$$(x+y)^3 = (x+y)\cdot(x+y)\cdot(x+y)$$

$$(x+y)^3 = (x+y)\cdot(x+y)\cdot(x+y)$$

xxx

$$(x+y)^3 = (x+y)\cdot(x+y)\cdot(x+y)$$

$$xxx + \underline{xxy}$$

$$(x+y)^3 = (x+y)\cdot(x+y)\cdot(x+y)$$

$$xxx + xxy + xyx$$

$$(x+y)^3 = (x+y)\cdot(x+y)\cdot(x+y)$$

$$xxx + xxy + xyx + xyy$$

etc

$$(x+y)^3 = (x+y)\cdot(x+y)\cdot(x+y)$$

$$xxx + xxy + xyx + xyy + yxx + yxy + yyx + yyy$$

$$(x+y)^3 = \underbrace{(x+y)}\cdot\underbrace{(x+y)}\cdot\underbrace{(x+y)}$$

3 choices

$$xxx + xxy + xyx + xyy + yxx + yxy + yyx + yyy$$

$$(x+y)^3 = \underbrace{(x+y)}\cdot\underbrace{(x+y)}\cdot\underbrace{(x+y)}$$

2 items to choose from

3 choices

$$xxx + xxy + xyx + xyy + yxx + yxy + yyx + yyy$$

$$(x+y)^3 = \underbrace{(x+y)}\cdot\underbrace{(x+y)}\cdot\underbrace{(x+y)}$$

$n=2$ items to choose from $k=3$ choices $n^k = 2^3 = 8$ ways

$$xxx + xxy + xyx + xyy + yxx + yxy + yyx + yyy$$

$$(x+y)^3 = \underbrace{(x+y)}\cdot\underbrace{(x+y)}\cdot\underbrace{(x+y)}$$

$n=2$ items to choose from $\qquad$ $k=3$ choices $\qquad$ $n^k = 2^3 = 8$ ways

$$xxx + xxy + xyx + xyy + yxx + yxy + yyx + yyy$$

$$000 \quad 001 \quad 010 \quad 011 \quad 100 \quad 101 \quad 110 \quad 111$$

$$(x+y)^3 = \underbrace{(x+y)}\cdot\underbrace{(x+y)}\cdot\underbrace{(x+y)}$$

$n=2$ items to choose from

$k=3$ choices

$n^k = 2^3 = 8$ ways

$$xxx + xxy + xyx + xyy + yxx + yxy + yyx + yyy$$

000	001	010	011	100	101	110	111
0	1	2	3	4	5	6	7

$$(x+y)^3 = \underbrace{(x+y)}\cdot\underbrace{(x+y)}\cdot\underbrace{(x+y)}$$

$n=2$ items to choose from $\quad k=3$ choices $\quad n^k = 2^3 = 8$ ways

$$xxx + xxy + xyx + xyy + yxx + yxy + yyx + yyy$$

000	001	010	011	100	101	110	111
0	1	2	3	4	5	6	7

---

how many ways to get $\rightarrow xy^2$?

$$(x+y)^3 = \underbrace{(x+y)}\cdot\underbrace{(x+y)}\cdot\underbrace{(x+y)}$$

$n=2$ items to choose from $\quad k=3$ choices $\quad n^k = 2^3 = 8$ ways

$$xxx + xxy + xyx + xyy + yxx + yxy + yyx + yyy$$

000	001	010	011	100	101	110	111
0	1	2	3	4	5	6	7

---

how many ways to get xy^2 ? 3

$$(x+y)^3 = \underbrace{(x+y)}\cdot\underbrace{(x+y)}\cdot\underbrace{(x+y)}$$

$n=2$ items to choose from $\quad k=3$ choices $\quad n^k = 2^3 = 8$ ways

$$\begin{array}{ccccccccccccccc} xxx & + & xxy & + & xyx & + & xyy & + & yxx & + & yxy & + & yyx & + & yyy \\ 000 & & 001 & & 010 & & 011 & & 100 & & 101 & & 110 & & 111 \\ 0 & & 1 & & 2 & & 3 & & 4 & & 5 & & 6 & & 7 \end{array}$$

---

how many ways to get xy^2 ? $\binom{3}{2} = 3$ choose two y's from three (1's)

$$(x+y)^3 = \underbrace{(x+y)}\cdot\underbrace{(x+y)}\cdot\underbrace{(x+y)}$$

$n=2$ items to choose from $\qquad$ $K=3$ choices $\qquad$ $n^k = 2^3 = 8$ ways

xxx +	xxy +	xyx +	xyy +	yxx +	yxy +	yyx +	yyy
000	001	010	011	100	101	110	111
0	1	2	3	4	5	6	7

---

how many ways to get $\left\{ \begin{array}{l} y^3\ ? \\ xy^2\ ? \\ x^2y\ ? \\ x^3\ ? \end{array} \right.$

$\binom{3}{2} = 3$ $\quad$ choose two y's from three (1's)

$$(x+y)^3 = \underbrace{(x+y)}\cdot\underbrace{(x+y)}\cdot\underbrace{(x+y)}$$

$n=2$ items to choose from $\qquad$ $k=3$ choices $\qquad$ $n^k = 2^3 = 8$ ways

$$xxx + xxy + xyx + xyy + yxx + yxy + yyx + yyy$$

xxx	xxy	xyx	xyy	yxx	yxy	yyx	yyy
000	001	010	011	100	101	110	111
0	1	2	3	4	5	6	7

---

how many ways to get
$$\left\{\begin{array}{lll} y^3\ ? & \binom{3}{3} & \\ xy^2\ ? & \binom{3}{2} = 3 & \text{choose two } y\text{'s from three} \\ & & (1\text{'s}) \\ x^2y\ ? & \binom{3}{1} & \\ x^3\ ? & \binom{3}{0} & \end{array}\right.$$

$$(x+y)^3 = \underbrace{(x+y)}\cdot\underbrace{(x+y)}\cdot\underbrace{(x+y)}$$

$n=2$ items to choose from $\quad$ $k=3$ choices $\quad$ $n^k = 2^3 = 8$ ways

$$xxx + xxy + xyx + xyy + yxx + yxy + yyx + yyy$$

000	001	010	011	100	101	110	111
0	1	2	3	4	5	6	7

---

how many ways to get

x^0y^3 ? $\quad \binom{3}{3} = 1$

x^1y^2 ? $\quad \binom{3}{2} = 3$ $\quad$ choose two y's from three (1's)

x^2y^1 ? $\quad \binom{3}{1} = 3$

x^3y^0 ? $\quad \binom{3}{0} = 1$

$$(x+y)^n = (x+y)\cdot(x+y)\cdot(x+y)\cdot(x+y)\cdot(x+y)\cdots$$

$$(x+y)^n = (x+y)\cdot(x+y)\cdot(x+y)\cdot(x+y)\cdot(x+y)\cdots$$

- 2^n terms (including duplicates) e.g. $\left.\begin{matrix} xxyyx \\ yxyxy \end{matrix}\right\} x^3y^2$

$$(x+y)^n = (x+y)\cdot(x+y)\cdot(x+y)\cdot(x+y)\cdot(x+y)\cdots$$

- 2^n terms (including duplicates) e.g. $\left.\begin{matrix} xxyyx \\ yxyxy \end{matrix}\right\} x^3y^2$

- each term has n x's & y's (combined)

$$(x+y)^n = (x+y)\cdot(x+y)\cdot(x+y)\cdot(x+y)\cdot(x+y)\cdots$$

- 2^n terms (including duplicates) e.g. $\left.\begin{matrix} xxyyx \\ yxyxy \end{matrix}\right\} x^3y^2$

- each term has n x's & y's (combined) : $x^{n-k}\cdot y^k$ $(0 \leq k \leq n)$

$$(x+y)^n = (x+y)\cdot(x+y)\cdot(x+y)\cdot(x+y)\cdot(x+y)\cdots$$

- 2^n terms (including duplicates) e.g. $\left.\begin{matrix} xxyyx \\ yxyxy \end{matrix}\right\} x^3y^2$

- each term has n x's & y's (combined) : $x^{n-k}\cdot y^k$ $(0 \leq k \leq n)$

- how many ways to get $x^{n-k}\cdot y^k$?

$$(x+y)^n = (x+y)\cdot(x+y)\cdot(x+y)\cdot(x+y)\cdot(x+y)\cdots$$

- 2^n terms (including duplicates) e.g. $\left.\begin{matrix} xxyyx \\ yxyxy \end{matrix}\right\} x^3y^2$

- each term has n x's & y's (combined) : $x^{n-k}\cdot y^k$ $(0 \le k \le n)$

- how many ways to get $x^{n-k}\cdot y^k$? $\left\{\begin{matrix} \text{choose } k \text{ } y\text{'s from } n \\ \binom{n}{k} \end{matrix}\right.$

$$\binom{n}{k} x^{n-k} \cdot y^{k}$$

$$\boxed{(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^{n-k} \cdot y^k}$$

$$\boxed{(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^{n-k} \cdot y^k}$$

$$= \sum_{k=0}^{n} \binom{n}{k} x^{k} \cdot y^{n-k}$$

(symmetry)

Binomial theorem

$x+y$: binomial

(polynomial with 2 terms)

binomial coefficient

$$\boxed{(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^{n-k} \cdot y^k}$$

binomial expression

(symmetry)

$$= \sum_{k=0}^{n} \binom{n}{k} x^{k} \cdot y^{n-k}$$

$$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^{n-k} \cdot y^k$$

$$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^{n-k} \cdot y^k$$

$$(1+1)^n = \sum_{k=0}^{n} \binom{n}{k} 1^{n-k} \cdot 1^k$$

$$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^{n-k} \cdot y^k$$

$$(1+1)^n = \sum_{k=0}^{n} \binom{n}{k} 1^{n-k} \cdot 1^k$$

$$\sum_{k=0}^{n} \binom{n}{k} = 2^n$$

$$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^{n-k} \cdot y^k$$

$$(1+1)^n = \sum_{k=0}^{n} \binom{n}{k} 1^{n-k} \cdot 1^k$$

$$\sum_{k=0}^{n} \binom{n}{k} = 2^n$$

number of binary numbers

number of ways to have precisely

$$\left\{ \begin{array}{l} \text{no } 1\text{'s}, \\ + \text{ one } 1, \\ \quad \vdots \\ + \text{ } n-1 \text{ } 1\text{'s}, \\ + \text{ } n \text{ } 1\text{'s} \end{array} \right\}$$

in a string with n digits

$$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^{n-k} \cdot y^k$$

$$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^{n-k} \cdot y^k$$

$$(x+y)^0 = 1$$

$$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^{n-k} \cdot y^k$$

$$(x+y)^0 = 1$$

$$(x+y)^1 = x + y$$

$$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^{n-k} \cdot y^k$$

$$(x+y)^0 = 1$$

$$(x+y)^1 = x^1 \cdot y^0 + x^0 \cdot y^1$$

$$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^{n-k} \cdot y^k$$

$$(x+y)^0 = 1$$

$$(x+y)^1 = 1x^1 \cdot y^0 + 1x^0 \cdot y^1$$

$$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^{n-k} \cdot y^k$$

$$(x+y)^0 = 1$$

$$(x+y)^1 = 1x^1 \cdot y^0 + 1x^0 \cdot y^1$$

$$(x+y)^2 = 1x^2 \cdot y^0 + 2x^1 \cdot y^1 + 1x^0 \cdot y^2$$

$$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^{n-k} \cdot y^k$$

$$(x+y)^0 = 1$$

$$(x+y)^1 = 1x^1 \cdot y^0 + 1x^0 \cdot y^1$$

$$(x+y)^2 = 1x^2 \cdot y^0 + 2x^1 \cdot y^1 + 1x^0 \cdot y^2$$

$$(x+y)^3 = 1x^3 y^0 + 3x^2 y^1 + 3x^1 y^2 + 1x^0 y^3$$

$$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^{n-k} \cdot y^k$$

$$(x+y)^0 = 1$$

$$(x+y)^1 = 1x^1 \cdot y^0 + 1x^0 \cdot y^1$$

$$(x+y)^2 = 1x^2 \cdot y^0 + 2x^1 \cdot y^1 + 1x^0 \cdot y^2$$

$$(x+y)^3 = 1x^3y^0 + 3x^2y^1 + 3x^1y^2 + 1x^0y^3$$

$$(x+y)^4 = 1x^4y^0 + 4x^3y^1 + 6x^2y^2 + 4x^1y^3 + 1x^0y^4$$

$$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^{n-k} \cdot y^k$$

$$(x+y)^0 = 1$$

$$(x+y)^1 = 1x^1 \cdot y^0 + 1x^0 \cdot y^1$$

$$(x+y)^2 = 1x^2 \cdot y^0 + 2x^1 \cdot y^1 + 1x^0 \cdot y^2$$

$$(x+y)^3 = 1x^3y^0 + 3x^2y^1 + 3x^1y^2 + 1x^0y^3$$

$$(x+y)^4 = 1x^4y^0 + 4x^3y^1 + 6x^2y^2 + 4x^1y^3 + 1x^0y^4$$

$$(x+y)^5 = 1x^5y^0 + 5x^4y^1 + 10x^3y^2 + 10x^2y^3 + 5x^1y^4 + 1x^0y^5$$

$$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^{n-k} \cdot y^k$$

$$(x+y)^0 = 1$$

$$(x+y)^1 = 1x^1 \cdot y^0 + 1x^0 \cdot y^1$$

$$(x+y)^2 = 1x^2 \cdot y^0 + 2x^1 \cdot y^1 + 1x^0 \cdot y^2$$

$$(x+y)^3 = 1x^3y^0 + 3x^2y^1 + 3x^1y^2 + 1x^0y^3$$

$$(x+y)^4 = 1x^4y^0 + 4x^3y^1 + 6x^2y^2 + 4x^1y^3 + 1x^0y^4$$

$$(x+y)^5 = 1x^5y^0 + 5x^4y^1 + 10x^3y^2 + 10x^2y^3 + 5x^1y^4 + 1x^0y^5$$

$$(x+y)^6 = \quad ? \quad ? \quad ? \quad ? \quad ? \quad ? \quad ? \quad ? \quad ?$$

$$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^{n-k} \cdot y^k$$

$$(x+y)^0 = 1$$

$$(x+y)^1 = 1x^1 \cdot y^0 + 1x^0 \cdot y^1$$

$$(x+y)^2 = 1x^2 \cdot y^0 + 2x^1 \cdot y^1 + 1x^0 \cdot y^2$$

$$(x+y)^3 = 1x^3y^0 + 3x^2y^1 + 3x^1y^2 + 1x^0y^3$$

$$(x+y)^4 = 1x^4y^0 + 4x^3y^1 + 6x^2y^2 + 4x^1y^3 + 1x^0y^4$$

$$(x+y)^5 = 1x^5y^0 + 5x^4y^1 + 10x^3y^2 + 10x^2y^3 + 5x^1y^4 + 1x^0y^5$$

$$(x+y)^6 = 1x^6y^0 + 6x^5y^1 + 15x^4y^2 + 20x^3y^3 + 15x^2y^4 + 6x^1y^5 + 1x^0y^6$$

$$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^{n-k} \cdot y^k$$

$$n=0 \qquad 1$$

$$n=1 \qquad 1x^1 \cdot y^0 + 1x^0 \cdot y^1$$

$$n=2 \qquad 1x^2 \cdot y^0 + 2x^1 \cdot y^1 + 1x^0 \cdot y^2$$

$$n=3 \qquad 1x^3y^0 + 3x^2y^1 + 3x^1y^2 + 1x^0y^3$$

$$n=4 \qquad 1x^4y^0 + 4x^3y^1 + 6x^2y^2 + 4x^1y^3 + 1x^0y^4$$

$$n=5 \qquad 1x^5y^0 + 5x^4y^1 + 10x^3y^2 + 10x^2y^3 + 5x^1y^4 + 1x^0y^5$$

$$n=6 \qquad 1x^6y^0 + 6x^5y^1 + 15x^4y^2 + 20x^3y^3 + 15x^2y^4 + 6x^1y^5 + 1x^0y^6$$

$$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^{n-k} \cdot y^k$$

$k=0$

$n=0$ $\quad 1x^0 \cdot y^0$

$n=1$ $\quad 1x^1 \cdot y^0 + 1x^0 \cdot y^1$

$n=2$ $\quad 1x^2 \cdot y^0 + 2x^1 \cdot y^1 + 1x^0 \cdot y^2$

$n=3$ $\quad 1x^3y^0 + 3x^2y^1 + 3x^1y^2 + 1x^0y^3$

$n=4$ $\quad 1x^4y^0 + 4x^3y^1 + 6x^2y^2 + 4x^1y^3 + 1x^0y^4$

$n=5$ $\quad 1x^5y^0 + 5x^4y^1 + 10x^3y^2 + 10x^2y^3 + 5x^1y^4 + 1x^0y^5$

$n=6$ $\quad 1x^6y^0 + 6x^5y^1 + 15x^4y^2 + 20x^3y^3 + 15x^2y^4 + 6x^1y^5 + 1x^0y^6$

$$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^{n-k} \cdot y^k$$

$k=0$

$k=1$

$n=0 \qquad 1x^0 \cdot y^0$

$n=1 \qquad 1x^1 \cdot y^0 + 1x^0 \cdot y^1$

$n=2 \qquad 1x^2 \cdot y^0 + 2x^1 \cdot y^1 + 1x^0 \cdot y^2$

$n=3 \qquad 1x^3y^0 + 3x^2y^1 + 3x^1y^2 + 1x^0y^3$

$n=4 \qquad 1x^4y^0 + 4x^3y^1 + 6x^2y^2 + 4x^1y^3 + 1x^0y^4$

$n=5 \qquad 1x^5y^0 + 5x^4y^1 + 10x^3y^2 + 10x^2y^3 + 5x^1y^4 + 1x^0y^5$

$n=6 \qquad 1x^6y^0 + 6x^5y^1 + 15x^4y^2 + 20x^3y^3 + 15x^2y^4 + 6x^1y^5 + 1x^0y^6$

$$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^{n-k} \cdot y^k$$

$k=0$ $k=1$ $k=2$ $k=3$ $k=4$ $k=5$ $k=6$

$n=0$ $$1x^0 \cdot y^0$$

$n=1$ $$1x^1 \cdot y^0 + 1x^0 \cdot y^1$$

$n=2$ $$1x^2 \cdot y^0 + 2x^1 \cdot y^1 + 1x^0 \cdot y^2$$

$n=3$ $$1x^3y^0 + 3x^2y^1 + 3x^1y^2 + 1x^0y^3$$

$n=4$ $$1x^4y^0 + 4x^3y^1 + 6x^2y^2 + 4x^1y^3 + 1x^0y^4$$

$n=5$ $$1x^5y^0 + 5x^4y^1 + 10x^3y^2 + 10x^2y^3 + 5x^1y^4 + 1x^0y^5$$

$n=6$ $$1x^6y^0 + 6x^5y^1 + 15x^4y^2 + 20x^3y^3 + 15x^2y^4 + 6x^1y^5 + 1x^0y^6$$

$$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^{n-k} \cdot y^k$$

$k=0$ $k=1$ $k=2$ $k=3$ $k=4$ $k=5$ $k=6$

$n=0$ $\quad 1x^0 \cdot y^0$

$n=1$ $\quad 1x^1 \cdot y^0 + 1x^0 \cdot y^1$

$n=2$ $\quad 1x^2 \cdot y^0 + 2x^1 \cdot y^1 + 1x^0 \cdot y^2$

$n=3$ $\quad 1x^3y^0 + 3x^2y^1 + 3x^1y^2 + 1x^0y^3$

$n=4$ $\quad 1x^4y^0 + 4x^3y^1 + 6x^2y^2 + 4x^1y^3 + 1x^0y^4$

$n=5$ $\quad 1x^5y^0 + 5x^4y^1 + 10x^3y^2 + 10x^2y^3 + 5x^1y^4 + 1x^0y^5$

$n=6$ $\quad 1x^6y^0 + 6x^5y^1 + 15x^4y^2 + 20x^3y^3 + 15x^2y^4 + 6x^1y^5 + 1x^0y^6$

$$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^{n-k} \cdot y^k$$

	k=0	k=1	k=2	k=3	k=4	k=5	k=6
n=0	1						
n=1	1	1					
n=2	1	2	1				
n=3	1	3	3	1			
n=4	1	4	6	4	1		
n=5	1	5	10	10	5	1	
n=6	1	6	15	20	15	6	1

$$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^{n-k} \cdot y^k$$

k=0
k=1
k=2
k=3
k=4
k=5
k=6

n = 0   1

n = 1   1   1

n = 2   1   2   1

n = 3   1   3   3   1

n = 4   1   4   6   4   1

n = 5   1   5   10   10   5   1

n = 6   1   6   15   20   15   6   1

+
+
+

$$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^{n-k} \cdot y^k$$

$k=0$

$n=0$ — $\binom{0}{0}$

$k=1$

$k=2$

$n=1$

$k=3$

$n=2$

$k=4$

$n=3$

$k=5$

$n=4$

$k=6$

$n=5$

$n=6$

$$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^{n-k} \cdot y^k$$

$k=0$ $k=1$ $k=2$ $k=3$ $k=4$ $k=5$ $k=6$

$n=0$ $\binom{0}{0}$

$n=1$ $\binom{1}{0}$

$n=2$ $\binom{2}{0}$

$n=3$ $\binom{3}{0}$

$n=4$ $\binom{4}{0}$

$n=5$ $\binom{5}{0}$

$n=6$ $\binom{6}{0}$

$$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^{n-k} \cdot y^k$$

$k=0$ $\quad k=1$ $\quad k=2$ $\quad k=3$ $\quad k=4$ $\quad k=5$ $\quad k=6$

$n=0 \qquad \binom{0}{0}$

$n=1 \qquad \binom{1}{0} \qquad \binom{1}{1}$

$n=2 \qquad \binom{2}{0} \qquad \binom{2}{2}$

$n=3 \qquad \binom{3}{0} \qquad \binom{3}{3}$

$n=4 \qquad \binom{4}{0} \qquad \binom{4}{4}$

$n=5 \qquad \binom{5}{0} \qquad \binom{5}{5}$

$n=6 \qquad \binom{6}{0} \qquad \binom{6}{6}$

$$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^{n-k} \cdot y^k$$

$k=0 \quad k=1 \quad k=2 \quad k=3 \quad k=4 \quad k=5 \quad k=6$

$n=0 \qquad \binom{0}{0}$

$n=1 \qquad \binom{1}{0} \quad \binom{1}{1}$

$n=2 \qquad \binom{2}{0} \quad \binom{2}{1} \quad \binom{2}{2}$

$n=3 \qquad \binom{3}{0} \quad \binom{3}{1} \quad \binom{3}{2} \quad \binom{3}{3}$

$n=4 \qquad \binom{4}{0} \quad \binom{4}{1} \quad \binom{4}{2} \quad \binom{4}{3} \quad \binom{4}{4}$

$n=5 \qquad \binom{5}{0} \quad \binom{5}{1} \quad \binom{5}{2} \quad \binom{5}{3} \quad \binom{5}{4} \quad \binom{5}{5}$

$n=6 \qquad \binom{6}{0} \quad \binom{6}{1} \quad \binom{6}{2} \quad \binom{6}{3} \quad \binom{6}{4} \quad \binom{6}{5} \quad \binom{6}{6}$

$$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^{n-k} \cdot y^k$$

$k=0$ $k=1$ $k=2$ $k=3$ $k=4$ $k=5$ $k=6$

$n=0$: $\binom{0}{0}$

$n=1$: $\binom{1}{0}$ $\binom{1}{1}$

$n=2$: $\binom{2}{0}$ $\binom{2}{1}$ $\binom{2}{2}$

$n=3$: $\binom{3}{0}$ $\binom{3}{1}$ $\binom{3}{2}$ $\binom{3}{3}$

$n=4$: $\binom{4}{0}$ $\binom{4}{1}$ $\binom{4}{2}$ $\binom{4}{3}$ $\binom{4}{4}$

$n=5$: $\binom{5}{0}$ $\binom{5}{1}$ $\binom{5}{2}$ $\binom{5}{3}$ $\binom{5}{4}$ $\binom{5}{5}$

$n=6$: $\binom{6}{0}$ $\binom{6}{1}$ $\binom{6}{2}$ $\binom{6}{3}$ $\binom{6}{4}$ $\binom{6}{5}$ $\binom{6}{6}$

$$\binom{4}{1} + \binom{4}{2} \to \binom{5}{2}$$

$$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^{n-k} \cdot y^k$$

$k=0$ $\quad k=1$ $\quad k=2$ $\quad k=3$ $\quad k=4$ $\quad k=5$ $\quad k=6$

$n=0 \qquad \binom{0}{0}$

$n=1 \qquad \binom{1}{0} \quad \binom{1}{1}$

$n=2 \qquad \binom{2}{0} \quad \binom{2}{1} \quad \binom{2}{2}$

$n=3 \qquad \binom{3}{0} \quad \binom{3}{1} \quad \binom{3}{2} \quad \binom{3}{3}$

$n=4 \qquad \binom{4}{0} \quad \binom{n-1}{k-1} \quad + \quad \binom{n-1}{k} \quad \binom{4}{3} \quad \binom{4}{4}$

$n=5 \qquad \binom{5}{0} \quad \binom{5}{1} \quad \binom{n}{k} \quad \binom{5}{3} \quad \binom{5}{4} \quad \binom{5}{5}$

$n=6 \qquad \binom{6}{0} \quad \binom{6}{1} \quad \binom{6}{2} \quad \binom{6}{3} \quad \binom{6}{4} \quad \binom{6}{5} \quad \binom{6}{6}$

Pascal's identity

$$\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$$

$$\begin{tikzcd}
\binom{n-1}{k-1} \arrow[dr] & + & \binom{n-1}{k} \arrow[dl] \\
& \binom{n}{k} &
\end{tikzcd}$$

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\end{tikzcd}$$

$$\binom{n-1}{k-1} + \binom{n-1}{k} = \quad ?$$

Pascal's identity

$$\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$$

$$\begin{tikzcd} \binom{n-1}{k-1} \arrow[dr] & + & \binom{n-1}{k} \arrow[dl] \\ & \binom{n}{k} & \end{tikzcd}$$

$$\binom{n-1}{k-1} + \binom{n-1}{k} = \frac{(n-1)!}{(n-1-(k-1))!\cdot(k-1)!} + \frac{(n-1)!}{(n-1-k)!\cdot k!}$$

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$$= \frac{(n-1)!}{(n-k)!\cdot(k-1)!} + \frac{(n-1)!}{(n-1-k)!\cdot k!}$$

Pascal's identity

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$$\begin{tikzcd}[column sep=small] \binom{n-1}{k-1} \arrow[dr] & + & \binom{n-1}{k} \arrow[dl] \\ & \binom{n}{k} & \end{tikzcd}$$

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$$= \frac{(n-1)!}{(n-k)!\cdot(k-1)!} + \frac{(n-1)!}{(n-1-k)!\cdot k!}$$

$$\boxed{\begin{array}{l} k \neq 0 \\ n-k \neq 0 \end{array}}$$

$$= \frac{(n-1)!\cdot k}{(n-k)!\cdot(k-1)!\cdot k} + \frac{(n-1)!\cdot(n-k)}{(n-1-k)!\cdot k!\cdot(n-k)}$$

Pascal's identity

$$\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$$

$$\begin{tikzcd} \binom{n-1}{k-1} \arrow[dr] & + & \binom{n-1}{k} \arrow[dl] \\ & \binom{n}{k} & \end{tikzcd}$$

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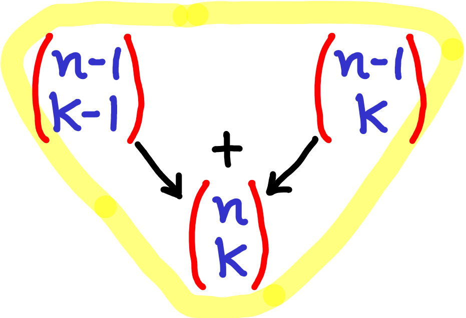
$$= \frac{(n-1)!\cdot k}{(n-k)!\cdot k!} + \frac{(n-1)!\cdot(n-k)}{(n-k)!\cdot k!}$$

Pascal's identity

$$\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$$



$$\binom{n-1}{k-1} + \binom{n-1}{k} = \frac{(n-1)!}{(n-1-(k-1))!\cdot(k-1)!} + \frac{(n-1)!}{(n-1-k)!\cdot k!}$$

$$\boxed{\begin{array}{c} k \neq 0 \\ n-k \neq 0 \end{array}}$$

$$= \frac{(n-1)!}{(n-k)!\cdot(k-1)!} + \frac{(n-1)!}{(n-1-k)!\cdot k!}$$

$$= \frac{(n-1)!\cdot k}{(n-k)!\cdot(k-1)!\cdot k} + \frac{(n-1)!\cdot(n-k)}{(n-1-k)!\cdot k!\cdot(n-k)}$$

$$= \frac{(n-1)!\cdot k}{(n-k)!\cdot k!} + \frac{(n-1)!\cdot(n-k)}{(n-k)!\cdot k!}$$

$$= \frac{(n-1)!\cdot n}{(n-k)!\cdot k!}$$

Pascal's identity

$$\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$$

$$\begin{tikzcd}
\binom{n-1}{k-1} \arrow[dr] & + & \binom{n-1}{k} \arrow[dl] \\
& \binom{n}{k} &
\end{tikzcd}$$

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$$= \frac{(n-1)!}{(n-k)!\cdot(k-1)!} + \frac{(n-1)!}{(n-1-k)!\cdot k!}$$

$$= \frac{(n-1)!\cdot k}{(n-k)!\cdot(k-1)!\cdot k} + \frac{(n-1)!\cdot(n-k)}{(n-1-k)!\cdot k!\cdot(n-k)}$$

$$= \frac{(n-1)!\cdot k}{(n-k)!\cdot k!} + \frac{(n-1)!\cdot(n-k)}{(n-k)!\cdot k!}$$

$$= \frac{(n-1)!\cdot n}{(n-k)!\cdot k!} = \frac{n!}{(n-k)!\cdot k!}$$

Pascal's identity

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$$\binom{n-1}{k-1} + \binom{n-1}{k} = \frac{(n-1)!}{(n-1-(k-1))! \cdot (k-1)!} + \frac{(n-1)!}{(n-1-k)! \cdot k!}$$

$$= \frac{(n-1)!}{(n-k)! \cdot (k-1)!} + \frac{(n-1)!}{(n-1-k)! \cdot k!}$$

$$= \frac{(n-1)! \cdot k}{(n-k)! \cdot (k-1)! \cdot k} + \frac{(n-1)! \cdot (n-k)}{(n-1-k)! \cdot k! \cdot (n-k)}$$

$$= \frac{(n-1)! \cdot k}{(n-k)! \cdot k!} + \frac{(n-1)! \cdot (n-k)}{(n-k)! \cdot k!}$$

$$= \frac{(n-1)! \cdot n}{(n-k)! \cdot k!} = \frac{n!}{(n-k)! \cdot k!} = \binom{n}{k}$$

Pascal's identity

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\binom{n-1}{k-1} \arrow[dr] & + & \binom{n-1}{k} \arrow[dl] \\
& \binom{n}{k} &
\end{tikzcd}$$

$$\left.\begin{array}{l} k = 0 \\ n = k \end{array}\right\} \text{base cases}$$

$$\boxed{\begin{array}{l} k \neq 0 \\ n-k \neq 0 \end{array}}$$

$$\binom{n-1}{k-1} + \binom{n-1}{k} = \frac{(n-1)!}{(n-1-(k-1))!\cdot(k-1)!} + \frac{(n-1)!}{(n-1-k)!\cdot k!}$$

$$= \frac{(n-1)!}{(n-k)!\cdot(k-1)!} + \frac{(n-1)!}{(n-1-k)!\cdot k!}$$

$$= \frac{(n-1)!\cdot k}{(n-k)!\cdot(k-1)!\cdot k} + \frac{(n-1)!\cdot(n-k)}{(n-1-k)!\cdot k!\cdot(n-k)}$$

$$= \frac{(n-1)!\cdot k}{(n-k)!\cdot k!} + \frac{(n-1)!\cdot(n-k)}{(n-k)!\cdot k!}$$

$$= \frac{(n-1)!\cdot n}{(n-k)!\cdot k!} = \frac{n!}{(n-k)!\cdot k!} = \binom{n}{k}$$

Pascal's identity

$$\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$$

$$\left.\begin{matrix} k = 0 \\ n = k \end{matrix}\right\} \text{base cases}$$

$k=0$ $\quad k=1$ $\quad k=2$ $\quad k=3$ $\quad k=4$ $\quad k=5$ $\quad k=6$

$n=0$ $\quad \binom{0}{0}$

$n=1$ $\quad \binom{1}{0} \quad \binom{1}{1}$

$n=2$ $\quad \binom{2}{0} \quad \binom{2}{1} \quad \binom{2}{2}$

$n=3$ $\quad \binom{3}{0} \quad \binom{3}{1} \quad \binom{3}{2} \quad \binom{3}{3}$

$n=4$ $\quad \binom{4}{0} \quad \binom{n-1}{k-1} \quad \binom{n-1}{k} \quad \binom{4}{3} \quad \binom{4}{4}$

$+$

$n=5$ $\quad \binom{5}{0} \quad \binom{5}{1} \quad \binom{n}{k} \quad \binom{5}{3} \quad \binom{5}{4} \quad \binom{5}{5}$

$n=6$ $\quad \binom{6}{0} \quad \binom{6}{1} \quad \binom{6}{2} \quad \binom{6}{3} \quad \binom{6}{4} \quad \binom{6}{5} \quad \binom{6}{6}$

Pascal's identity

$$\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$$

sometimes written as

$$\binom{n+1}{k} = \binom{n}{k-1} + \binom{n}{k}$$

$$\left.\begin{matrix} k = 0 \\ n = k \end{matrix}\right\} \text{base cases}$$

$k=0$, $k=1$, $k=2$, $k=3$, $k=4$, $k=5$, $k=6$

$n=0$: $\binom{0}{0}$

$n=1$: $\binom{1}{0}$ $\binom{1}{1}$

$n=2$: $\binom{2}{0}$ $\binom{2}{1}$ $\binom{2}{2}$

$n=3$: $\binom{3}{0}$ $\binom{3}{1}$ $\binom{3}{2}$ $\binom{3}{3}$

$n=4$: $\binom{4}{0}$ $\binom{n-1}{k-1}$ $+$ $\binom{n-1}{k}$ $\binom{4}{3}$ $\binom{4}{4}$

$n=5$: $\binom{5}{0}$ $\binom{5}{1}$ $\binom{n}{k}$ $\binom{5}{3}$ $\binom{5}{4}$ $\binom{5}{5}$

$n=6$: $\binom{6}{0}$ $\binom{6}{1}$ $\binom{6}{2}$ $\binom{6}{3}$ $\binom{6}{4}$ $\binom{6}{5}$ $\binom{6}{6}$

Pascal's identity

$$\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$$

Alternate proof

Pascal's identity

$$\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$$

Alternate proof

- You are one of n people.
- How many ways to form a committee of size k?

Pascal's identity

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- You are one of n people.
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$$\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$$

Alternate proof $\longrightarrow$

- You are one of n people.
- How many ways to form a committee of size k? $\Big\}$ $\binom{n}{k}$

$\hookrightarrow$ precisely 2 disjoint scenarios:

You are on the committee OR You are off the committee

Pascal's identity

$$\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$$

Alternate proof →

- You are one of n people.
- How many ways to form a committee of size k? $\Big\}\ \binom{n}{k}$

*↳ precisely 2 disjoint scenarios:

You are on the committee OR You are off the committee

base cases (exceptions)*
($\nexists$ 2 scenarios)

k = 0 $\quad\nexists$ committee
(you're off)

n = k $\quad$ all on committee

Pascal's identity

$$\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$$

Alternate proof $\longrightarrow$

- You are one of n people.
- How many ways to form a committee of size k? $\Big\}\ \binom{n}{k}$

* precisely 2 disjoint scenarios:

You are on the committee OR You are off the committee

either way, n-1 people remain

base cases (exceptions)*
($\not\exists$ 2 scenarios)

$k=0$ $\quad$ $\not\exists$ committee (you're off)

$n=k$ $\quad$ all on committee

Pascal's identity

$$\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$$

Alternate proof →

- You are one of n people.
- How many ways to form a committee of size k? } $\binom{n}{k}$

* precisely 2 disjoint scenarios:

You are on the committee OR You are off the committee

either way, n-1 people remain

must still choose k

base cases (exceptions)*
($\nexists$ 2 scenarios)

$k=0$ $\nexists$ committee (you're off)

$n=k$ all on committee

Pascal's identity

$$\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$$

Alternate proof $\longrightarrow$

- You are one of n people.
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* precisely 2 disjoint scenarios:

You are on the committee OR You are off the committee

either way, $n-1$ people remain

must still choose k

$$\binom{n-1}{k}$$

base cases (exceptions)*
($\nexists$ 2 scenarios)

$k=0$ $\nexists$ committee (you're off)

$n=k$ all on committee

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choose k-1 remaining

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$$\binom{n-1}{k}$$

base cases (exceptions)*
($\cancel{\exists}$ 2 scenarios)

$k=0$ $\quad \cancel{\exists}$ committee (you're off)

$n=k$ $\quad$ all on committee

Pascal's identity

$$\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$$

Alternate proof $\longrightarrow$

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- How many ways to form a committee of size k? $\Big\}\ \binom{n}{k}$

*precisely 2 disjoint scenarios:

You are on the committee OR You are off the committee

either way, n-1 people remain

choose k-1 remaining + must still choose k

$$\binom{n-1}{k-1} + \binom{n-1}{k}$$

base cases (exceptions)*
($\cancel{\exists}$ 2 scenarios)

$k=0$ $\cancel{\exists}$ committee (you're off)

$n=k$ all on committee