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Signal Recovery
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HW 3

Due: 28 March 2017 in class

1. Let $x_0 \in \mathbb{R}^m$. Let $A \in \mathbb{R}^{n\times m}$ with $m \geq n$ have rank n. Let $b \in \mathbb{R}^n$. Use Lagrange multipliers to find an analytical expression for the solution to

$$\min_{x\in\mathbb{R}^m} \|x - x_0\|_2 \text{ subject to } Ax = b.$$

2. Let $X_0 \in \mathbb{R}^{n\times n}$ and $X_0 \succeq 0$. Let $A_i \in \mathbb{R}^{n\times n}$ for $i = 1 \dots m$. Find the dual program to

$$\min 0 \text{ subject to } X \succeq 0, \langle A_i, X\rangle = \langle A_i, X_0\rangle, i = 1 \dots m.$$

Show all the work involved in computing the infimum of the augmented Lagrangian. That is to say, your answer should derive the dual feasibility conditions instead of simply stating them.

3. Consider the space of $n \times n$ real symmetric matrices. Let $I_S(X) = \begin{cases} 0 & X \in S, \\ \infty & \text{otherwise.} \end{cases}$

Show that $\partial I_{\{X\succeq 0\}}(0) + \partial I_{\{X_{11}=0\}}(0) \neq \partial I_{\{X\succeq 0, X_{11}=0\}}(0)$.

4. (a) Let $f : \mathbb{R}^2 \to \mathbb{R}$ be given by $f(x) = |x_1 - x_2|$ for $x = (x_1, x_2)$. What is $\partial f(x)$? Make sure your answer is presented for all possible values of $x \in \mathbb{R}^2$.

   (b) Let $Z \in \mathbb{R}^{nm}$. Let z_i be the ith block of size n of Z. That is $z = \begin{pmatrix} z_1 \\ z_2 \\ z_3 \\ \vdots \\ z_m \end{pmatrix}$, where $z_i \in \mathbb{R}^n$.

   Let $f(Z) = \sum_{i=1}^m \sum_{j=1}^m \|z_i - z_j\|_2$. What is $\partial f(Z)$? Make sure your answer is presented for all possible values of $Z \in \mathbb{R}^{nm}$.