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Analysis I
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HW 3

Due: 13 Sep 2016

The problems are written in the format 'chapter.section.problem-number' from Lang's book. Practice problems are not to be handed in. The HW problems will be graded thoroughly and may be revised once, by the Tuesday after they were returned. Please submit each problem on a detached sheet of paper with your name on it.

Practice problems:

1. Directly prove that $f(x) = x^2$ is Riemann integrable on $[0, 1]$ and that the value of the Riemann integral is $1/3$. Do this by showing that the supremum of all lower sums equals the infimum of all upper sums equals $1/3$.

2. Evaluate
$$\lim_{n\to\infty} \frac{\log n + \log(n+1) + \cdots + \log(2n-1)}{n} - \log n$$
by viewing it as a Riemann sum. Feel free to use elementary techniques of integration to evaluate the value of the resulting integral. Make sure to justify why the limit exists.

Homework problems:

P7. Let $f(x) = \begin{cases} 1 & \text{if } x = 1/n \text{ for some } n \in \mathbb{N}, \\ 0 & \text{otherwise.} \end{cases}$
Is f Riemann integrable on $[0, 1]$? Prove it. If so, what is the value of the integral.

P8. Let $f(x) = \begin{cases} 0 & \text{if } x = 0, \\ \sin(1/x) & \text{otherwise.} \end{cases}$
Is f Riemann integrable on $[0, 1]$? Prove it.

P9. Suppose that the sequence of *Riemann integrable* functions f_n converges uniformly to f on $[a, b]$. That is, suppose that $\lim_{n\to\infty} \sup_{x\in[a,b]} |f_n(x) - f(x)| = 0$. Prove that f is Riemann integrable and that
$$\lim_{n\to\infty} \int_a^b f_n(x)dx = \int_a^b \lim_{n\to\infty} f_n(x)dx.$$
Feel free to use facts like $|\int_a^b g(x)dx| \leq \int_a^b |g(x)|dx$.