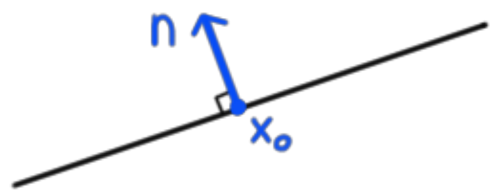
Lecture 2 - Lines, planes, determinants, cross products - 7/2/2014 — Interphase 2014 Calc 3

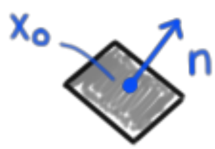
11. Lines and planes

    a. In 2d, any line can be written $\mathbf{n} \cdot \mathbf{x} = b$ for some b. Here $\mathbf{n}$ is a normal vector to the line.

    b. In 2d, the line going through $\mathbf{x}_0$ with normal vector $\mathbf{n}$ is given by $\mathbf{n} \cdot \mathbf{x} = \mathbf{n} \cdot \mathbf{x}_0$.



    c. In 3d, to specify a plane, we need a point $\mathbf{x_0}$ and a normal vector $\mathbf{n}$.



    d. In 3d, any plane can be written $\mathbf{n} \cdot \mathbf{x} = b$ for some b. Here $\mathbf{n}$ is a normal vector the the plane.

    e. In 3d, the plane going through $\mathbf{x}_0$ with normal vector $\mathbf{n}$ is given by $\mathbf{n} \cdot \mathbf{x} = \mathbf{n} \cdot \mathbf{x}_0$.

12. Cross product

    a. The determinant of a 2×2 matrix is

$$\det \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

    b. The cross product of $\mathbf{x} = \langle x_1, x_2, x_3 \rangle$ and $\mathbf{y} = \langle y_1, y_2, y_3 \rangle$ is

$$\mathbf{x} \times \mathbf{y} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \end{vmatrix} = \begin{vmatrix} x_2 & x_3 \\ y_2 & y_3 \end{vmatrix} \mathbf{i} - \begin{vmatrix} x_1 & x_3 \\ y_1 & y_3 \end{vmatrix} \mathbf{j} + \begin{vmatrix} x_1 & x_2 \\ y_1 & y_2 \end{vmatrix} \mathbf{k}$$

    c. The cross product of two three-dimensional vectors is a three-dimensional vector perpendicular to both.

d. The right hand rule helps qualitatively determine the direction of a cross product:

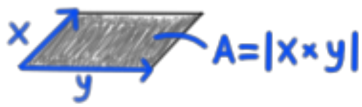
> *If you align the fingers of your right hand with the vector* **a** *and you can curl them directly to align with* **b***, then your right thumb points in the direction of* $\mathbf{a} \times \mathbf{b}$.

e. If **x** and **y** are vectors and θ is the angle between them, then the length of their cross product is

$$|\mathbf{a} \times \mathbf{b}| = |\mathbf{a}||\mathbf{b}| \sin(\theta)$$

f. Two vectors have zero cross product if they are multiples of each other.

g. The area of the parallelogram spanned by the vectors **x** and **y** is $|\mathbf{x} \times \mathbf{y}|$.



h. Algebra of cross products:

$$\begin{aligned}
\mathbf{a} \times \mathbf{a} &= 0 \\
\mathbf{a} \times \mathbf{b} &= -\mathbf{b} \times \mathbf{a} \\
\mathbf{a} \times (\mathbf{b} + \mathbf{c}) &= \mathbf{a} \times \mathbf{b} + \mathbf{a} \times \mathbf{c} \\
(c\mathbf{a}) \times \mathbf{b} &= \mathbf{a} \times (c\mathbf{b}) = c(\mathbf{a} \times \mathbf{b})
\end{aligned}$$