Lecture 12 - Fundamental theorem of line integrals, Green's theorem - 7/30/2014 — Interphase 2014 Calc 3

32. Fundamental theorem of line integrals

    a. The fundamental theorem of line integrals:

    If C is any curve connecting $\mathbf{x}$ to $\mathbf{y}$, then $\int_C \nabla\phi \cdot d\mathbf{r} = \phi(\mathbf{y}) - \phi(\mathbf{x})$.

    b. Line integrals of conservative vector fields are independent of path. Only the endpoints affect the value of the line integral.

    c. The line integral of a conservative vector field over a closed curve is 0.

    d. If $\mathbf{F}$ is conservative, then $\int_C \mathbf{F} \cdot d\mathbf{r}$ can be evaluated by

    - Finding the potential function ϕ such that $\mathbf{F} = \nabla\phi$.
    - Using the fundamental theorem of line integrals

33. Green's theorem

    a. A curve $\mathbf{r}(t)$ for $a \leq t \leq b$ is *closed* if $\mathbf{r}(b) = \mathbf{r}(a)$.

    b. A curve is *simple* if it has no self-intersections.

    c. A curve is *positively-oriented* if its interior is always on the left of the path relative to the direction traversed.

    d. The line integral over a closed curve C with positive orientation is written $\oint_C \nabla\phi \cdot d\mathbf{r}$.

    e. If $\mathbf{F} = \langle P, Q \rangle$ and $d\mathbf{r} = \langle dx, dy \rangle$, then $\int_C \mathbf{F} \cdot d\mathbf{r} = \int_C P dx + Q dy$.

    f. Green's Theorem:

    Let C be a positively-oriented, piecewise smooth, simple, and closed curve that bounds the region R. Then

$$\oint_C P dx + Q dy = \iint_R (\partial_x Q - \partial_y P) dA.$$

34. Evaluating line integrals

    a. Three ways to integrate line integrals are

    - Directly
    - By the fundamental theorem of line integrals
    - By Green's theorem