COMBINATIONS & PERMUTATIONS etc

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n items to choose from, k choices to make

COMBINATIONS & PERMUTATIONS etc

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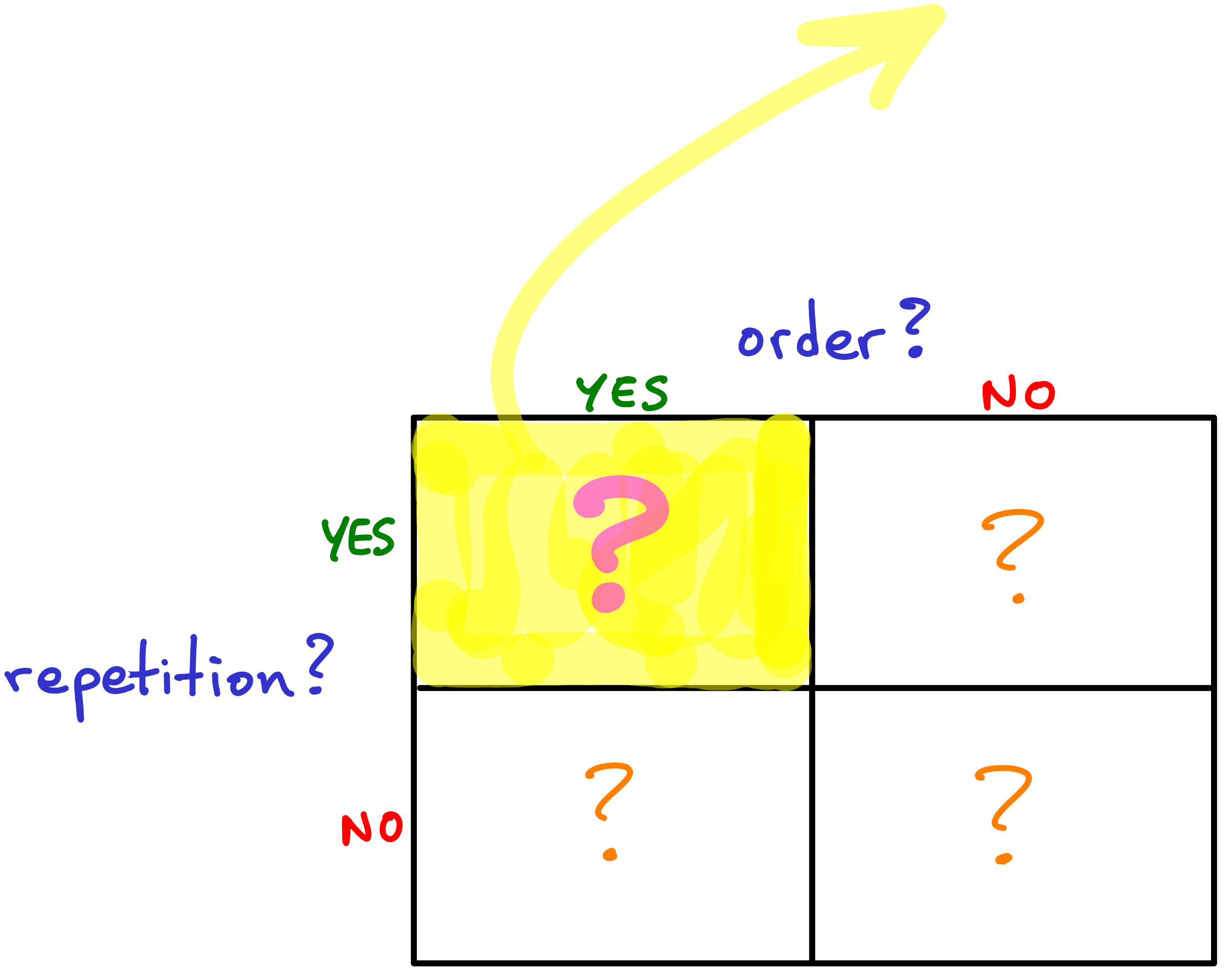
n items to choose from, k choices to make

does order matter?

is repetition allowed?

	YES	NO
YES	?	?
NO	?	?

n items, k choices : repetition allowed, order matters



h hats, t shirts, s shoes, j jeans

colors

- how many ways to dress?

h hats, t shirts, s shoes, j jeans

colors

- how many ways to dress?

a simple multiplication rule

$$\underset{\text{choice 1}}{h}\cdot t\cdot s\cdot \underset{\text{choice 4}}{j}$$

a simple multiplication rule

h hats, t shirts, s shoes, j jeans

colors

- how many ways to dress?

$h \cdot t \cdot s \cdot j$

choice 1 choice 4

- what if only 2 hats?

a simple multiplication rule

h hats, t shirts, s shoes, j jeans

colors

- how many ways to dress?

$h \cdot t \cdot s \cdot j$

choice 1 choice 4

- what if only 2 hats? → $n = 2$ items to choose from

$k = 1$ (choose a hat)

2 ways to dress

a simple multiplication rule

h hats, t shirts, s shoes, j jeans

colors

- how many ways to dress?

$h \cdot t \cdot s \cdot j$

choice 1 choice 4

- what if only 2 hats? → $n = 2$ items to choose from

↳ on 7 consecutive days?

a simple multiplication rule

h hats, t shirts, s shoes, j jeans

colors

- how many ways to dress?

$h \cdot t \cdot s \cdot j$

choice 1 choice 4

- what if only 2 hats? → $n=2$ items to choose from

↳ on 7 consecutive days?

repetition allowed, order matters.

a simple multiplication rule

h hats, t shirts, s shoes, j jeans

colors

- how many ways to dress?

$h \cdot t \cdot s \cdot j$

choice 1 … choice 4

- what if only 2 hats? → $n=2$ items to choose from

↳ on 7 consecutive days?

$k=7$ choices to make

repetition allowed, order matters.

a simple multiplication rule

h hats, t shirts, s shoes, j jeans

colors

- how many ways to dress?

$h \cdot t \cdot s \cdot j$

choice 1 ... choice 4

- what if only 2 hats? → $n = 2$ items to choose from

↳ on 7 consecutive days?

$k = 7$ choices to make

↳ same as 7 people, 2 hats each

repetition allowed, order matters.

a simple multiplication rule

h hats, t shirts, s shoes, j jeans

colors

- how many ways to dress?

$h \cdot t \cdot s \cdot j$

choice 1 choice 4

- what if only 2 hats? → $n = 2$ items to choose from

↳ on 7 consecutive days? $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$

$k = 7$ choices to make: choice 1 choice 7

↳ same as 7 people, 2 hats each

repetition allowed, order matters.

a simple multiplication rule

h hats, t shirts, s shoes, j jeans

colors

- how many ways to dress?

$$\underset{\text{choice 1}}{h}\cdot t\cdot s\cdot \underset{\text{choice 4}}{j}$$

- what if only 2 hats? → $n = 2$ items to choose from

↳ on 7 consecutive days?

$$\underset{\text{choice 1}}{2}\cdot 2\cdot 2\cdot 2\cdot 2\cdot 2\cdot \underset{\text{choice 7}}{2}$$

$$2^7$$

$k = 7$ choices to make: choice 1 choice 7

↳ same as 7 people, 2 hats each

repetition allowed, order matters.

a simple multiplication rule

h hats, t shirts, s shoes, j jeans

colors

- how many ways to dress?

$h \cdot t \cdot s \cdot j$

choice 1 choice 4

- what if only 2 hats? → $n = 2$ items to choose from

↳ on 7 consecutive days? $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$

$k = 7$ choices to make: choice 1 choice 7

↳ same as 7 people, 2 hats each

repetition allowed, order matters.

$$2^7 = n^k$$

		order?	
		YES	NO
repetition?	YES	n^k	?
	NO	?	?

n items to choose from, **k choices** to make

↳ repetition allowed, order matters

order?

repetition?	YES	NO
YES	n^k	?
NO	?	?

n items to choose from, k choices to make

↳ repetition allowed, order matters

		order?	
		YES	NO
repetition?	YES	n^k	?
	NO	?	?

- how many binary numbers have length k?

  e.g., 100110 , 000101, $\underbrace{000000}_{k=6}$

n items to choose from, k choices to make

↳ repetition allowed, order matters

repetition? \ order?	YES	NO
YES	n^k	?
NO	?	?

- how many binary numbers have length k?

  e.g., 100110, 000101, $\underbrace{000000}_{k=6}$

- how many binary numbers have length $\leq k$?

  e.g., $\underbrace{100110}_{k=6}$, 101, $\underset{<k}{0}$

n items to choose from, k choices to make

↳ repetition allowed, order matters

		order?	
		YES	NO
repetition?	YES	n^k	?
	NO	?	?

- how many binary numbers have length k?

e.g., 100110, 000101, 000000

k ordered positions

$000101 \neq 010001$

n items to choose from, k choices to make

↳ repetition allowed, order matters

		order?	
		YES	NO
repetition?	YES	n^k	?
	NO	?	?

- how many binary numbers have length k?

e.g., 100110, 000101, 000000

$n = 2$

k ordered positions, for each we can choose 0 or 1

n items to choose from, k choices to make

↳ repetition allowed, order matters

order?

repetition?	YES	NO
YES	n^k	?
NO	?	?

- how many binary numbers have length k?

e.g., 100110, 000101, 000000

$n=2$

k ordered positions, for each we can choose 0 or 1

can repeat choices

n items to choose from, k choices to make

↳ repetition allowed, order matters

order?

repetition?	YES	NO
YES	n^k	?
NO	?	?

- how many binary numbers have length k?

e.g., 100110, 000101, 000000

k ordered positions, for each we can choose $\overset{n=2}{\underbrace{0 \text{ or } 1}_{\text{can repeat choices}}}$

↳ like hats & days

n items to choose from, k choices to make

↳ repetition allowed, order matters

		order?	
		YES	NO
repetition?	YES	n^k	?
	NO	?	?

- how many binary numbers have length k? → $\boxed{2^k}$

e.g., 100110, 000101, 000000

k ordered positions, for each we can choose $\underbrace{\overset{n=2}{0 \text{ or } 1}}_{\text{can repeat choices}}$

↳ like hats & days

- how many binary numbers have length ≤ k?

↳ context: no padding with 0's. e.g., $\underbrace{1001}_{K=6}$, 101, $\underset{<K}{0}$

- how many binary numbers have length ≤ k ?

↳ context: no padding with 0's. e.g., $\underbrace{1001}_{k=6}$, 101, $\underset{<k}{0}$

consider lengths k, k-1, k-2, etc → disjoint counts, can add results

- how many binary numbers have length $\leq k$?

↳ context: no padding with 0's.

e.g., $\underbrace{1001}_{k=6}$, 101, 0

$<k$

consider lengths k, $k-1$, $k-2$, etc → disjoint counts, can add results

↳ for each scenario, start with 1.

length

e.g., $k=4$: 1???

- how many binary numbers have length $\leq k$?

↳ context: no padding with 0's.   e.g., $\underbrace{1001}_{k=6}$, 101, $\underset{<k}{0}$

consider lengths k, k-1, k-2, etc → disjoint counts, can add results

↳ for each scenario, start with 1.

length   numbers

e.g., k=4:   1??? 2^3 8

- how many binary numbers have length $\leq k$?

↳ context: no padding with 0's. e.g., $\underbrace{1001}_{k=6}$, 101, 0 $<k$

consider lengths k, $k-1$, $k-2$, etc → disjoint counts, can add results

↳ for each scenario, start with 1.

	length		numbers	
e.g.,	$k=4$:	1???	2^3	8
	3 :	1??	2^2	4

- how many binary numbers have length $\leq k$?

↳ context: no padding with 0's. e.g., $\underbrace{1001}_{k=6}$, 101, $\underset{<k}{0}$

consider lengths k, k-1, k-2, etc → disjoint counts, can add results

↳ for each scenario, start with 1.

	length		numbers	
e.g.,	k=4 :	1???	2^3	8
	3 :	1??	2^2	4
	2 :	1?	2^1	2

- how many binary numbers have length ≤ k?

↳ context: no padding with 0's. e.g., $\underbrace{1001}_{k=6}$, 101, $\underset{<k}{0}$

consider lengths k, k-1, k-2, etc → disjoint counts, can add results

↳ for each scenario, start with 1.

	length		numbers	
e.g.,	k=4 :	1???	2^3	8
	3 :	1??	2^2	4
	2 :	1?	2^1	2
	1 :	?	?	

- how many binary numbers have length $\leq k$?

↳ context: no padding with 0's. e.g., $\underbrace{1001}_{k=6}$, 101, $\underset{<k}{0}$

consider lengths k, k-1, k-2, etc → disjoint counts, can add results

↳ for each scenario, start with 1.

	length		numbers	
e.g.,	k=4 :	1???	2^3	8
	3 :	1??	2^2	4
	2 :	1?	2^1	2
combine with nothing →	1 :	1{ }		

- how many binary numbers have length ≤ k?

↳ context: no padding with 0's. e.g., $\underbrace{1001}_{k=6}$, 101, $\underset{<k}{0}$

consider lengths k, k-1, k-2, etc → disjoint counts, can add results

↳ for each scenario, start with 1.

	length		numbers	
e.g.,	k=4:	1???	2^3	8
	3:	1??	2^2	4
	2:	1?	2^1	2
combine with nothing →	1:	1{ }	2^0	1

- how many binary numbers have length $\leq k$?

↳ context: no padding with 0's. e.g., $\underbrace{1001}_{k=6}$, 101, 0

$<k$

consider lengths k, $k-1$, $k-2$, etc → disjoint counts, can add results

↳ for each scenario, start with 1,

but don't forget 0

	length		numbers	
e.g.,	$k=4$:	1???	2^3	8
	3 :	1??	2^2	4
	2 :	1?	2^1	2
combine with nothing →	1 :	$1\{\}$	2^0	1

not done

- how many binary numbers have length ≤ k?

↳ context: no padding with 0's. e.g., $\underbrace{1001}_{k=6}$, 101, 0

<k

consider lengths k, k-1, k-2, etc → disjoint counts, can add results

↳ for each scenario, start with 1,

but don't forget 0

		length		numbers	
	e.g.,	k=4:	1???	2^3	8
		3:	1??	2^2	4
		2:	1?	2^1	2
combine with nothing →		1:	1{ }	2^0	1
no 1's, combine with nothing →		0:	{ }{ }		

- how many binary numbers have length $\le k$?

↳ context: no padding with 0's. e.g., $\underbrace{1001}_{k=6}$, 101, 0 $<k$

consider lengths k, k-1, k-2, etc → disjoint counts, can add results

↳ for each scenario, start with 1.

but don't forget 0

	length		numbers	
e.g.,	k=4:	1???	2^3	8
	3:	1??	2^2	4
	2:	1?	2^1	2
combine with nothing →	1:	1{ }	2^0	1
no 1's, combine with nothing →	0:	{ }{ }	1	1

- how many binary numbers have length ≤ k?

↳ context: no padding with 0's. e.g., $\underbrace{1001}_{k=6}$, 101, $\underset{<k}{0}$

consider lengths k, $k-1$, $k-2$, etc → disjoint counts, can add results

↳ for each scenario, start with 1.

but don't forget 0

e.g.,

length		numbers	2^k 16
$k=4$:	1???	2^3	8
3:	1??	2^2	4
2:	1?	2^1	2
1:	1{ }	2^0	1
0:	{ }{ }	1	1

combine with nothing → (length 1: 1{ })

no 1's, combine with nothing → (length 0: { }{ })

n items to choose from, k choices to make

↳ repetition allowed, order matters

order?

repetition?	YES	NO
YES	n^k	?
NO	?	?

- how many decimal numbers have length k?

n items to choose from, k choices to make

↳ repetition allowed, order matters

		order?	
		YES	NO
repetition?	YES	n^k	?
	NO	?	?

- how many decimal numbers have length k? → 10^k

k ordered positions, for each we can choose $\overset{n=10}{0, 1, 2, \ldots, 9}$

- how many decimal numbers have length 0 ?

- how many decimal numbers have length 0?

↳ choosing from n hats on 0 consecutive days.

- how many decimal numbers have length 0? → $10^k = 10^0 = 1$?

↳ choosing from n hats on 0 consecutive days. (absurd? 0? 1?)

- how many decimal numbers have length 0? $\rightarrow 10^k = 10^0 = 1$?

↳ choosing from n hats on 0 consecutive days. (absurd? 0? 1?)

Is it just a phrasing issue? A matter of definition?

- how many decimal numbers have length 0? → $10^k = 10^0 = 1$?

↳ choosing from n hats on 0 consecutive days. (absurd? 0? 1?)

Is it just a phrasing issue? A matter of definition?

Given the set of all decimal strings, how many elements have length 0?

- how many decimal numbers have length 0? → $10^k = 10^0 = 1$?

↳ choosing from n hats on 0 consecutive days. (absurd? 0? 1?)

Is it just a phrasing issue? A matter of definition?

Given the set of all decimal strings, how many elements have length 0?

↳ 1 empty string

- how many decimal numbers have length 0 ? → $10^k = 10^0 = 1$?

↳ choosing from n hats on 0 consecutive days. (absurd? 0? 1?)

Is it just a phrasing issue? A matter of definition?

Given the set of all decimal strings, how many elements have length 0?

↳ 1 empty string

How many ways can we form an ordered list of 0 items?

- how many decimal numbers have length 0? → $10^k = 10^0 = 1$?

↳ choosing from n hats on 0 consecutive days. (absurd? 0? 1?)

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Given the set of all decimal strings, how many elements have length 0?

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How many ways can we form an ordered list of 0 items?

↳ 1 empty list

- how many decimal numbers have length 0? → $10^k = 10^0 = 1$?

↳ choosing from n hats on 0 consecutive days. (absurd? 0? 1?)

Is it just a phrasing issue? A matter of definition?

Given the set of all decimal strings, how many elements have length 0?

↳ 1 empty string

How many ways can we form an ordered list of 0 items?

↳ 1 empty list

know what choices are valid $\left\{ \begin{array}{l} \text{if numbers can't have length 0, require } k \geq 1 \\ \text{for strings, lists, sets, } k=0 \text{ is ok} \end{array} \right.$

n items to choose from, k choices to make

↳ repetition allowed, order matters

	order?	
repetition?	YES	NO
YES	n^k	?
NO	?	?

- how many ways to assign k tasks to a group of n people?

n items to choose from, k choices to make

↳ repetition allowed, order matters

		order?	
		YES	NO
repetition?	YES	n^k	?
	NO	?	?

- how many ways to assign k tasks to a group of n people?
  - each task is handled by one person.

    ↳ first decide who does task 1, then decide for task 2, etc

n items to choose from, k choices to make

↳ repetition allowed, order matters

		order?	
		YES	NO
repetition?	YES	n^k	?
	NO	?	?

- how many ways to assign k tasks to a group of n people?
  - each task is handled by one person.

    ↳ first decide who does task 1, then decide for task 2, etc
  - one person could handle multiple tasks.

    ↳ the task group (committee) has $\leq k$ people

n items to choose from, k choices to make

↳ repetition allowed, order matters

order?

repetition?	YES	NO
YES	n^k	?
NO	?	?

- how many ways to assign k tasks to a group of n people?
  - each task is handled by one person.
    ↳ first decide who does task 1, then decide for task 2, etc
  - one person could handle multiple tasks.
    ↳ the task group (committee) has $\leq k$ people

n items to choose from, k choices to make

↳ repetition allowed, order matters

		order?	
		YES	NO
repetition?	YES	n^k	?
	NO	?	?

- how many ways to assign k tasks to a group of n people?
  - each task is handled by one person.

    ↳ first decide who does task 1, then decide for task 2, etc
  - one person could handle multiple tasks.

    ↳ the task group (committee) has $\leq k$ people

- how many ways to assign 0 tasks to a group of $n \geqslant 1$ people?

n items to choose from, k choices to make

↳ repetition allowed, order matters

		order?	
		YES	NO
repetition?	YES	n^k	?
	NO	?	?

- how many ways to assign k tasks to a group of n people?
  - each task is handled by one person.
    ↳ first decide who does task 1, then decide for task 2, etc
  - one person could handle multiple tasks.
    ↳ the task group (committee) has $\leq k$ people
- how many ways to assign 0 tasks to a group of $n \geq 1$ people? → $n^0 = 1$

n items to choose from, k choices to make

↳ repetition allowed, order matters

		order?	
		YES	NO
repetition?	YES	n^k	?
	NO	?	?

- how many ways to assign k tasks to a group of n people?
  - each task is handled by one person.

    ↳ first decide who does task 1, then decide for task 2, etc
  - one person could handle multiple tasks.

    ↳ the task group (committee) has $\leq k$ people

- how many ways to assign 0 tasks to a group of $n \geq 1$ people? → $n^0 = 1$
- how many ways to assign 0 tasks to a group of 0 people?

n items to choose from, k choices to make

↳ repetition allowed, order matters

		order?	
		YES	NO
repetition?	YES	n^k	?
	NO	?	?

- how many ways to assign k tasks to a group of n people?
  - each task is handled by one person.
    ↳ first decide who does task 1, then decide for task 2, etc
  - one person could handle multiple tasks.
    ↳ the task group (committee) has $\leq k$ people

- how many ways to assign 0 tasks to a group of $n \geq 1$ people? $\to n^0 = 1$
- how many ways to assign 0 tasks to a group of 0 people? $\to 0^0 = 1$

$\left\{\right.$ combinatorics definition

n items to choose from, k choices to make

↳ repetition allowed, order matters

		order?	
		YES	NO
repetition?	YES	n^k	?
	NO	?	?

- how many ways to assign k tasks to a group of n people?
  - each task is handled by one person.
    - ↳ first decide who does task 1, then decide for task 2, etc
  - one person could handle multiple tasks.
    - ↳ the task group (committee) has $\leq k$ people

- how many ways to assign 0 tasks to a group of $n \geqslant 1$ people? → $n^0 = 1$
- how many ways to assign 0 tasks to a group of 0 people? → $0^0 = 1$

doing nothing counts as something

combinatorics definition

n items to choose from, k choices to make

does order matter?

is repetition allowed?

	YES	NO
YES	n^k	?
NO	?	?

n items, k choices: repetition <u>not</u> allowed, order matters.

n items, k choices: repetition not allowed, order matters. $(k \leq n)$

n items, k choices: repetition not allowed, order matters. $(k \leq n)$

easy: $k=1$: n ways to choose 1 item from n

n items, k choices : repetition <u>not</u> allowed, order matters. $(k \le n)$

easy: $k=1$: n ways to choose 1 item from n

familiar: $k=n$: ? ways to choose n ordered items from n

n items, k choices: repetition not allowed, order matters. $(k \leq n)$

easy: $k=1$: n ways to choose 1 item from n

familiar: $k=n$: ? ways to choose n ordered items from n

- 7 hats, 7 days, must wear a different one each day

↳ how many ways?

n items, k choices: repetition not allowed, order matters. $(k \le n)$

easy: $k=1$: n ways to choose 1 item from n

familiar: $k=n$: ? ways to choose n ordered items from n

- 7 hats, 7 days, must wear a different one each day

  7 options on day 1

n items, k choices: repetition not allowed, order matters. $(k \le n)$

easy: $k=1$: n ways to choose 1 item from n

familiar: $k=n$: ? ways to choose n ordered items from n

- 7 hats, 7 days, must wear a different one each day

  7 options on day 1, 6 remaining options on day 2

n items, k choices: repetition not allowed, order matters. $(k \le n)$

easy: $k=1$: n ways to choose 1 item from n

familiar: $k=n$: $n!$ ways to choose n ordered items from n

- 7 hats, 7 days, must wear a different one each day → $7!$

7 options on day 1, 6 remaining options on day 2, 5 on day 3, etc

---

n items, k choices: repetition <u>not</u> allowed, order matters. $(k \le n)$

easy: $k=1$: n ways to choose 1 item from n

familiar: $k=n$: $n!$ ways to choose n ordered items from n

- 7 hats, 7 days, must wear a different one each day → $7!$

  7 options on day 1, 6 remaining options on day 2, 5 on day 3, etc

- 7 hats, 3 days?

  n k

n items, k choices: repetition not allowed, order matters. $(k \leq n)$

easy: $k=1$: n ways to choose 1 item from n

familiar: $k=n$: $n!$ ways to choose n ordered items from n

- 7 hats, 7 days, must wear a different one each day → $7!$

  7 options on day 1, 6 remaining options on day 2, 5 on day 3, etc

- 7 hats, 3 days?

  n k

  7 options on day 1, 6 remaining options on day 2, 5 on day 3, ~~etc~~

n items, k choices: repetition not allowed, order matters. $(k \leq n)$

easy: $k=1$: n ways to choose 1 item from n

familiar: $k=n$: $n!$ ways to choose n ordered items from n

- 7 hats, 7 days, must wear a different one each day → $7!$

  7 options on day 1, 6 remaining options on day 2, 5 on day 3, etc

- $\underset{n}{7}$ hats, $\underset{k}{3}$ days?

  7 options on day 1, 6 remaining options on day 2, 5 on day 3, ~~etc~~

  $7 \cdot 6 \cdot 5 \cdot \cancel{4 \cdot 3 \cdot 2 \cdot 1}$

n items, k choices: repetition not allowed, order matters. $(k \leq n)$

easy: $k=1$: n ways to choose 1 item from n

familiar: $k=n$: $n!$ ways to choose n ordered items from n

- 7 hats, 7 days, must wear a different one each day → $7!$

  7 options on day 1, 6 remaining options on day 2, 5 on day 3, etc

---

- $\underset{n}{7}$ hats, $\underset{k}{3}$ days?

  7 options on day 1, 6 remaining options on day 2, 5 on day 3, ~~etc~~

$$7\cdot6\cdot5\cdot\cancel{4\cdot3\cdot2\cdot1} = \frac{7\cdot6\cdot5\cdot4\cdot3\cdot2\cdot1}{4\cdot3\cdot2\cdot1}$$

n items, k choices: repetition not allowed, order matters. $(k \leq n)$

easy: $k=1$: n ways to choose 1 item from n

familiar: $k=n$: $n!$ ways to choose n ordered items from n

- 7 hats, 7 days, must wear a different one each day → $7!$

  7 options on day 1, 6 remaining options on day 2, 5 on day 3, etc

---

- $\underset{n}{7}$ hats, $\underset{k}{3}$ days?

  7 options on day 1, 6 remaining options on day 2, 5 on day 3, ~~etc~~

$$7 \cdot 6 \cdot 5 \cdot \cancel{4 \cdot 3 \cdot 2 \cdot 1} = \frac{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{4 \cdot 3 \cdot 2 \cdot 1} = \frac{7!}{4!}$$

n items, k choices: repetition not allowed, order matters. $(k \leq n)$

easy: $k=1$: n ways to choose 1 item from n

familiar: $k=n$: $n!$ ways to choose n ordered items from n

- 7 hats, 7 days, must wear a different one each day → $7!$

  7 options on day 1, 6 remaining options on day 2, 5 on day 3, etc

- 7 hats, 3 days?

  n k

  7 options on day 1, 6 remaining options on day 2, 5 on day 3, ~~etc~~

$$7 \cdot 6 \cdot 5 \cdot \cancel{4 \cdot 3 \cdot 2 \cdot 1} = \frac{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{4 \cdot 3 \cdot 2 \cdot 1} = \frac{7!}{4!} = \frac{7!}{(7-3)!}$$

n items, k choices: repetition not allowed, order matters. $(k \leq n)$

easy: $k=1$: n ways to choose 1 item from n

familiar: $k=n$: $n!$ ways to choose n ordered items from n

- 7 hats, 7 days, must wear a different one each day → $7!$

  7 options on day 1, 6 remaining options on day 2, 5 on day 3, etc

- $\underset{n}{7}$ hats, $\underset{k}{3}$ days?

  7 options on day 1, 6 remaining options on day 2, 5 on day 3, ~~etc~~

$$7\cdot6\cdot5\cdot\cancel{4\cdot3\cdot2\cdot1} = \frac{7\cdot6\cdot5\cdot4\cdot3\cdot2\cdot1}{4\cdot3\cdot2\cdot1} = \frac{7!}{4!} = \frac{7!}{(7-3)!} = \frac{n!}{(n-k)!}$$

n items, k choices: repetition not allowed, order matters. $(k \leq n)$

easy: $k=1$: n ways to choose 1 item from n

familiar: $k=n$: $n!$ ways to choose n ordered items from n

- 7 hats, 7 days, must wear a different one each day → 7!

  7 options on day 1, 6 remaining options on day 2, 5 on day 3, etc

- $\underset{n}{7}$ hats, $\underset{k}{3}$ days?

  7 options on day 1, 6 remaining options on day 2, 5 on day 3, ~~etc~~

$$7\cdot6\cdot5\cdot\cancel{4\cdot3\cdot2\cdot1} = \frac{7\cdot6\cdot5\cdot4\cdot3\cdot2\cdot1}{4\cdot3\cdot2\cdot1} = \frac{7!}{4!} = \frac{7!}{(7-3)!} = \frac{n!}{(n-k)!}$$

Notation: $(7)_3 = (n)_k = P(n,k)$ $\boxed{\text{permutation}}$

n items, k choices : repetition not allowed, order matters

- 12 people, must choose (i) president (ii) vice president (iii) treasurer

3 "tasks"

n items, k choices : repetition not allowed, order matters

- 12 people, must choose (i) president (ii) vice president (iii) treasurer

  – each person can hold only one position

n items, k choices : repetition not allowed, order matters

- 12 people, must choose (i) president (ii) vice president (iii) treasurer

12 items, 3 choices – each person can hold only one position

n $\qquad\qquad$ k

n items, k choices : repetition not allowed, order matters

- 12 people, must choose (i) president (ii) vice president (iii) treasurer

12 items, 3 choices – each person can hold only one position

n k

$$12 \cdot 11 \cdot 10 = \frac{12!}{(12-3)!}$$

n items, k choices : repetition not allowed, order matters

- 12 people, must choose (i) president (ii) vice president (iii) treasurer

12 items, 3 choices – each person can hold only one position
n k

$$12\cdot 11\cdot 10 = \frac{12!}{(12-3)!} = \overset{(n-0)}{\underset{\substack{0\\1}}{n}}\cdot\underset{\substack{1\\2}}{(n-1)}\cdot\underset{\substack{2\\3}}{(n-2)}\cdot\underset{\substack{\cdots\\\cdots}}{\cdots}\cdot\underset{\substack{k-1\\k}}{(n-(k-1))}$$

$$n! = (n)_n = P(n,n)$$

$n! = (n)_n = P(n,n)$

$n>0$: $(n)_0 = P(n,0) = ?$

$n! = (n)_n = P(n,n)$

$n>0$: $(n)_0 = P(n,0) = ?$

choose 0 items from n

$$n! = (n)_n = P(n,n)$$

$n>0$: $(n)_0 = P(n,0) = 1$ 1 way to choose 0 items from n

$n! = (n)_n = P(n,n)$

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$0! = (0)_0 = P(0,0) = ?$ choose 0 items from 0

$$n! = (n)_n = P(n,n)$$

$n>0$: $(n)_0 = P(n,0) = 1$ 1 way to choose 0 items from n

$0! = (0)_0 = P(0,0) = 1$ 1 way to choose 0 items from 0

definition

$n! = (n)_n = P(n,n)$

$n>0$: $(n)_0 = P(n,0) = 1$ 1 way to choose 0 items from n

$0! = (0)_0 = P(0,0) = 1$ 1 way to choose 0 items from 0

definition

it helps to think of lists or sets:

how many ways can we form an ordered list of 0 items?

$$n! = (n)_n = P(n,n)$$

$n>0$: $(n)_0 = P(n,0) = 1$ 1 way to choose 0 items from n

$0! = (0)_0 = P(0,0) = 1$ 1 way to choose 0 items from 0

definition

it helps to think of lists or sets:

how many ways can we form an ordered list of 0 items?

⟷ 1 way: the empty list

$n! = (n)_n = P(n,n)$

$n>0$: $(n)_0 = P(n,0) = 1$ 1 way to choose 0 items from n

$0! = (0)_0 = P(0,0) = 1$ 1 way to choose 0 items from 0

definition

it helps to think of lists or sets:

how many ways can we form an ordered list of 0 items?

↳ 1 way: the empty list

for $n \geq 1$: $n! = \prod_{i=1}^{n} i$

$$n! = (n)_n = P(n,n)$$

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definition

it helps to think of lists or sets:

how many ways can we form an ordered list of 0 items?

↳ 1 way: the empty list

for $n \geq 1$: $n! = \prod_{i=1}^{n} i = n \cdot (n-1)!$

$n! = (n)_n = P(n,n)$

$n > 0$: $(n)_0 = P(n,0) = 1$ 1 way to choose 0 items from n

$0! = (0)_0 = P(0,0) = 1$ 1 way to choose 0 items from 0

definition

it helps to think of lists or sets:

how many ways can we form an ordered list of 0 items?

↳ 1 way: the empty list

for $n \geq 1$: $n! = \prod_{i=1}^{n} i = n \cdot (n-1)!$ thus for $n=1$, $1! = 1 \cdot 0! = 1$

$n!$ is related to choosing without repetition,
but also to rearranging:

- how many ways to rearrange (i.e. permute) the letters in "WORD"?

1 2 3 4

$4!$

will revisit this

n items to choose from, k choices to make

does order matter?

is repetition allowed?

	YES	NO
YES	n^k	?
NO	$\frac{n!}{(n-k)!}$ $P(n,k)$	?

n items, k choices: repetition not allowed, order doesn't matter

combination without replacement

n items, k choices: repetition not allowed, order doesn't matter

- how many ways can we form a committee of 3, from 12 people?

n items, k choices : repetition not allowed, order doesn't matter

---

- how many ways can we form a committee of 3, from 12 people ?

  ↳ all duties (tasks) shared

n items, k choices : repetition not allowed, order doesn't matter

- how many ways can we form a committee of 3, from 12 people ?

  ↳ all duties (tasks) shared, but we need 3 distinct people

n items, k choices : repetition not allowed, order doesn't matter

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- how many ways can we form a committee of 3, from 12 people ?

  ↳ all duties (tasks) shared, but we need 3 distinct people

↳ not the same as : (i) president (ii) vice president (iii) treasurer

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- how many ways can we form a committee of 3, from 12 people ?

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- lets assign order* temporarily

solution*: $P(12,3) = \frac{12!}{(12-3)!}$

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- lets assign order* temporarily

solution*: $P(12,3) = \frac{12!}{(12-3)!}$

- for any triple of people A, B, C that might be chosen:
  - ↳ could get (A,B,C) (A,C,B) (B,A,C) (B,C,A) (C,A,B) (C,B,A)

n items, k choices : repetition not allowed, order doesn't matter

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  - ↳ could get (A, B, C) (A, C, B) (B, A, C) (B, C, A) (C, A, B) (C, B, A) 3! ways

n items, k choices : repetition not allowed, order doesn't matter

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- solution: eliminate overcounting : $\frac{12!}{(12-3)! \cdot 3!}$

n items, k choices: repetition not allowed, order doesn't matter

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- solution: eliminate overcounting: $\frac{12!}{(12-3)!\cdot 3!}$

$$\frac{P(n,k)}{k!} = \frac{n!}{(n-k)!\cdot k!}$$

n items, k choices: repetition not allowed, order doesn't matter

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"n choose k"

$$C(n,k) = \binom{n}{k} = \frac{P(n,k)}{k!} = \frac{n!}{(n-k)! \cdot k!}$$

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- solution: eliminate overcounting: $\frac{12!}{(12-3)!\cdot 3!}$

"n choose k"

$$C(n,k) = \binom{n}{k} = \frac{P(n,k)}{k!} = \frac{n!}{(n-k)!\cdot k!} = \binom{n}{n-k}$$

how many ways to leave 9 people out?

n items, k choices : repetition not allowed, order doesn't matter

- how many 8-bit binary numbers have exactly 3 1's ?

n items, k choices : repetition not allowed, order doesn't matter

- how many 8-bit binary numbers have exactly 3 1's ?

11100000, 01110000, 01010001 etc

n items, k choices : repetition not allowed, order doesn't matter

- how many $\underset{n}{8}$-bit binary numbers have exactly $\underset{k}{3}$ 1's ?

11100000, 01110000, 01010001 etc

↳ like committee

n items, k choices: repetition not allowed, order doesn't matter

---

- how many 8-bit binary numbers have exactly 3 1's ?

  n ... k

11100000, 01110000, 01010001 etc

↳ like committee

$$\binom{8}{3} = \frac{8!}{(8-3)! \cdot 3!} = 56$$

n items, k choices: repetition not allowed, order doesn't matter

$$C(n,1) = \binom{n}{1} = ?$$

n items, k choices: repetition not allowed, order doesn't matter

---

$$C(n,1) = \binom{n}{1} = \frac{n!}{(n-1)!\,1!} = n$$

n items, k choices : repetition not allowed, order doesn't matter

---

$$C(n,1) = \binom{n}{1} = \frac{n!}{(n-1)!\,1!} = n \qquad = P(n,1)$$

n items, k choices: repetition not allowed, order doesn't matter

$$C(n,1) = \binom{n}{1} = \frac{n!}{(n-1)!\,1!} = n \qquad = P(n,1)$$

$$C(n,0) = \binom{n}{0} = ?$$

n items, k choices : repetition not allowed, order doesn't matter

$$C(n,1) = \binom{n}{1} = \frac{n!}{(n-1)!\,1!} = n \qquad = P(n,1)$$

$$C(n,0) = \binom{n}{0} = \frac{n!}{(n-0)!\,0!} = 1 \qquad = P(n,0)$$

n items, k choices : repetition not allowed, order $\underline{\text{doesn't}}$ matter

---

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n items, k choices: repetition not allowed, order doesn't matter

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$$C(n,n) = \binom{n}{n} = \frac{n!}{(n-n)!\,n!} = 1 \qquad = P(n,n)\ ?$$

n items, k choices: repetition not allowed, order doesn't matter

---

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$$C(n,n) = \binom{n}{n} = \frac{n!}{(n-n)!\,n!} = 1 \qquad\qquad \neq P(n,n)$$

n items, k choices : repetition not allowed, order doesn't matter

$$C(n,1) = \binom{n}{1} = \frac{n!}{(n-1)!\,1!} = n \qquad = P(n,1)$$

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$$C(n,n) = \binom{n}{n} = \frac{n!}{(n-n)!\,n!} = 1 \qquad \neq P(n,n)$$

$$C(n,2) = \binom{n}{2} = ?$$

n items, k choices: repetition not allowed, order doesn't matter

$$C(n,1) = \binom{n}{1} = \frac{n!}{(n-1)!\,1!} = n \qquad = P(n,1)$$

$$C(n,0) = \binom{n}{0} = \frac{n!}{(n-0)!\,0!} = 1 \qquad = P(n,0)$$

$$C(n,n) = \binom{n}{n} = \frac{n!}{(n-n)!\,n!} = 1 \qquad \neq P(n,n)$$

$$C(n,2) = \binom{n}{2} = \frac{n!}{(n-2)!\,2!} = \frac{n(n-1)}{2}$$

n items, k choices: repetition not allowed, order doesn't matter

$$C(n,1) = \binom{n}{1} = \frac{n!}{(n-1)!\,1!} = n \qquad = P(n,1)$$

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$P(n,2)$ double-counts

n items, k choices: repetition not allowed, order doesn't matter

---

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$P(n,2)$ double-counts

$$C(n,n+1) = \binom{n}{n+1} = ?$$

n items, k choices: repetition not allowed, order doesn't matter

$$C(n,1) = \binom{n}{1} = \frac{n!}{(n-1)!\,1!} = n \qquad = P(n,1)$$

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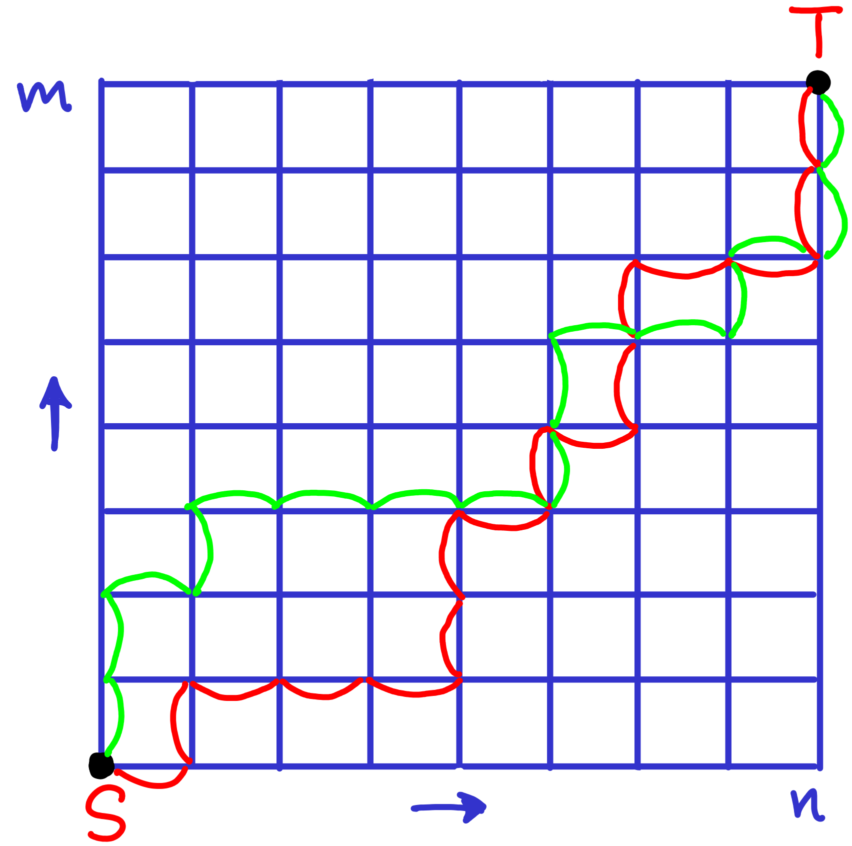
$$C(n,2) = \binom{n}{2} = \frac{n!}{(n-2)!\,2!} = \frac{n(n-1)}{2}$$

P(n,2) double-counts

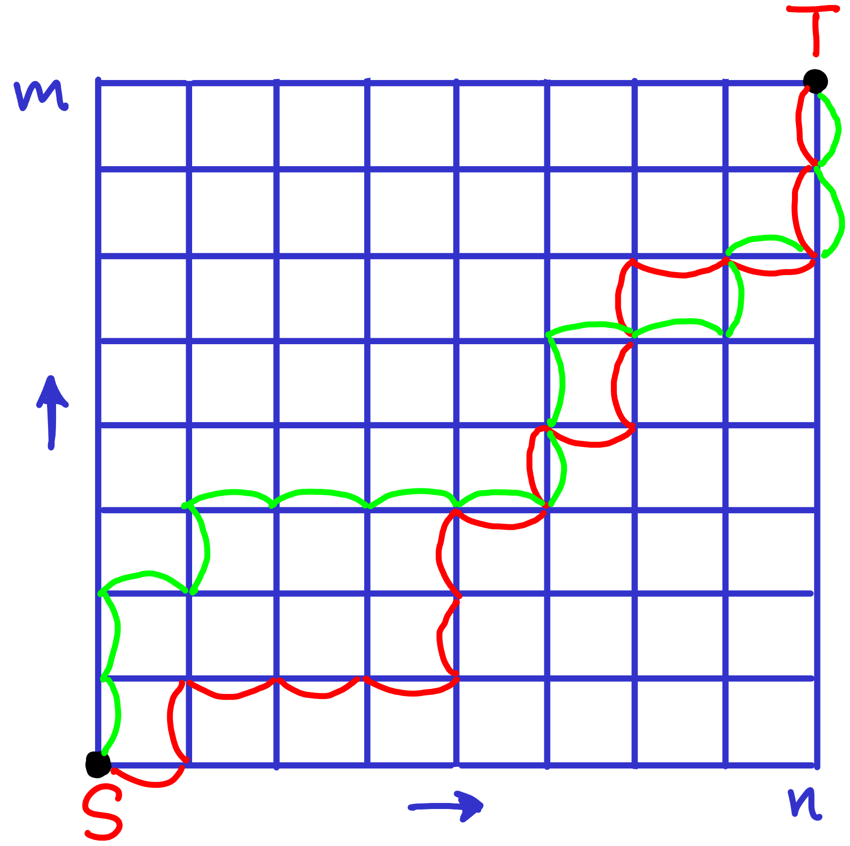
$$C(n,n+1) = \binom{n}{n+1} = 0 = \binom{n}{-1}$$

How many ways are there to get from $S=(0,0)$ to $T=(n,m)$ if you always take a step up or to the right?



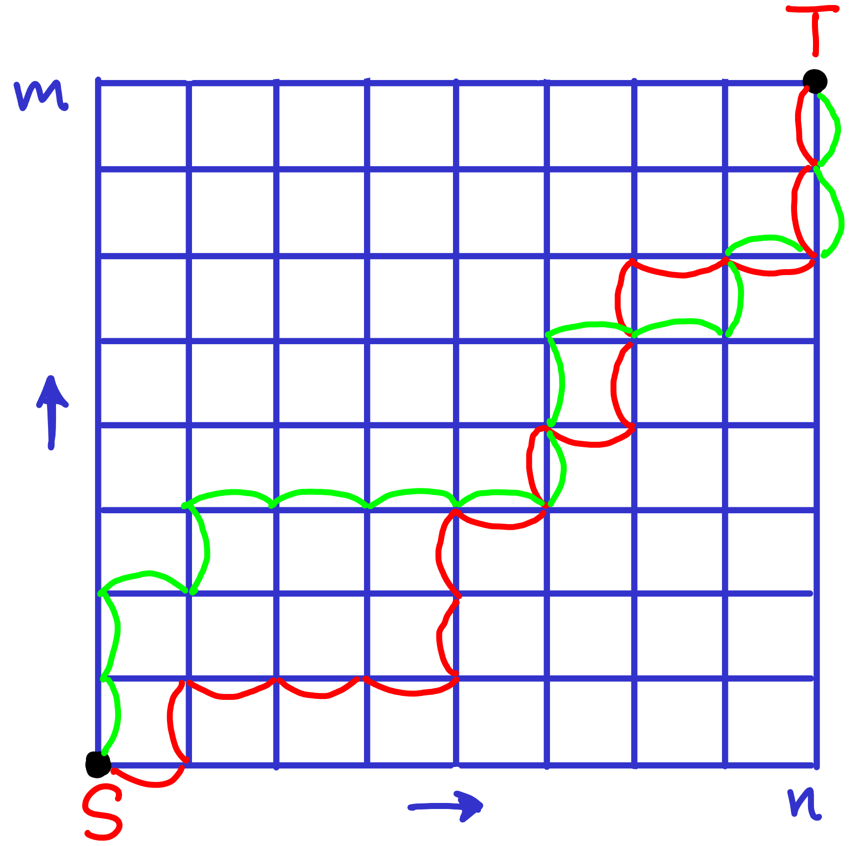
How many ways are there to get from S=(0,0) to T=(n,m) if you always take a step up or to the right?

- S→T must have n+m steps.



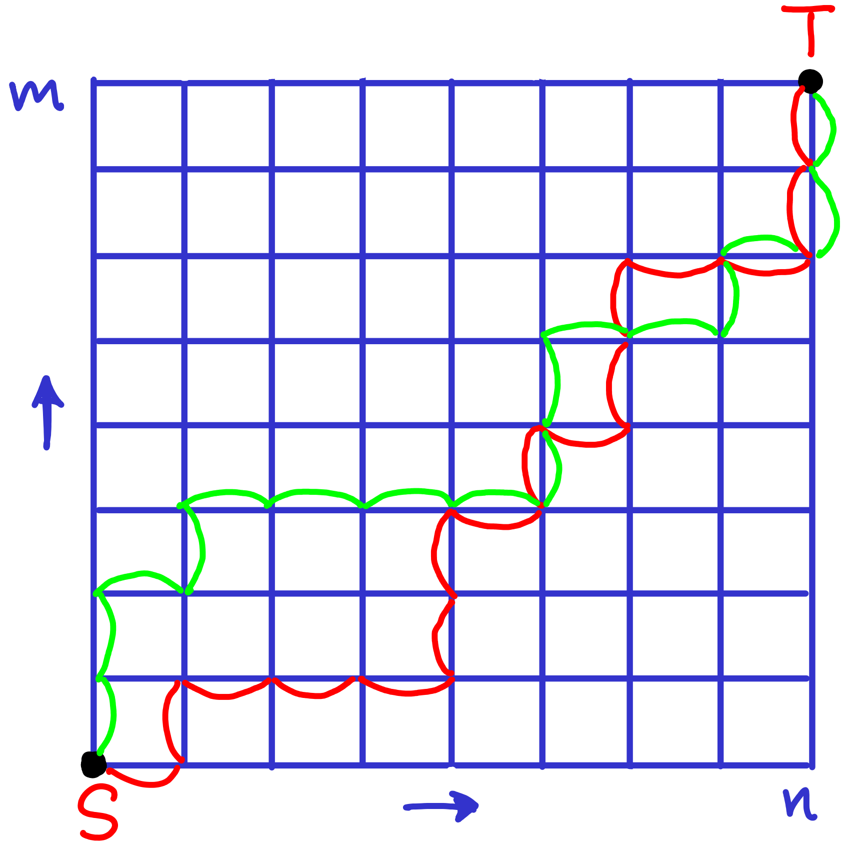
How many ways are there to get from $S=(0,0)$ to $T=(n,m)$ if you always take a step up or to the right?

- $S \to T$ must have $n+m$ steps.
- m of these steps go up.



How many ways are there to get from $S=(0,0)$ to $T=(n,m)$ if you always take a step up or to the right?

- $S \to T$ must have $n+m$ steps.
- m of these steps go up.
- We must choose when to go up.



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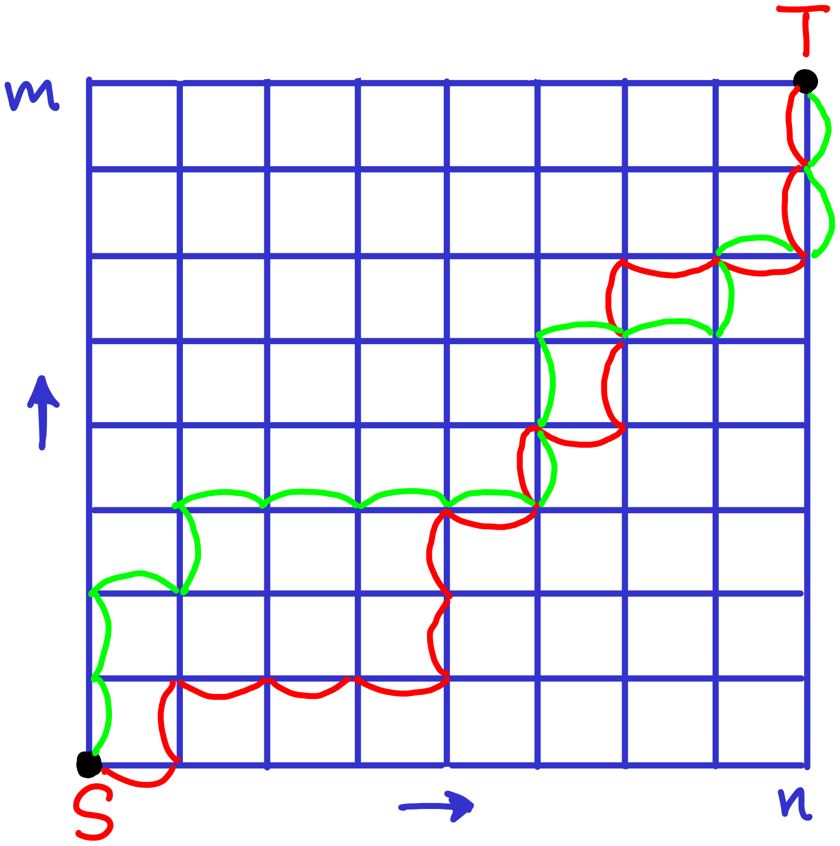
- m of these steps go up.
- We must choose when to go up.

~ binary number with length $\underline{n+m}$ and exactly $\underline{m}$ 1's.



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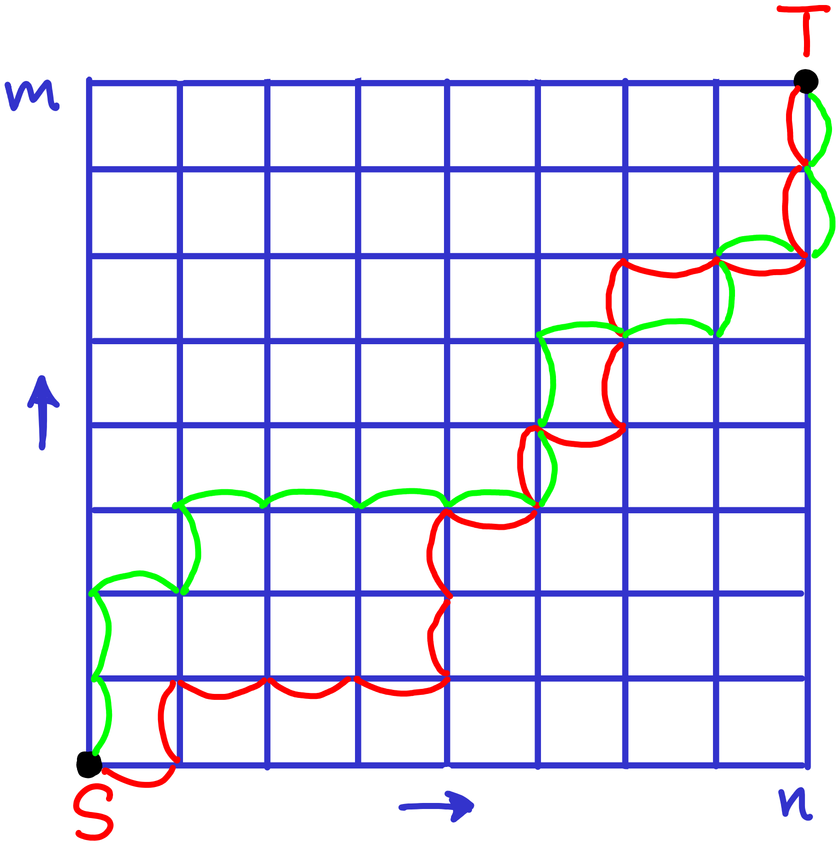

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~ binary number with length $n+m$ and exactly m 1's.

$$\binom{n+m}{m}$$

How many ways are there to get from $S=(0,0)$ to $T=(n,m)$ if you always take a step up or to the right?

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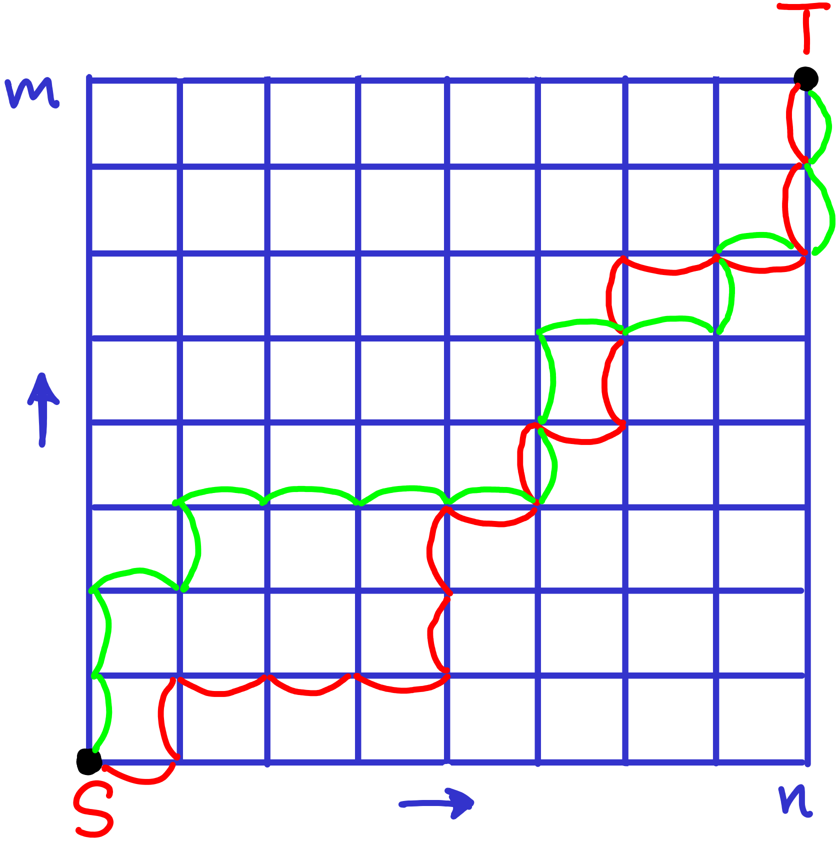
~ binary number with length $n+m$ and exactly m 1's.

$$\binom{n+m}{m} = \frac{(n+m)!}{((n+m)-m)!\,m!}$$



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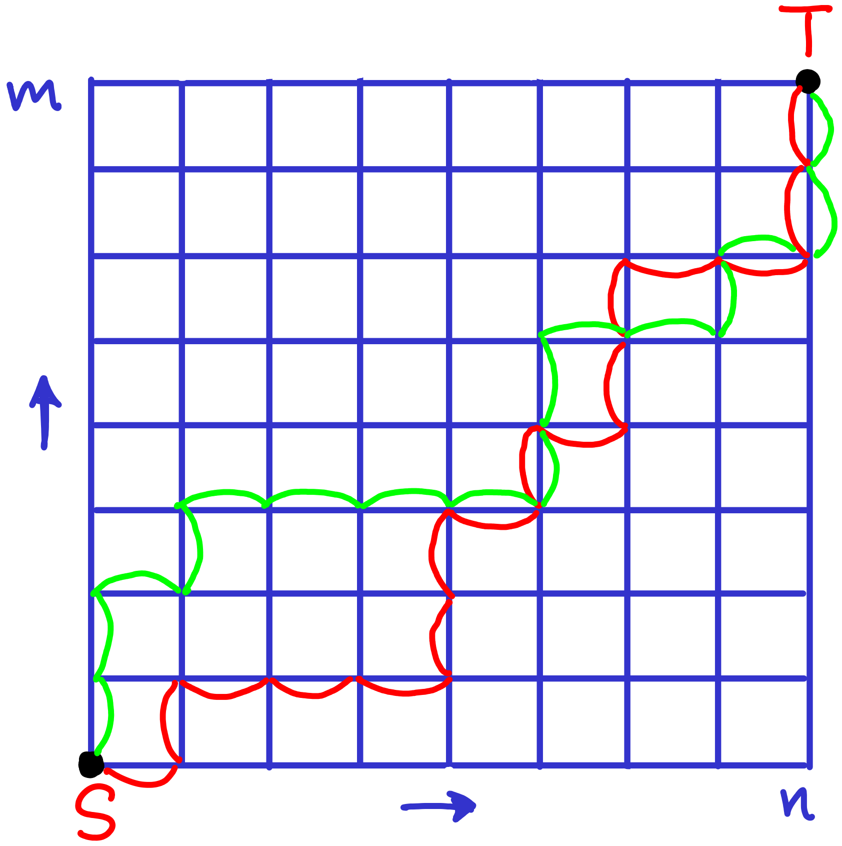
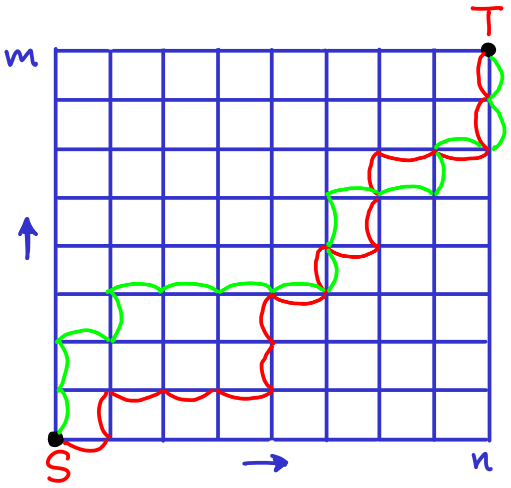
$$\binom{n+m}{m} = \frac{(n+m)!}{((n+m)-m)!\,m!} = \frac{(n+m)!}{n!\,m!}$$



CONTEXT

repetition allowed? order matters?

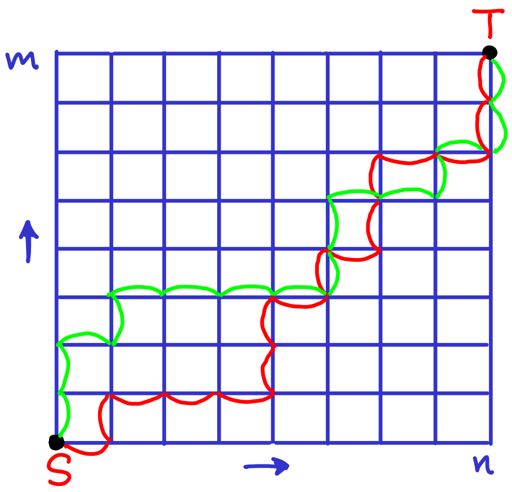


CONTEXT

repetition allowed? order matters?

it looks like:

- we can (must) move up repeatedly

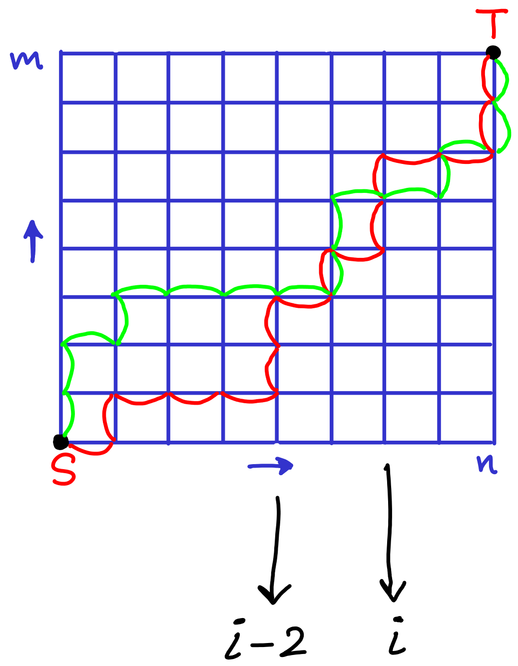


CONTEXT

repetition allowed? order matters?

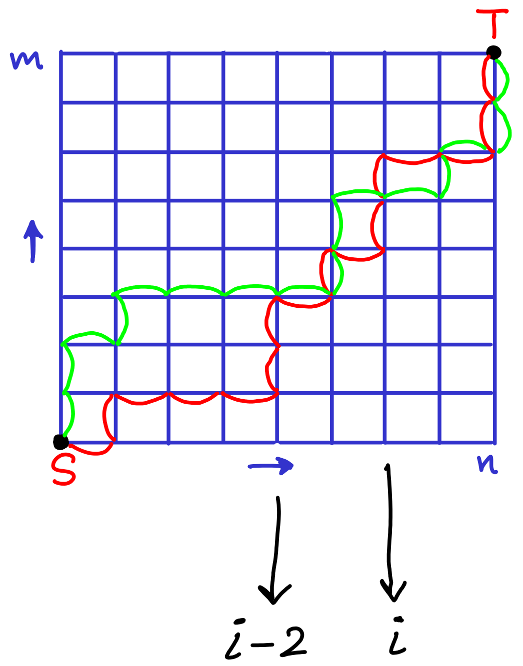
it looks like:

- we can (must) move up repeatedly
- a move up at position i comes after a move up at position $< i$



CONTEXT

repetition allowed? order matters?



it looks like:

- we can (must) move up repeatedly
- a move up at position i comes after a move up at position < i

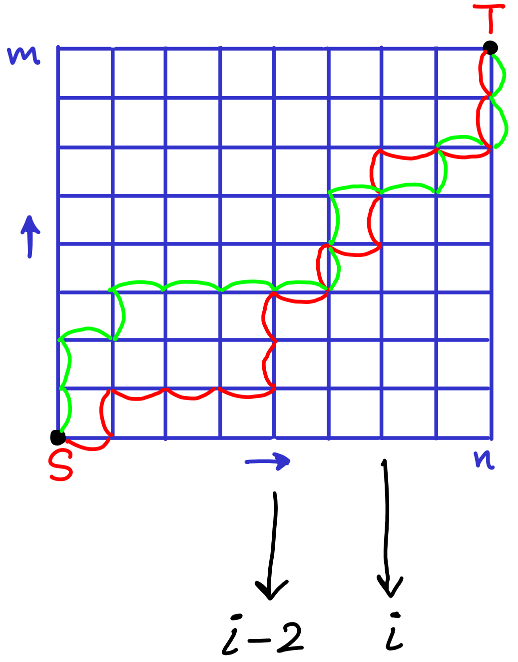
both observations are out of context

CONTEXT

repetition allowed? order matters?

it looks like:

- we can (must) move up repeatedly
- a move up at position i comes after a move up at position < i



There are n+m steps

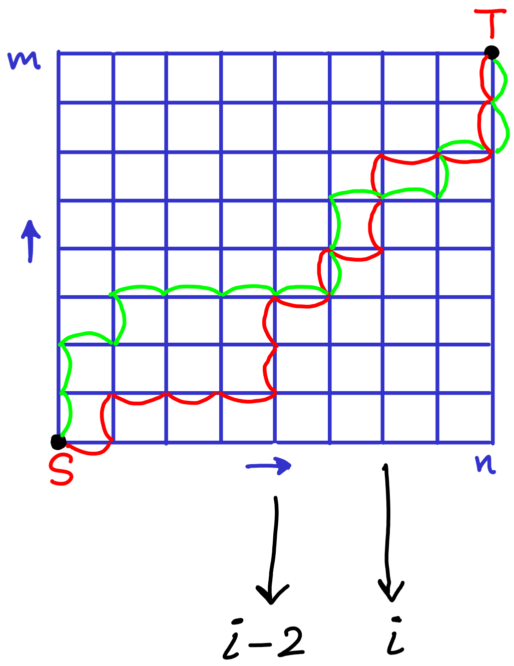
CONTEXT

repetition allowed? order matters?

NO

it looks like:

- ~~we can (must) move up repeatedly~~
- a move up at position i comes after a move up at position < i



There are n+m steps

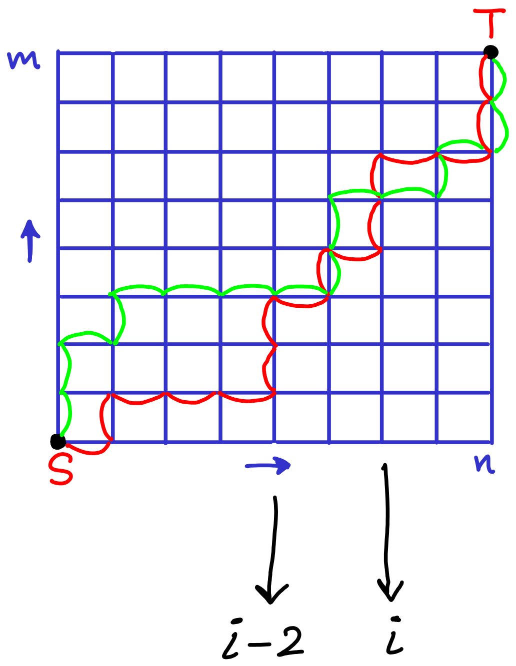
- we can't repeat a move up at the same step

CONTEXT

repetition allowed? NO   order matters? NO

~~it looks like:~~

- ~~we can (must) move up repeatedly~~
- ~~a move up at position i comes after a move up at position < i~~



There are n+m steps

- we can't repeat a move up at the same step
- we care only about the set of steps where we move up.

n items to choose from, k choices to make

		does order matter?	
		YES	NO
is repetition allowed?	YES	n^k	?
	NO	$\frac{n!}{(n-k)!}$ $P(n,k)$	$\frac{n!}{(n-k)!k!}$ $C(n,k)$

n items, k choices : repetition allowed, order doesn't matter

combination with replacement

n items, k choices : repetition allowed, order doesn't matter

- 5 types of donuts d_1 d_2 d_3 d_4 d_5 unlimited supply

n items, k choices : repetition allowed, order doesn't matter

- 5 types of donuts d_1 d_2 d_3 d_4 d_5 unlimited supply
- how many ways can you choose 12 donuts ?

n items, k choices: repetition allowed, order doesn't matter

- 5 types of donuts d_1 d_2 d_3 d_4 d_5 unlimited supply
  n
- how many ways can you choose $\overset{k}{12}$ donuts?

n items, k choices: repetition allowed, order doesn't matter

- $\underset{n}{5}$ types of donuts d_1 d_2 d_3 d_4 d_5 unlimited supply
- how many ways can you choose $\overset{k}{12}$ donuts?

e.g. $\underset{d_1}{\circ\circ\circ\circ\circ}$ $\underset{d_2}{}$ $\underset{d_3}{\circ\circ}$ $\underset{d_4}{\circ}$ $\underset{d_5}{\circ\circ\circ\circ}$

n items, k choices: repetition allowed, order doesn't matter

- 5 types of donuts d_1 d_2 d_3 d_4 d_5 unlimited supply
  n

  k
- how many ways can you choose 12 donuts?

e.g.

ooooo		oo	o	oooo
d_1	d_2	d_3	d_4	d_5

choose 5 d_1 then 2 d_3

= choose 2d_1 then d_3 then d_1 again, etc

n items, k choices : repetition allowed, order doesn't matter

- 5 types of donuts d_1 d_2 d_3 d_4 d_5 unlimited supply

  n

  k
- how many ways can you choose 12 donuts ?

e.g.

n items, k choices : repetition allowed, order doesn't matter

- 5 types of donuts d_1 d_2 d_3 d_4 d_5 unlimited supply
  n

k

- how many ways can you choose 12 donuts ?

e.g. ooooo | | oo | o | oooo
d_1 d_2 d_3 d_4 d_5

need 12 o's, include 4 borders
k $n-1$

n items, k choices : repetition allowed, order doesn't matter

- $\underset{n}{5}$ types of donuts $\quad d_1 \ d_2 \ d_3 \ d_4 \ d_5$ $\quad$ unlimited supply
- how many ways can you choose $\overset{k}{12}$ donuts ?

e.g. $\quad \underset{d_1}{00000} \ 1 \ \underset{d_2}{} \ 1 \ \underset{d_3}{00} \ 1 \ \underset{d_4}{0} \ 1 \ \underset{d_5}{0000}$

↳ encoding: need $\underset{k}{12}$ 0's, include $\underset{n-1}{4}$ borders (1's)

n items, k choices: repetition allowed, order doesn't matter

- $\underset{n}{5}$ types of donuts d_1 d_2 d_3 d_4 d_5 unlimited supply
- how many ways can you choose $\overset{k}{12}$ donuts?

e.g.
$$\begin{array}{ccccccccc} 00000 & | & & | & 00 & | & 0 & | & 0000 \\ d_1 & & d_2 & & d_3 & & d_4 & & d_5 \end{array}$$

↳ encoding: need 12 0's, include 4 borders (1's)

- how many 16-bit binary numbers have exactly 4 1's ?

n items, k choices: repetition allowed, order doesn't matter

- $\underset{n}{5}$ types of donuts d_1 d_2 d_3 d_4 d_5 unlimited supply

- how many ways can you choose $\overset{k}{12}$ donuts?

  e.g. $\underset{d_1}{00000} \mid \underset{d_2}{} \mid \underset{d_3}{00} \mid \underset{d_4}{0} \mid \underset{d_5}{0000}$

↳ encoding: need $\underset{k}{12}$ 0's, include $\underset{n-1}{4}$ borders (1's)

- how many $\underset{k+(n-1)}{16}$-bit binary numbers have exactly $\underset{n-1}{4}$ 1's ?

n items, k choices: repetition allowed, order doesn't matter

- $\underset{n}{5}$ types of donuts d_1 d_2 d_3 d_4 d_5 unlimited supply
- how many ways can you choose $\overset{k}{12}$ donuts?

e.g. $\underset{d_1}{00000} \mid \underset{d_2}{} \mid \underset{d_3}{00} \mid \underset{d_4}{0} \mid \underset{d_5}{0000}$

↪ encoding: need $\underset{k}{12}$ 0's, include $\underset{n-1}{4}$ borders (1's)

- how many $\underset{k+(n-1)}{16}$-bit binary numbers have exactly $\underset{n-1}{4}$ 1's? $\binom{16}{4}$

$$C(k+n-1, n-1) = \binom{k+n-1}{n-1}$$

n items, k choices: repetition allowed, order doesn't matter

- $\underset{n}{5}$ types of donuts d_1 d_2 d_3 d_4 d_5 unlimited supply
- how many ways can you choose $\overset{k}{12}$ donuts?

e.g. $\underset{d_1}{00000} \mid \underset{d_2}{} \mid \underset{d_3}{00} \mid \underset{d_4}{0} \mid \underset{d_5}{0000}$

↳ encoding: need $\underset{k}{12}$ 0's, include $\underset{n-1}{4}$ borders (1's)

- how many $\underset{k+(n-1)}{16}$-bit binary numbers have exactly $\underset{n-1}{4}$ 1's? $\binom{16}{4}$

$$C(k+n-1,\, n-1) = \binom{k+n-1}{n-1} = C(k+n-1,\, k) = \binom{k+n-1}{k}$$

n items, k choices: repetition allowed, order doesn't matter

- $\underset{n}{5}$ types of donuts d_1 d_2 d_3 d_4 d_5 unlimited supply

- how many ways can you choose $\overset{k}{12}$ donuts?

e.g. $\underset{d_1}{00000} \;|\; \underset{d_2}{} \;|\; \underset{d_3}{00} \;|\; \underset{d_4}{0} \;|\; \underset{d_5}{0000}$

↪ encoding: need $\underset{k}{12}$ 0's, include $\underset{n-1}{4}$ borders (1's)

- how many $\underset{k+(n-1)}{16}$-bit binary numbers have exactly $\underset{n-1}{4}$ 1's? $\binom{16}{4}$

$$\boxed{\left(\!\!\binom{n}{k}\!\!\right)} = C(k+n-1, n-1) = \binom{k+n-1}{n-1} = C(k+n-1, k) = \binom{k+n-1}{k}$$

n items, k choices : repetition allowed, order doesn't matter

---

- $\underset{n}{5}$ types of donuts $\quad d_1 \; d_2 \; d_3 \; d_4 \; d_5 \quad$ unlimited supply
- how many ways can you choose $\overset{k}{12}$ donuts ? "n multichoose k"

e.g. $\underset{d_1}{00000} \mid \underset{d_2}{} \mid \underset{d_3}{00} \mid \underset{d_4}{0} \mid \underset{d_5}{0000}$

↪ encoding: need $\underset{k}{12}$ 0's, include $\underset{n-1}{4}$ borders (1's)

- how many $\underset{k+(n-1)}{16}$-bit binary numbers have exactly $\underset{n-1}{4}$ 1's ? $\binom{16}{4}$

$$\left(\!\!\binom{n}{k}\!\!\right) = C(k+n-1, n-1) = \binom{k+n-1}{n-1} = C(k+n-1, k) = \binom{k+n-1}{k}$$

n items, k choices: repetition allowed, order doesn't matter

---

committees instead of donuts: $\binom{k+n-1}{n-1}$

n items, k choices: repetition allowed, order doesn't matter

$\binom{k+n-1}{n-1}$

committees instead of donuts:

- form a committee of k people, from n countries

  (each country has >k people, no restrictions on diversity)

n items, k choices: repetition allowed, order doesn't matter

$$\binom{k+n-1}{n-1}$$

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n items, k choices: repetition allowed, order doesn't matter

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"what is the size of the set of country representations?"

(in a committee of k people, from n countries)

n items, k choices: repetition allowed, order doesn't matter

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"what is the size of the set of country representations?"

(in a committee of k people, from n countries)

* in a committee of 0 people, from 0 countries?

n items, k choices : repetition allowed, order doesn't matter

committees instead of donuts :

$$\binom{k+n-1}{n-1}$$

- form a committee of k people, from n countries

  (each country has >k people, no restrictions on diversity)

- how many ways could countries be represented?

"what is the size of the set of country representations?"

(in a committee of k people, from n countries)

* in a committee of 0 people, from 0 countries? → 1

empty set

n items, k choices : repetition allowed, order doesn't matter

---

$$\binom{k+n-1}{n-1} \qquad\qquad \binom{k+n-1}{k}$$

n items, k choices : repetition allowed, order doesn't matter

$\binom{k+n-1}{n-1}$ $\binom{k+n-1}{k}$

- 1 type of donut, choose $k \geq 1$ donuts ?

n items, k choices: repetition allowed, order doesn't matter

	$\binom{k+n-1}{n-1}$	$\binom{k+n-1}{k}$
- 1 type of donut, choose $k \geq 1$ donuts	$\binom{k+1-1}{1-1}$	$\binom{k+1-1}{k}$

n items, k choices: repetition allowed, order doesn't matter

	$\binom{k+n-1}{n-1}$	$\binom{k+n-1}{k}$
• 1 type of donut, choose $k \geq 1$ donuts	$\binom{k+1-1}{1-1} = \binom{k}{0} = \frac{k!}{(k-0!)0!} = 1$	$\binom{k+1-1}{k}$

n items, k choices: repetition allowed, order doesn't matter

	$\binom{k+n-1}{n-1}$	$\binom{k+n-1}{k}$
• 1 type of donut, choose $k \geq 1$ donuts	$\binom{k+1-1}{1-1} = \binom{k}{0} = \frac{k!}{(k-0!)0!} = 1$	$\binom{k+1-1}{k}$
• 0 type of donut, choose $k \geq 1$ donuts	?	

n items, k choices: repetition allowed, order doesn't matter

	$\binom{k+n-1}{n-1}$	$\binom{k+n-1}{k}$
• 1 type of donut, choose $k \geq 1$ donuts	$\binom{k+1-1}{1-1} = \binom{k}{0} = \frac{k!}{(k-0!)0!} = 1$	$\binom{k+1-1}{k}$
• 0 type of donut, choose $k \geq 1$ donuts	$\binom{k+0-1}{0-1} = \binom{k-1}{-1} = ?$	$\binom{k+0-1}{k}$

n items, k choices: repetition allowed, order doesn't matter

	$\binom{k+n-1}{n-1}$	$\binom{k+n-1}{k}$
• 1 type of donut, choose $k \geq 1$ donuts	$\binom{k+1-1}{1-1} = \binom{k}{0} = \frac{k!}{(k-0!)0!} = 1$	$\binom{k+1-1}{k}$
• 0 type of donut, choose $k \geq 1$ donuts	$\binom{k+0-1}{0-1} = \binom{k-1}{-1} = 0$	$\binom{k+0-1}{k}$

- $n \geq 1$ types of donuts, choose 0 donuts ?

n items, k choices: repetition allowed, order doesn't matter

	$\binom{k+n-1}{n-1}$	$\binom{k+n-1}{k}$
• 1 type of donut, choose $k \geq 1$ donuts	$\binom{k+1-1}{1-1} = \binom{k}{0} = \frac{k!}{(k-0!)0!} = 1$	$\binom{k+1-1}{k}$
• 0 type of donut, choose $k \geq 1$ donuts	$\binom{k+0-1}{0-1} = \binom{k-1}{-1} = 0$	$\binom{k+0-1}{k}$
• $n \geq 1$ types of donuts, choose 0 donuts	$\binom{0+n-1}{n-1} = ?$	$\binom{0+n-1}{0}$

n items, k choices: repetition allowed, order doesn't matter

	$\binom{k+n-1}{n-1}$	$\binom{k+n-1}{k}$
• 1 type of donut, choose $k \geq 1$ donuts	$\binom{k+1-1}{1-1} = \binom{k}{0} = \frac{k!}{(k-0!)0!} = 1$	$\binom{k+1-1}{k}$
• 0 type of donut, choose $k \geq 1$ donuts	$\binom{k+0-1}{0-1} = \binom{k-1}{-1} = 0$	$\binom{k+0-1}{k}$
• $n \geq 1$ types of donuts, choose 0 donuts	$\binom{0+n-1}{n-1} = \frac{(n-1)!}{(n-1-0)!0!} = 1$	$\binom{0+n-1}{0}$

- 0 type of donut, choose 0 donuts

?

n items, k choices: repetition allowed, order doesn't matter

	$\binom{k+n-1}{n-1}$	$\binom{k+n-1}{k}$
• 1 type of donut, choose k≥1 donuts	$\binom{k+1-1}{1-1} = \binom{k}{0} = \frac{k!}{(k-0!)0!} = 1$	$\binom{k+1-1}{k}$
• 0 type of donut, choose k≥1 donuts	$\binom{k+0-1}{0-1} = \binom{k-1}{-1} = 0$	$\binom{k+0-1}{k}$
• n≥1 types of donuts, choose 0 donuts	$\binom{0+n-1}{n-1} = \frac{(n-1)!}{(n-1-0)!0!} = 1$	$\binom{0+n-1}{0}$
• 0 type of donut, choose 0 donuts	$\binom{0+0-1}{0-1} = ?$	

n items, k choices: repetition allowed, order doesn't matter

	$\binom{k+n-1}{n-1}$	$\binom{k+n-1}{k}$
• 1 type of donut, choose k≥1 donuts	$\binom{k+1-1}{1-1} = \binom{k}{0} = \frac{k!}{(k-0!)0!} = 1$	$\binom{k+1-1}{k}$
• 0 type of donut, choose k≥1 donuts	$\binom{k+0-1}{0-1} = \binom{k-1}{-1} = 0$	$\binom{k+0-1}{k}$
• n≥1 types of donuts, choose 0 donuts	$\binom{0+n-1}{n-1} = \frac{(n-1)!}{(n-1-0)!0!} = 1$	$\binom{0+n-1}{0}$
• 0 type of donut, choose 0 donuts	$\binom{0+0-1}{0-1} = \binom{-1}{-1}$ requires extended definition of C(n,k)	

n items, k choices: repetition allowed, order doesn't matter

	$\binom{k+n-1}{n-1}$	$\binom{k+n-1}{k}$
• 1 type of donut, choose $k \geq 1$ donuts	$\binom{k+1-1}{1-1} = \binom{k}{0} = \frac{k!}{(k-0!)0!} = 1$	$\binom{k+1-1}{k}$
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n items, k choices : repetition allowed, order doesn't matter

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n items to choose from, k choices to make

does order matter?

is repetition allowed?

	YES	NO
YES	n^k	$\binom{k+n-1}{n-1}$ $\left(\!\binom{n}{k}\!\right) = C(k+n-1, n-1)$
NO	$\frac{n!}{(n-k)!}$ $P(n,k)$	$\frac{n!}{(n-k)!k!}$ $\binom{n}{k} = C(n,k)$