DS 4400

Machine Learning and Data Mining I

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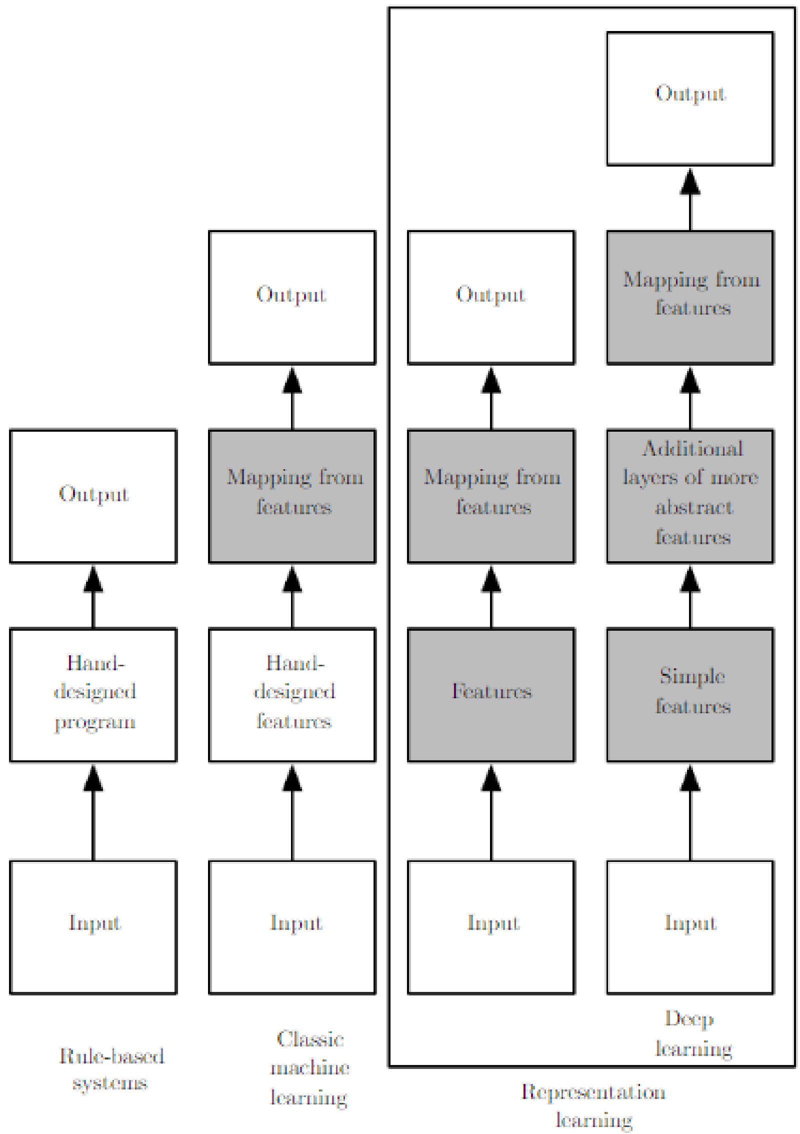
Outline

- Deep Learning
  - Success stories
  - Types of architectures
- Feed-Forward architectures
  - Terminology
  - Non-linear activations
  - Multi-Layer Perceptron
  - Multi-class classification (softmax unit)
  - Representing Boolean functions

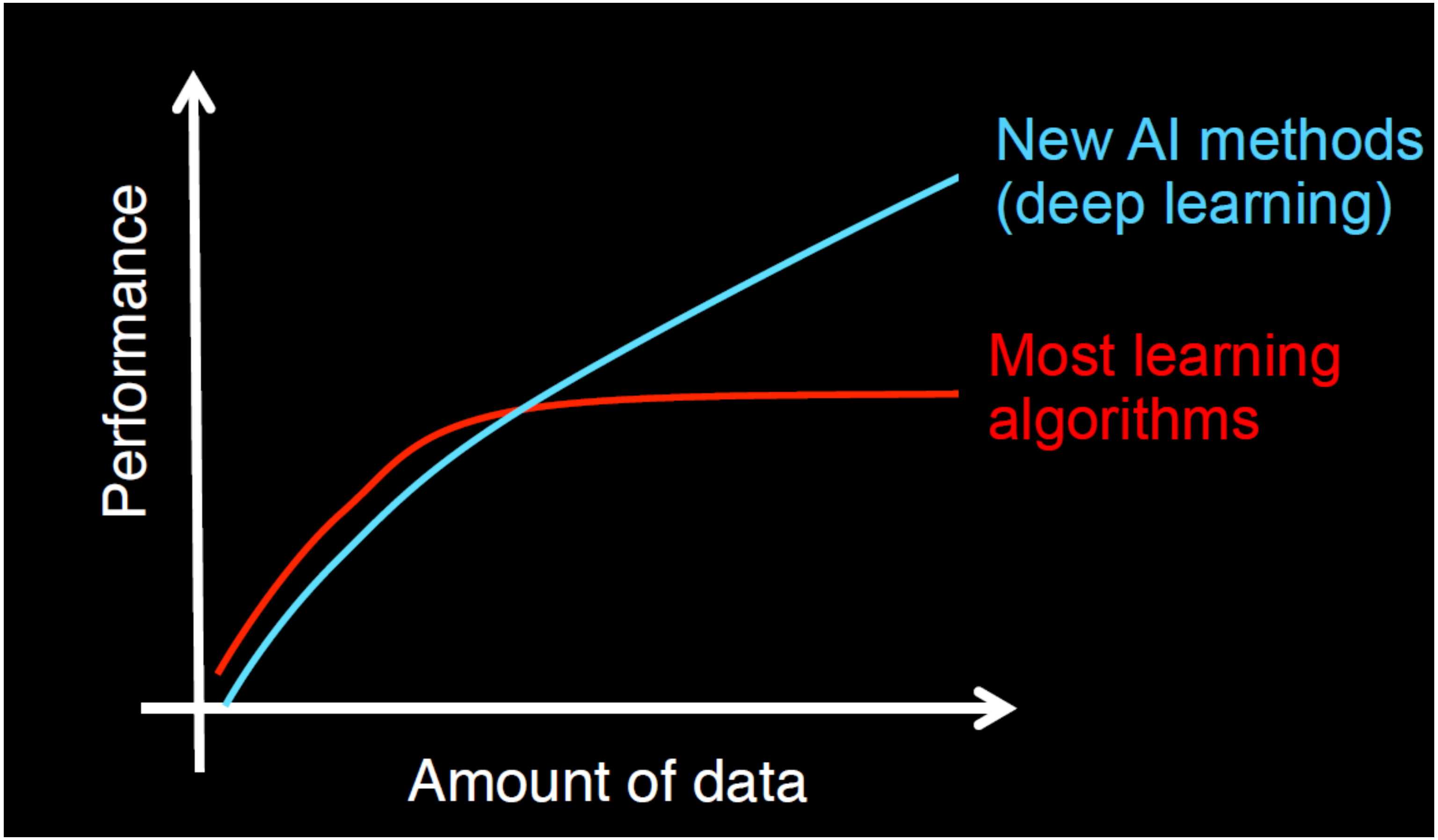
Deep Learning vs Traditional Learning



Neural Networks

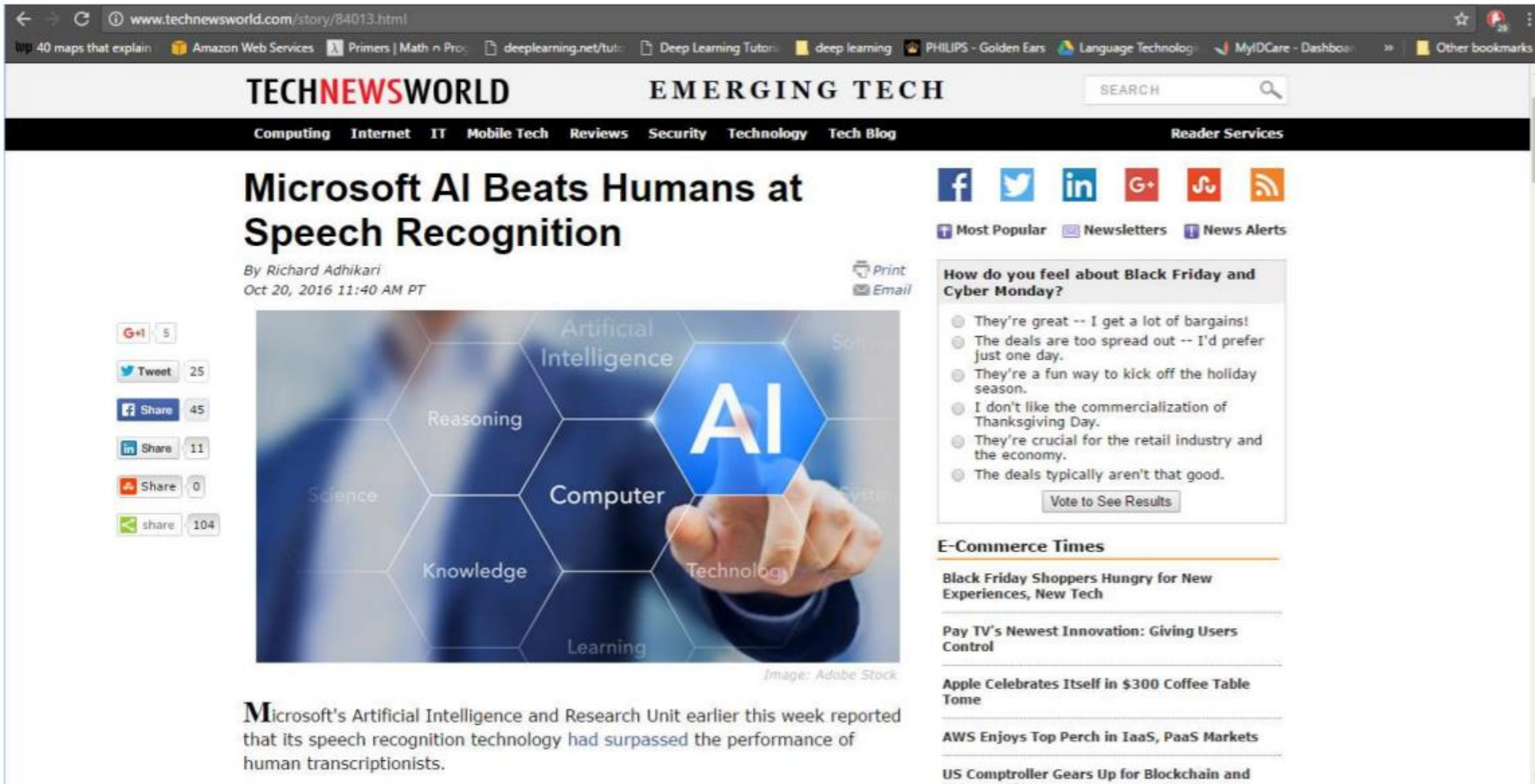
- Origins: Algorithms that try to mimic the brain.
- Very widely used in 80s and early 90s; popularity diminished in late 90s.
- Recent resurgence: State-of-the-art technique for many applications
- Artificial neural networks are not nearly as complex or intricate as the actual brain structure

Performance of Deep Learning

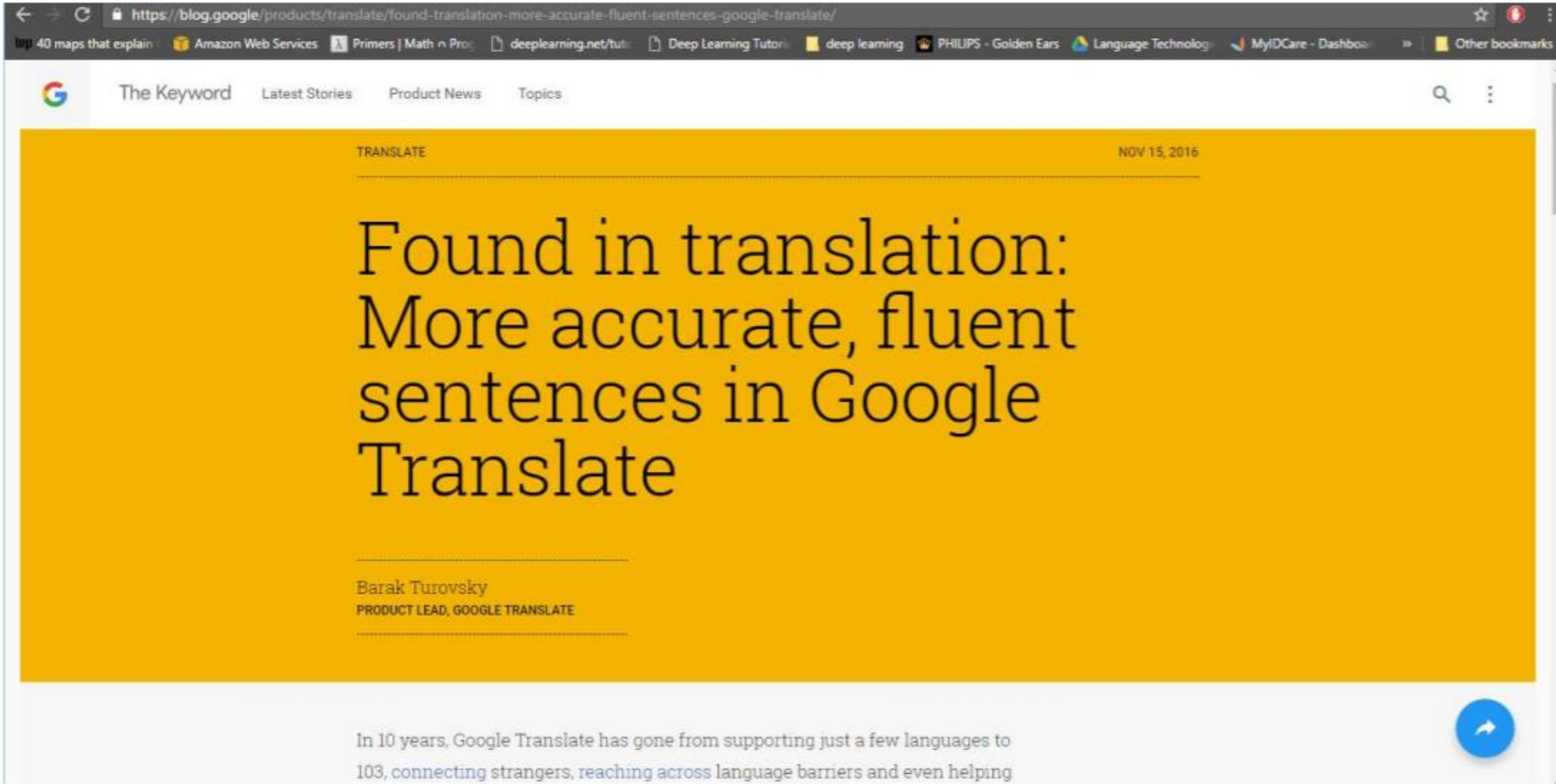


Deep Learning Applications

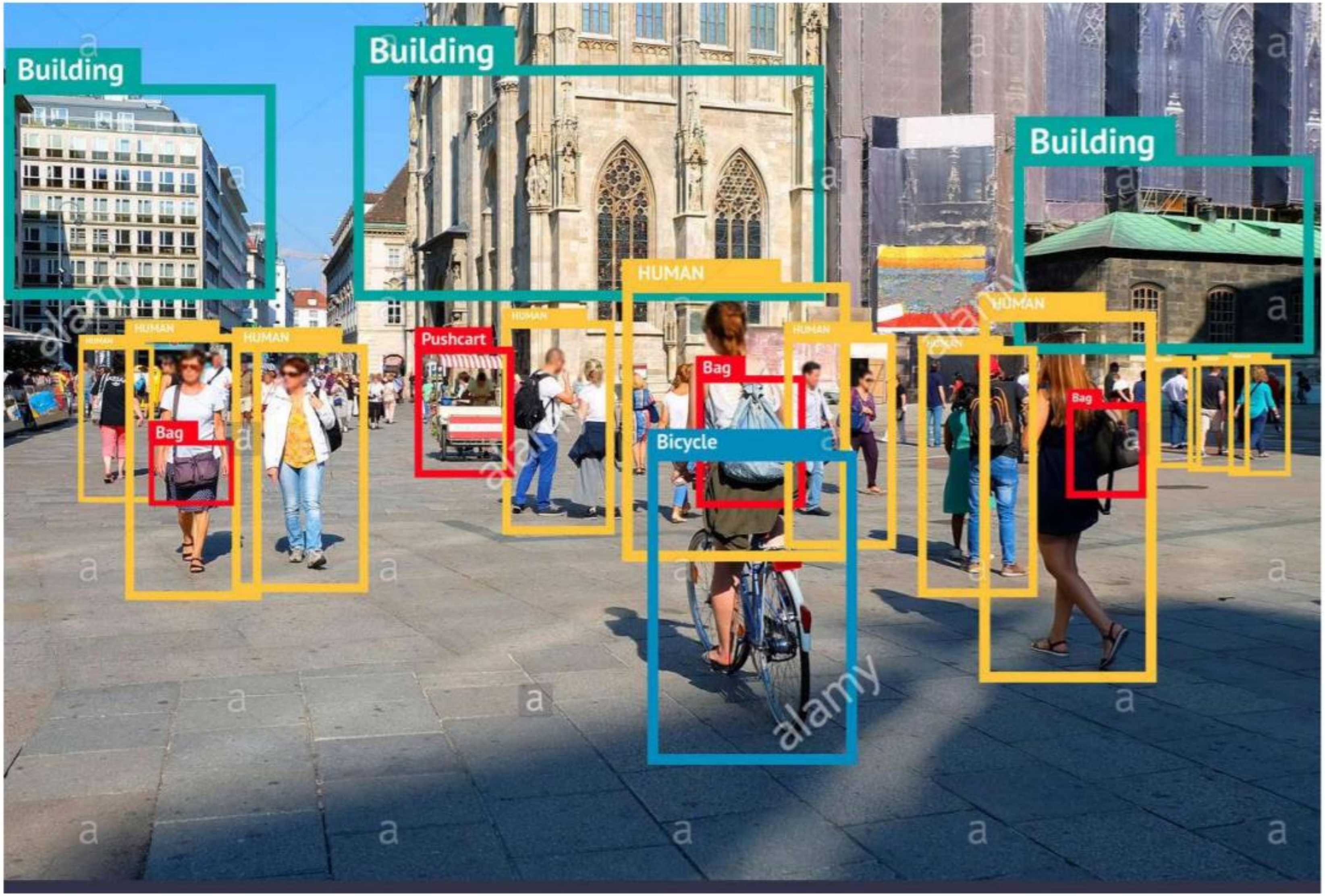
Success stories: Speech recognition



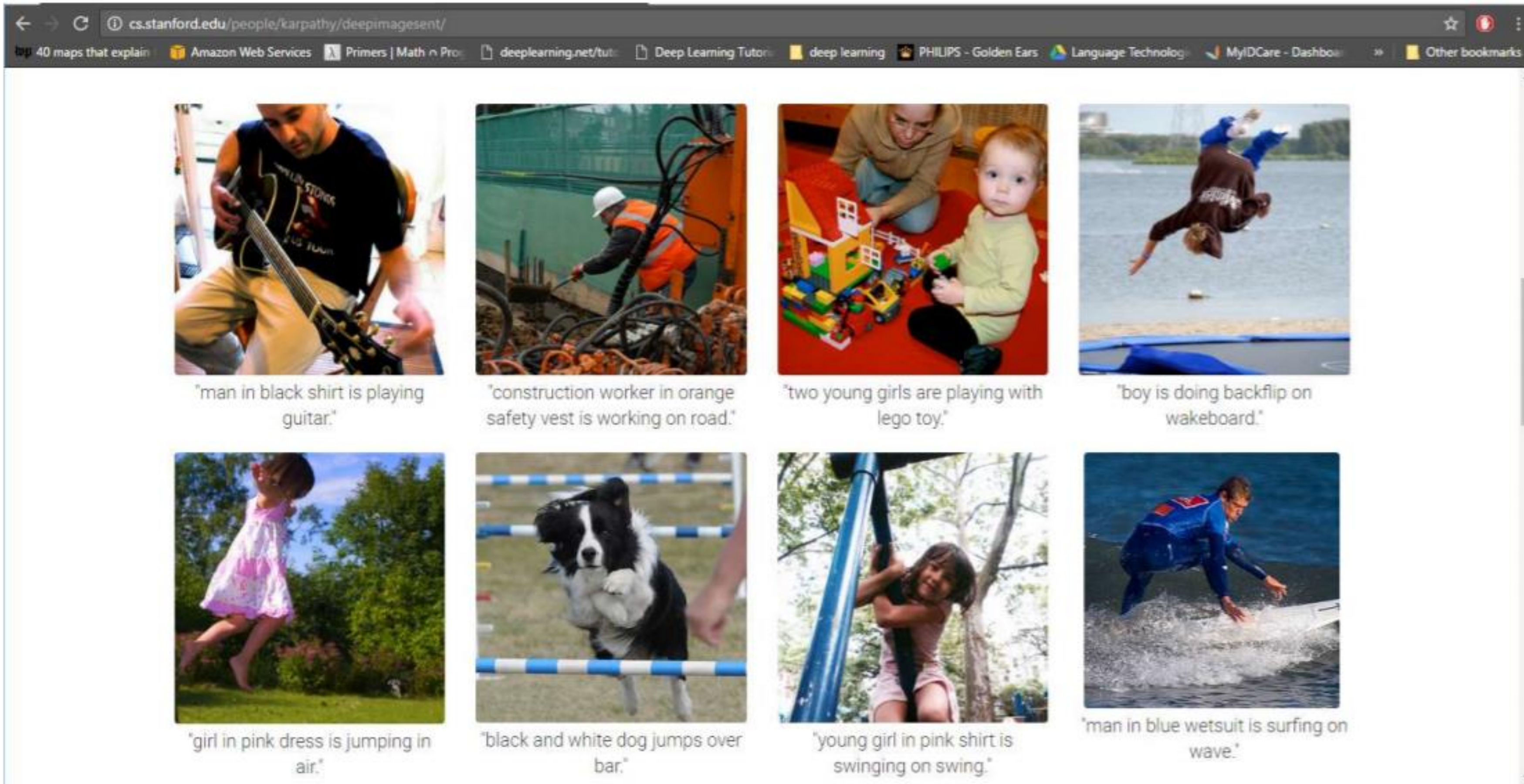
Success stories: Machine Translation



Success stories: Image segmentation



Success stories: Image captioning



References

- Deep Learning book
  - https://www.deeplearningbook.org/
- Stanford notes on deep learning
  - http://cs229.stanford.edu/notes/cs229-notes-deep_learning.pdf
- History of Deep Learning
  - https://beamandrew.github.io/deeplearning/2017/02/23/deep_learning_101_part1.html

Example

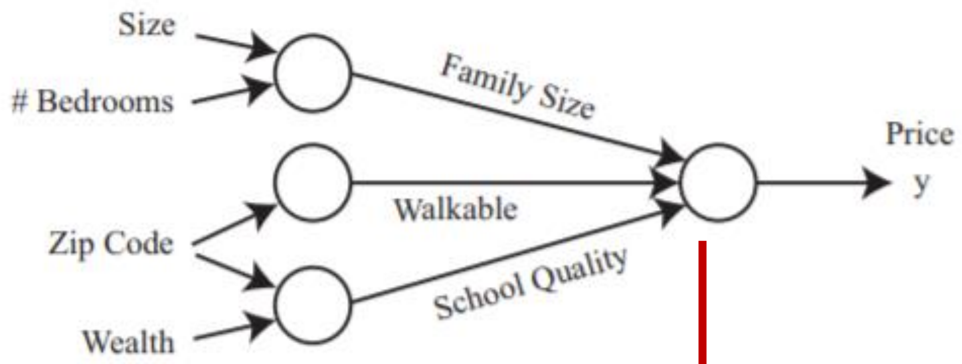
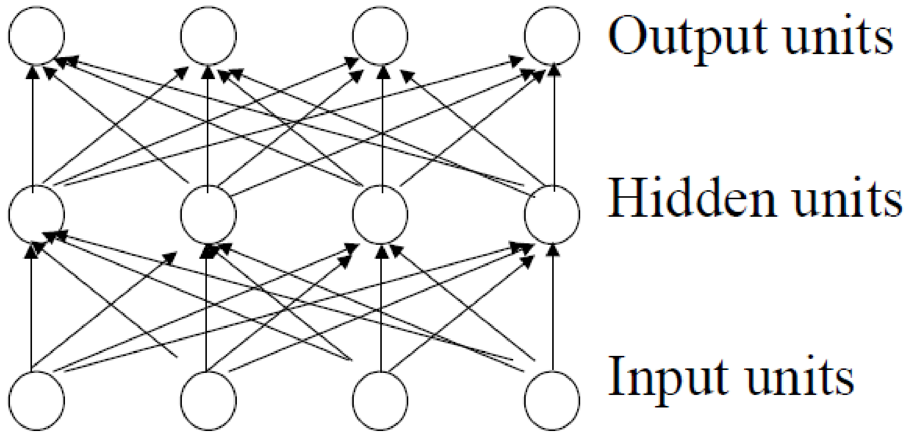


Figure 2: Diagram of a small neural network for predicting housing prices.

Intermediate Features

- Provide as input only training data: input and label
- Neural Networks automatically learn intermediate features!

Neural Networks



Layered feed-forward network

Training labels

Learned during training

Training data

- Neural networks are made up of **nodes** or **units**, connected by **links**
- Each link has an associated **weight** and **activation level**
- Each node has an **input function** (typically summing over weighted inputs), an **activation function**, and an **output**

Recall: The Perceptron

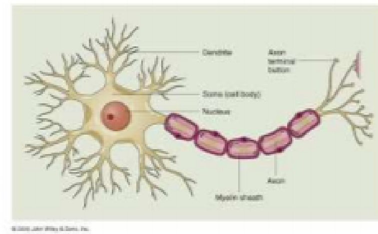
$$h(\boldsymbol{x}) = \operatorname{sign}(\boldsymbol{\theta}^\intercal \boldsymbol{x}) \quad \text{where} \quad \operatorname{sign}(z) = \begin{cases} 1 & \text{if } z \geq 0 \\ -1 & \text{if } z < 0 \end{cases}$$

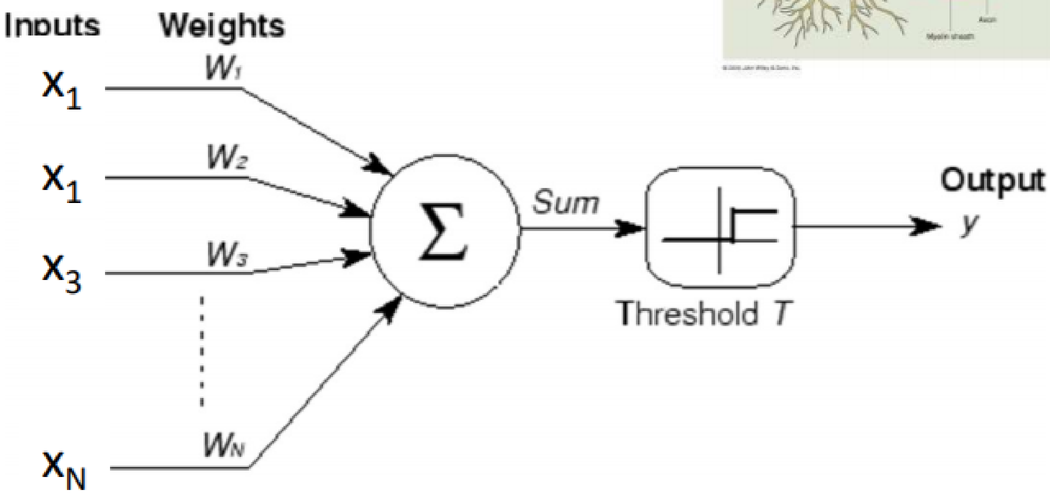
- The perceptron uses the following update rule each time it receives a new training instance $(\boldsymbol{x}^{(i)}, y^{(i)})$

$$\theta_j \leftarrow \theta_j - \frac{1}{2} \underbrace{\left(h_{\boldsymbol{\theta}}\left(\boldsymbol{x}^{(i)}\right) - y^{(i)} \right)}_{\text{either 2 or -2}} x_j^{(i)}$$

  - If the prediction matches the label, make no change
  - Otherwise, adjust $\boldsymbol{\theta}$

Perceptron

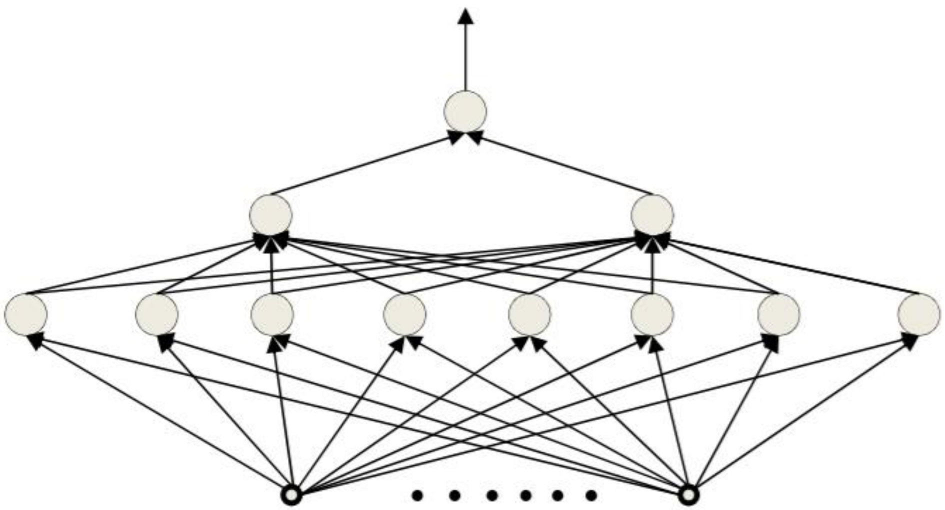


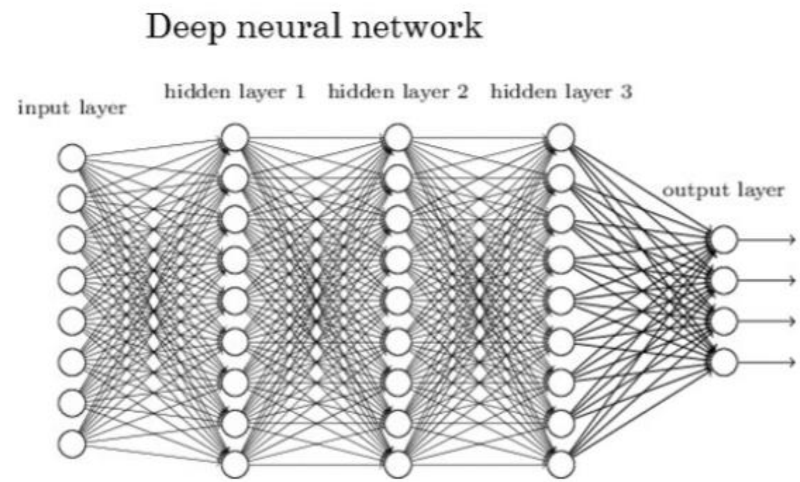


$$y = \begin{cases} 1 \; if \; \sum_i w_i x_i \geq T \\ 0 \; else \end{cases}$$

- A threshold unit
  - "Fires" if the weighted sum of inputs exceeds a threshold

Multi-Layer Perceptron



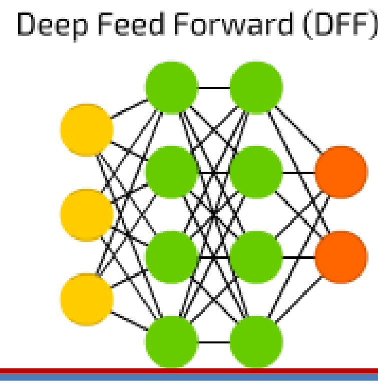


- A network of perceptrons
  - Generally "layered"

Neural Network Architectures

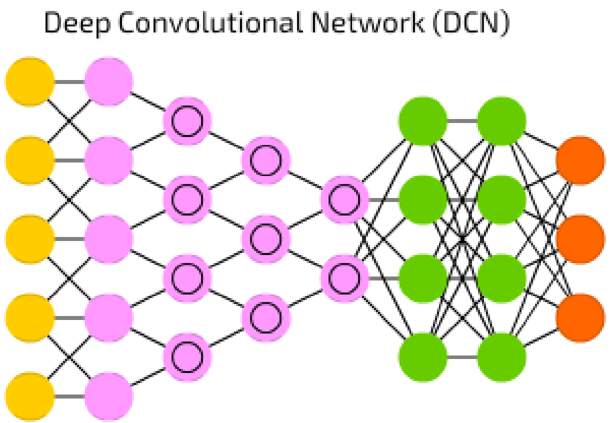
Feed-Forward Networks

- Neurons from each layer connect to neurons from next layer



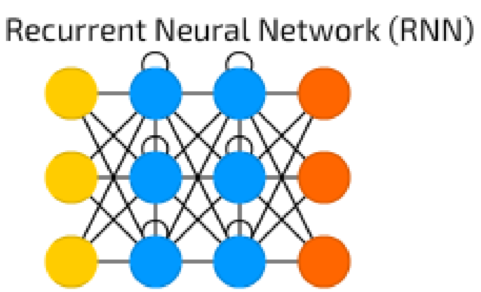
Convolutional Networks

- Includes convolution layer for feature reduction
- Learns hierarchical representations

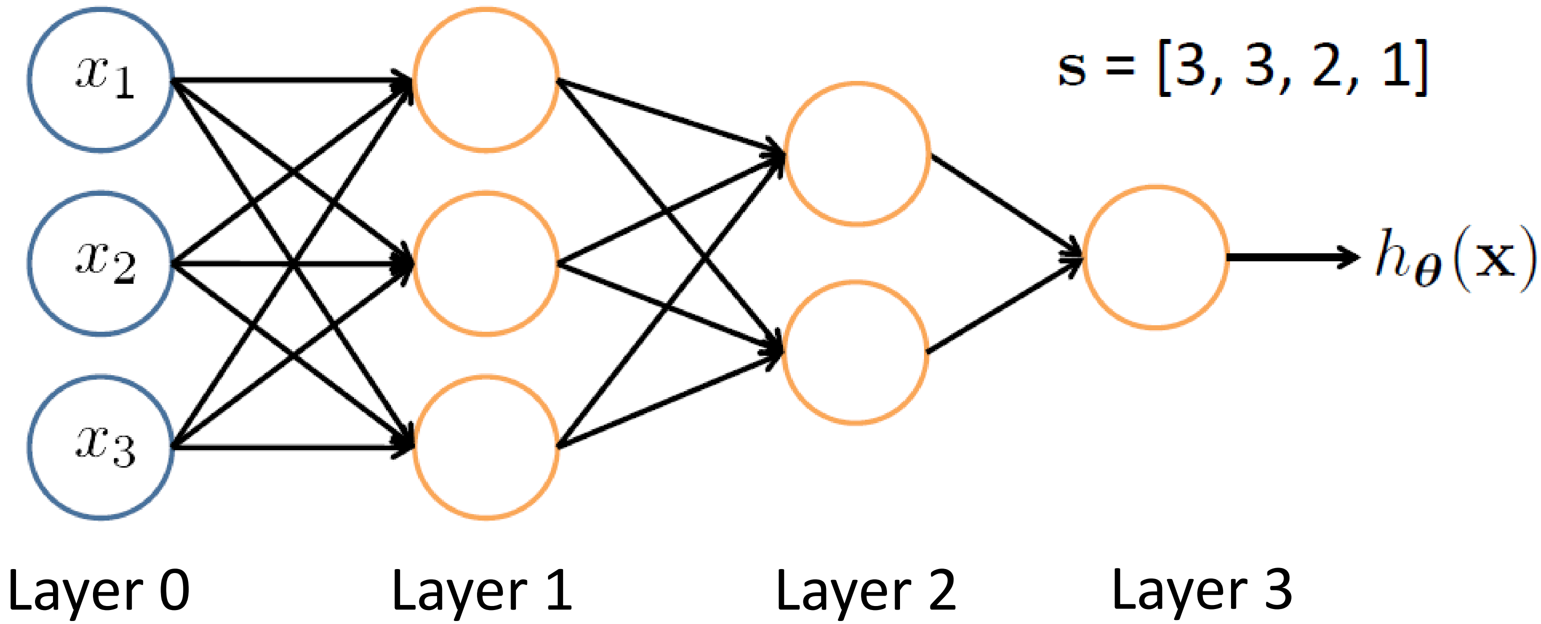


Recurrent Networks

- Keep hidden state
- Have cycles in computational graph



Feed-Forward Networks



L denotes the number of layers

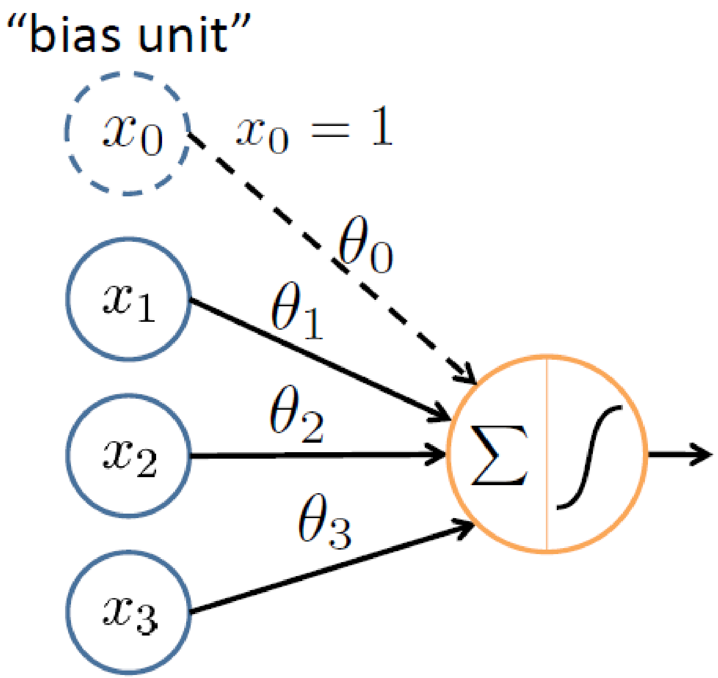
$\mathbf{s} \in \mathbb{N}^{+L}$ contains the numbers of nodes at each layer

- Not counting bias units
- Typically, $s_0 = d$ (# input features) and $s_{L\text{-}1} = K$ (# classes)

Feed-Forward NN

- Hyper-parameters
  - Number of layers
  - Architecture (how layers are connected)
  - Number of hidden units per layer
  - Number of units in output layer
  - Activation functions
- Other
  - Initialization
  - Regularization

Logistic Unit: A simple NN



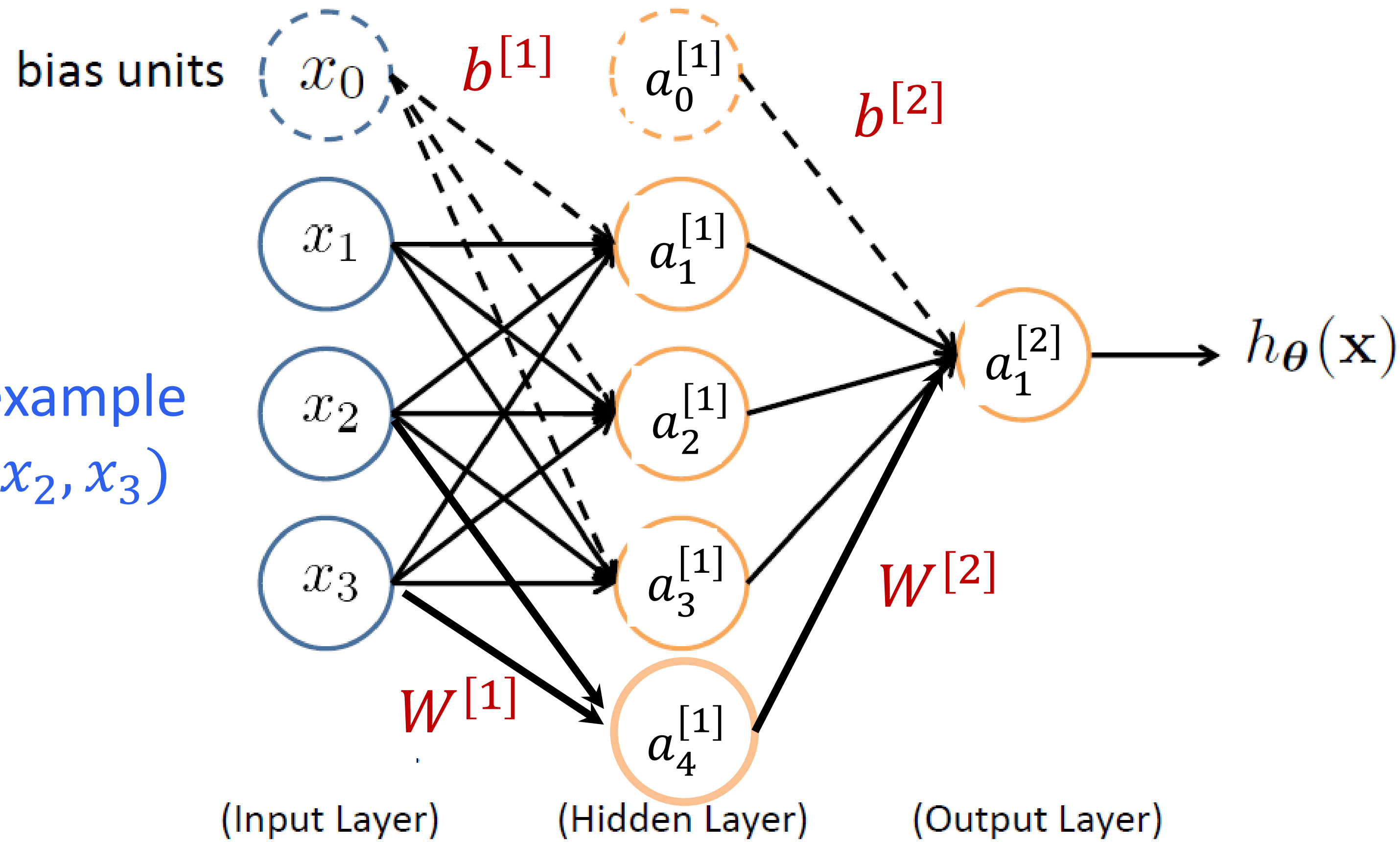
$$\mathbf{x} = \begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \end{bmatrix} \qquad \boldsymbol{\theta} = \begin{bmatrix} \theta_0 \\ \theta_1 \\ \theta_2 \\ \theta_3 \end{bmatrix}$$

$$h_{\boldsymbol{\theta}}(\mathbf{x}) = g\left(\boldsymbol{\theta}^\intercal \mathbf{x}\right) = g(w^T x + b)$$
$$= \frac{1}{1 + e^{-\boldsymbol{\theta}^\intercal \mathbf{x}}}$$

Sigmoid (logistic) activation function: $g(z) = \dfrac{1}{1 + e^{-z}}$

No hidden layers

Feed-Forward Neural Network



Training example
$x = (x_1, x_2, x_3)$

Layer 0 Layer 1 Layer 2

No cycles $\theta = (b^{[1]}, W^{[1]}, b^{[2]}, W^{[2]})$

Vectorization

$$z_1^{[1]} = W_1^{[1]T}x + b_1^{[1]} \quad \text{and} \quad a_1^{[1]} = g(z_1^{[1]})$$
$$\vdots \qquad\qquad \vdots \qquad\qquad \vdots$$
$$z_4^{[1]} = W_4^{[1]T}x + b_4^{[1]} \quad \text{and} \quad a_4^{[1]} = g(z_4^{[1]})$$

$$\boxed{\underbrace{\begin{bmatrix} z_1^{[1]} \\ \vdots \\ \vdots \\ z_4^{[1]} \end{bmatrix}}_{z^{[1]} \in \mathbb{R}^{4\times 1}} = \underbrace{\begin{bmatrix} — W_1^{[1]T} — \\ — W_2^{[1]T} — \\ \vdots \\ — W_4^{[1]T} — \end{bmatrix}}_{W^{[1]} \in \mathbb{R}^{4\times 3}} \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}}_{x \in \mathbb{R}^{3\times 1}} + \underbrace{\begin{bmatrix} b_1^{[1]} \\ b_2^{[1]} \\ \vdots \\ b_4^{[1]} \end{bmatrix}}_{b^{[1]} \in \mathbb{R}^{4\times 1}}}$$

$$z^{[1]} = W^{[1]}x + b^{[1]}$$

$$a^{[1]} = g(z^{[1]})$$

Vectorization

Output layer

$$z_1^{[2]} = W_1^{[2]^T} a^{[1]} + b_1^{[2]} \quad \text{and} \quad a_1^{[2]} = g(z_1^{[2]})$$

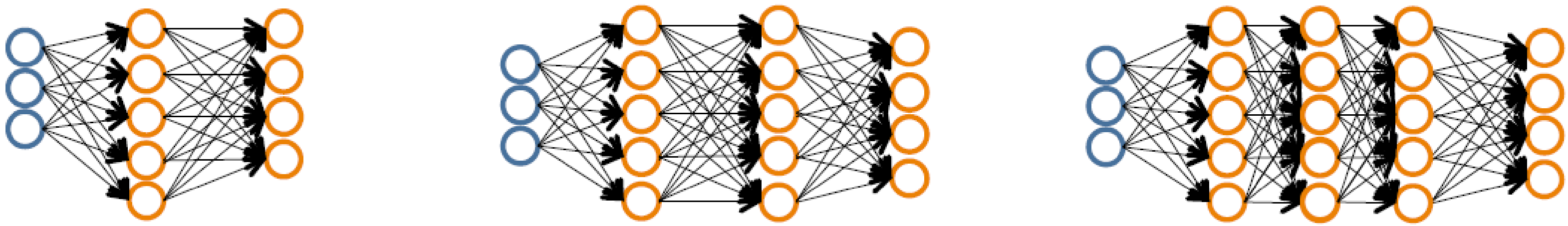
$$\boxed{\underbrace{z^{[2]}}_{1\times 1} = \underbrace{W^{[2]}}_{1\times 4}\underbrace{a^{[1]}}_{4\times 1} + \underbrace{b^{[2]}}_{1\times 1} \quad \text{and} \quad \underbrace{a^{[2]}}_{1\times 1} = g(\underbrace{z^{[2]}}_{1\times 1})}$$

Hidden Units

- Layer 1
  - First hidden unit:
    - Linear: $z_1^{[1]} = \mathrm{W}_1^{[1]\,\mathrm{T}} x + \mathrm{b}_1^{[1]}$
    - Non-linear: $a_1^{[1]} = \mathrm{g}(\mathrm{z}_1^{[1]})$
  - …
  - Fourth hidden unit:
    - Linear: $z_4^{[1]} = \mathrm{W}_4^{[1]\,\mathrm{T}} x + \mathrm{b}_4^{[1]}$
    - Non-linear: $a_4^{[1]} = \mathrm{g}(\mathrm{z}_4^{[1]})$
- Terminology
  - $a_i^{[j]}$ - Activation of unit i in layer j
  - g - Activation function
  - $W^{[j]}$ - Weight vector controlling mapping from layer j-1 to j
  - $b^{[j]}$ - Bias vector from layer j-1 to j

How to pick architecture?

Pick a network architecture (connectivity pattern between nodes)



- # input units = # of features in dataset
- # output units = # classes

Reasonable default: 1 hidden layer

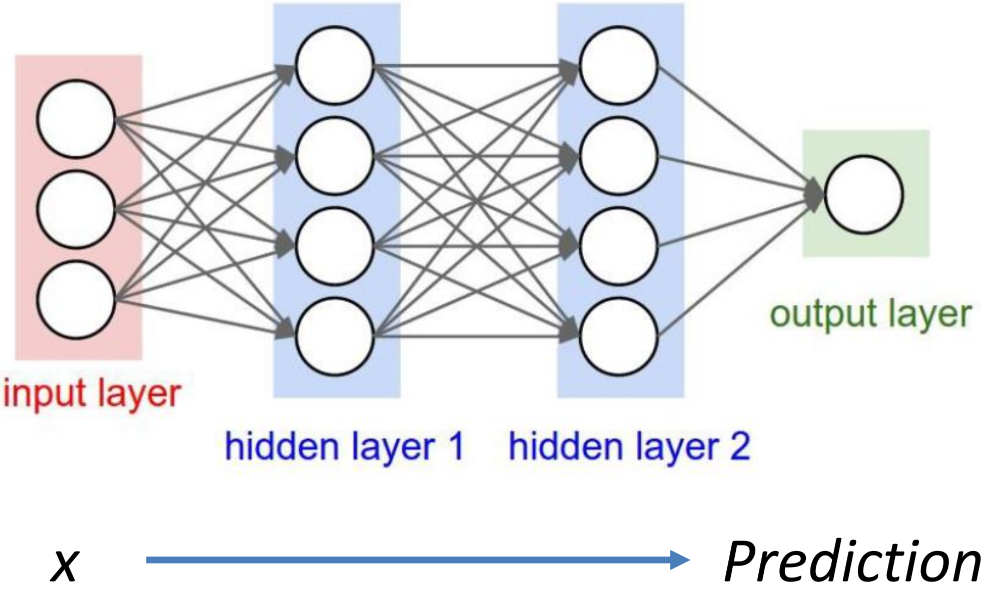
- or if >1 hidden layer, have same # hidden units in every layer (usually the more the better)

Training Neural Networks

- Input training dataset D
  - Number of features: d
  - Labels from K classes
- First layer has d+1 units (one per feature and bias)
- Output layer has K units
- Training procedure determines parameters that optimize loss function
  - Backpropagation
  - Learn optimal $W^{[i]}, b^{[i]}$ at layer *i*
- Testing done by forward propagation

Forward Propagation

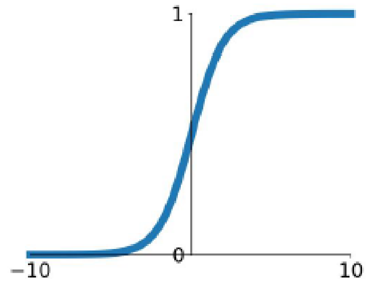
- The input neurons first receive the data features of the object. After processing the data, they send their output to the first hidden layer.
- The hidden layer processes this output and sends the results to the next hidden layer.
- This continues until the data reaches the final output layer, where the output value determines the object's classification.
- This entire process is known as Forward Propagation, or Forward prop.



Activation Functions

Sigmoid

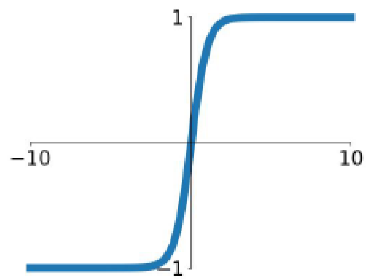
$$\sigma(x) = \frac{1}{1+e^{-x}}$$



Binary Classification

tanh

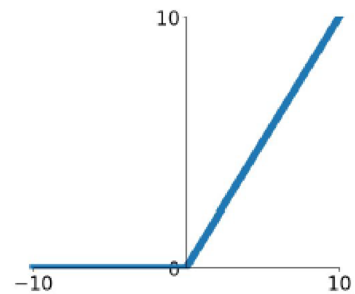
$$\tanh(x)$$



Regression

ReLU

$$\max(0, x)$$



Intermediary layers

Why Non-Linear Activations?

- Assume g is linear: $\mathrm{g}(z) = Uz$
  - At layer 1: $z^{[1]} = \mathrm{W}^{[1]} x + \mathrm{b}^{[1]}$
  - $a^{[1]} = \mathrm{g}\left(\mathrm{z}^{[1]}\right) = \mathrm{Uz}^{[1]} = \mathrm{UW}^{[1]}\mathrm{x} + \mathrm{Ub}^{[1]}$
- Layer 2:
  - $a^{[2]} = \mathrm{g}\left(\mathrm{z}^{[2]}\right) = \mathrm{Uz}^{[2]} = \mathrm{UW}^{[2]}\mathrm{a}^{[1]} + \mathrm{Ub}^{[2]} =$

  $= \mathrm{UW}^{[2]}\,\mathrm{UW}^{[1]}x + \mathrm{UW}^{[2]}\mathrm{Ub}^{[1]} + \mathrm{Ub}^{[2]}$
- Last layer
  - Output is linear in input!
  - Then NN will only learn linear functions

Acknowledgements

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  - Yann LeCun
  - Andrew Ng
  - Eric Eaton
  - David Sontag
  - Andrew Moore
- Thanks!