30 November 2016
Analysis I
Paul E. Hand
hand@rice.edu

Coverpage to Pledged HW 11

Time limit: 3 hours. You may not use your books, your homeworks, your notes, or any electronics during the exam. Please write the start and finish times on your paper. Each subproblem is worth 10 points. To receive full credit, you must name all major theorems and state definitions used in your arguments. All problems must be accompanied by a proof. You may cite results from class and well-known theorems.

This homework is pledged. On the first page, please write your signature and the Rice University pledge: "On my honor, I have neither given nor received any unauthorized aid on this homework."

Due: Thursday, 8 December 2016 by noon under my door.

[The exam is on the next page]

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1. (a) (7 points) Let $f : \mathbb{R}^2 \to \mathbb{R}^2$. Prove that if f is differentiable at $(0, 0)$ then all directional derivatives exist at $(0, 0)$.

   (b) (8 points) Find a continuous function $f : \mathbb{R}^2 \to \mathbb{R}^2$ such that all directional derivatives exist at $(0, 0)$, yet the function is not differentiable at $(0, 0)$.

2. (15 points) Show that the unit circle in $\mathbb{R}^2$ has content zero. (Recall that in $\mathbb{R}^2$, a set has content zero if for all $\varepsilon > 0$, it can be covered by a finite union of rectangles whose areas sum to less than ε).

3. (15 points) Let $f(x) = \sum_{n=1}^{\infty} \frac{1}{n^3 - x^3}$. What is $f'(x)$ for real valued x not equal to a positive integer? You may leave your answer as an infinite sum if you prove that the sum converges and is correct.

4. (15 points) Show the following statement is false by providing a counterexample: If $f : \mathbb{R}^n \to \mathbb{R}^n$ is differentiable for all x, $f'(0)$ is invertible, and $f(0) = 0$, then there exist open sets $U \subset \mathbb{R}^n$ containing 0 and $V \subset \mathbb{R}^n$ containing 0 and a function $g : V \to U$ such that $g(f(x)) = x$ for all $x \in U$ and $f(g(y)) = y$ for all $y \in V$.