14 July 2014
Calculus 3, Interphase 2014
Paul E. Hand
hand@math.mit.edu

Problem Set 3

Due: **Monday 21 July 2013** in class.

1. Find all critical points of $f(x, y) = x^4 + y^4 - 4xy$. Classify them as a local max, a local min, or a saddle point.

2. *Best fit line*. Consider the points $(0, 1), (1, 1), (2, 2), (3, 3)$. The residual of the point (x_i, y_i) with the line $y(x) = mx + b$ is defined as $y_i - y(x_i)$. The best-fit line is defined to be the line that minimizes the average of the squared residuals (aka the mean square error). Find m and b of the best fit line.

3. Use Lagrange multipliers to find the radius and the height of the cylinder with surface area S that has maximal volume.

4. Consider a current I going through three resistors in parallel. If each has resistance R_1, R_2, R_3, and the respective current through each resistor is I_1, I_2, I_3, then the power dissipated by each resistor is $I_1^2 R_1$, $I_2^2 R_2$, $I_3^2 R_3$. Note that $I = I_1 + I_2 + I_3$. The current splits up in a way that minimizes the total power dissipated by the resistors.

   (a) Using the constraint to remove one variable, show that the currents that minimize the total power dissipation are such that $I_1 R_1 = I_2 R_2 = I_3 R_3$.

   (b) Use Lagrange multipliers to show the same thing.