

CS 6140: Machine Learning — Fall 2021— Paul Hand

HW 1

Due: Wednesday September 22, 2021 at 2:30 PM Eastern time via [Gradescope](#).

Names: [Put Your Name(s) Here]

You can submit this homework either by yourself or in a group of 2. You may consult any and all resources. Make sure to justify your answers. If you are working alone, you may either write your responses in LaTeX or you may write them by hand and take a photograph of them. If you are working in a group of 2, you must type your responses in LaTeX. You are encouraged to use [Overleaf](#). Create a new project and replace the tex code with the tex file of this document, which you can find on the [course website](#). To share the document with your partner, click Share > Turn on link sharing, and send the link to your partner. When you upload your solutions to Gradescope, make sure to take each problem with the correct page or image.

Question 1. *The following matrix corresponds to a linear blur operation:*

$$M = \begin{pmatrix} 1/2 & 1/2 & 0 & 0 \\ 0 & 1/2 & 1/2 & 0 \\ 0 & 0 & 1/2 & 1/2 \\ 1/2 & 0 & 0 & 1/2 \end{pmatrix}.$$

- (a) Compute Mx , where $x = \begin{pmatrix} 2 \\ 0 \\ 1 \\ 0 \end{pmatrix}$.

Response:

- (b) Find a unit-norm vector in the null space of M .

Response:

- (c) Find a unit-norm eigenvector with eigenvalue 1.

Response:

Question 2. *You have a coin such that $P(\text{Heads}) = p$, for an unknown p . You flip the coin 10 times and get HHTHTHHHHT.*

- (a) What is the probability that you observed that exact sequence of tosses? Your answer should be an expression that depends on p .

Response:

- (b) Find the maximum likelihood estimate of p .

Response:

Question 3. You sample four numbers from a Uniform distribution over $[0, \theta]$ for an unknown positive θ . The numbers you get are $\{0.1, 2.0, 0.95, 1.2\}$.

- (a) What is the likelihood of the data you observed? Your answer should be an expression that depends on θ . Hint: use a piecewise function.

Response:

- (b) Find the maximum likelihood estimate of θ .

Response: