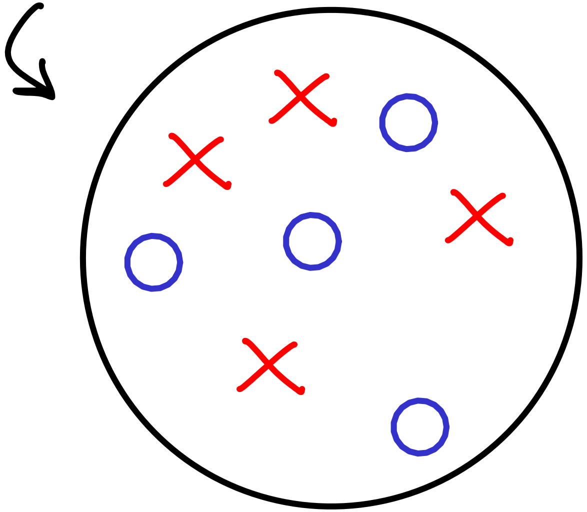
A, B, C are independent $\longleftrightarrow$ $\underbrace{P(A \cap B \cap C) = P(A) \cdot P(B) \cdot P(C)}_{\text{symmetry}}$

INDEPENDENCE

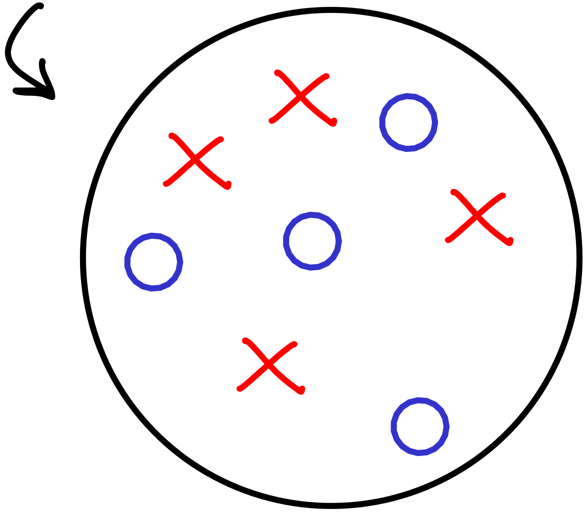
$$A, B, C \text{ are independent} \longleftrightarrow \underbrace{P(A \cap B \cap C) = P(A) \cdot P(B) \cdot P(C)}_{\text{symmetry}}$$

$$P(e_1 \cap \ldots \cap e_n) = P\left(\bigcap_{i=1}^{n} e_i\right) = \prod_{i=1}^{n} P(e_i)$$

Select 2 objects from this group (order matters, no replacement)



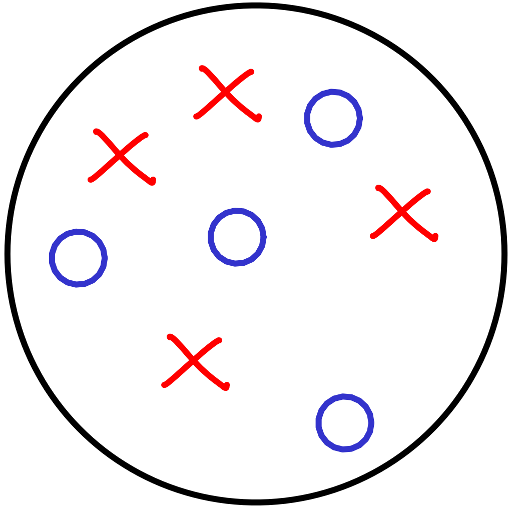
Select 2 objects from this group (order matters, no replacement)



event A: 1^{st} choice is X

event B: 2^{nd} choice is X

Select 2 objects from this group (order matters, no replacement)

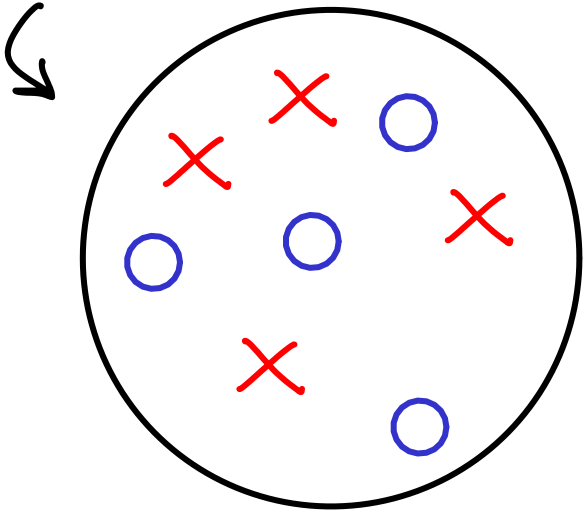


event A: 1^{st} choice is X

event B: 2^{nd} choice is X

$$P(A) = \frac{4}{8}$$

Select 2 objects from this group (order matters, no replacement)

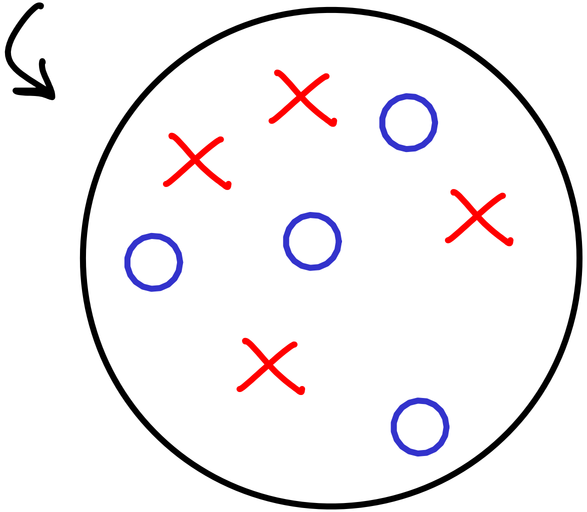


event A: 1st choice is X

event B: 2nd choice is X

$P(A) = \frac{4}{8}$ $\qquad$ $P(B) = ?$

Select 2 objects from this group (order matters, no replacement)

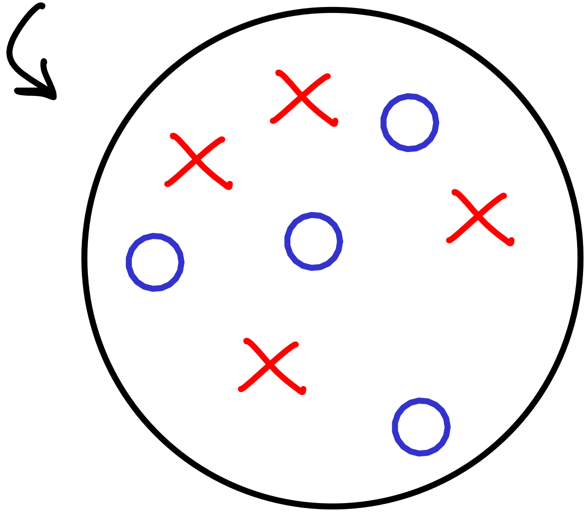


event A: 1^{st} choice is X

event B: 2^{nd} choice is X

$P(A) = \frac{4}{8}$ $P(B) = \frac{4}{8}$ symmetric

Select 2 objects from this group (order matters, no replacement)



$|\text{sample space}| = P(8,2) = 8 \cdot 7 = 56$

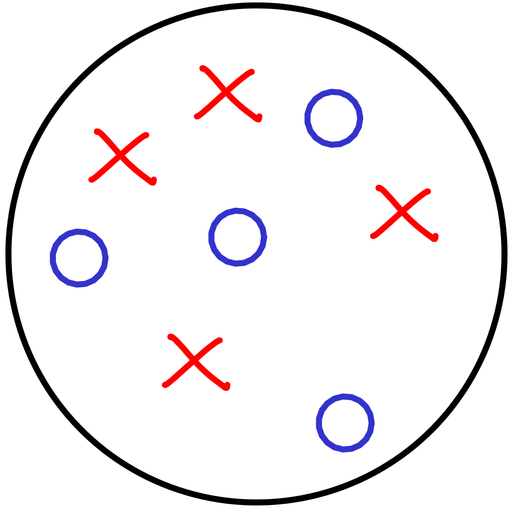
event A: 1^{st} choice is ×

event B: 2^{nd} choice is ×

$P(A) = \frac{4}{8}$ $P(B) = \frac{4}{8}$ symmetric

...or examine sample space

Select 2 objects from this group (order matters, no replacement)



$|\text{sample space}| = P(8,2) = 8 \cdot 7 = 56$

event A: 1st choice is X

event B: 2nd choice is X

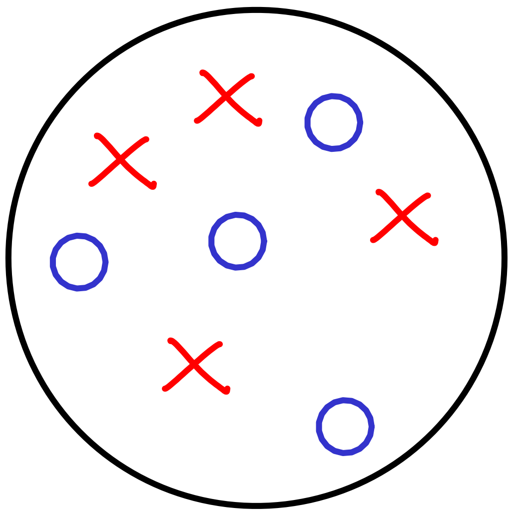
$P(A) = \frac{4}{8}$

$P(B) = \frac{4}{8}$ symmetric

...or examine sample space

$P(B|A) = ?$

Select 2 objects from this group (order matters, no replacement)



|sample space| $= P(8,2) = 8 \cdot 7 = 56$

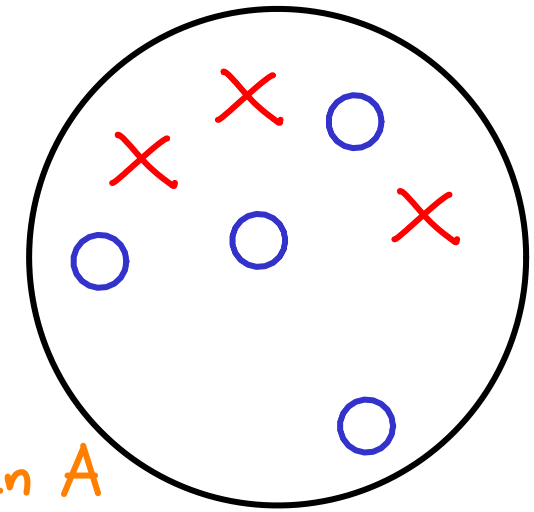
event A: 1^{st} choice is $\times$

event B: 2^{nd} choice is $\times$

$P(A) = \frac{4}{8}$

$P(B) = \frac{4}{8}$ symmetric

...or examine sample space

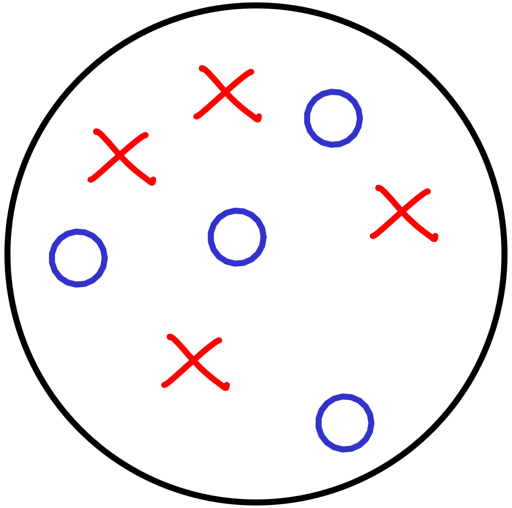


given A

$P(B|A) = ?$

Select 2 objects from this group (order matters, no replacement)

$|\text{sample space}| = P(8,2) = 8 \cdot 7 = 56$



event A: 1^{st} choice is ×

event B: 2^{nd} choice is ×

$P(A) = \frac{4}{8}$

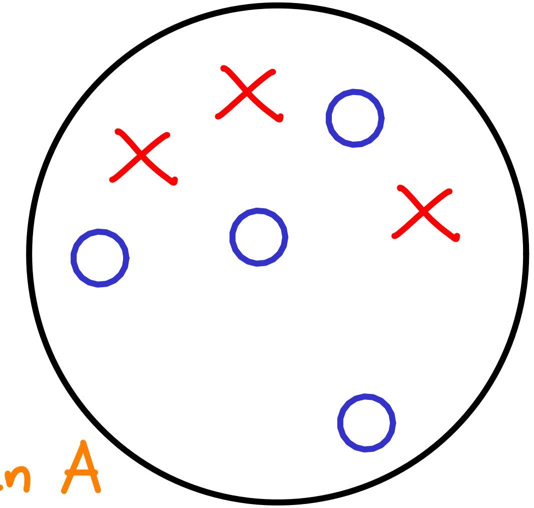
$P(B) = \frac{4}{8}$

symmetric

...or examine sample space

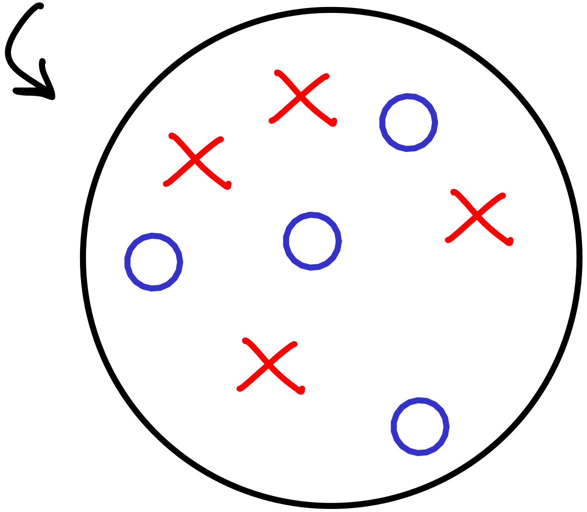
$\neq$

$P(B|A) = \frac{3}{7}$



given A

Select 2 objects from this group (order matters, no replacement)



|sample space| $= P(8,2) = 8 \cdot 7 = 56$

event A: 1^{st} choice is ×

event B: 2^{nd} choice is ×

} dependent

$P(A) = \frac{4}{8}$

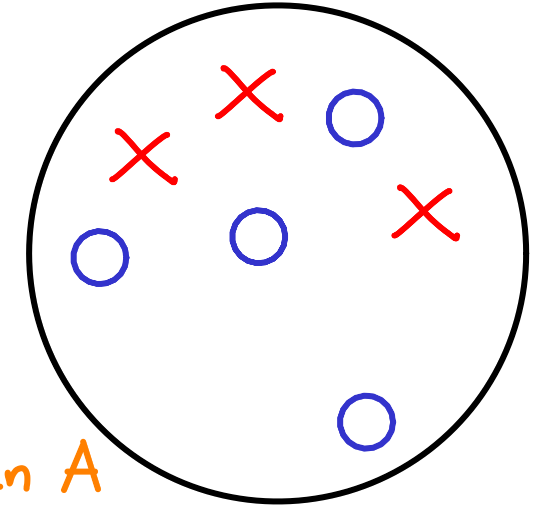
$P(B) = \frac{4}{8}$

symmetric

...or examine sample space

$\neq$

$P(B|A) = \frac{3}{7}$



given A