

25 August 2016
Analysis I
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HW 1

Due: 30 Aug 2016

The problems are written in the format ‘chapter.section.problem-number’ from Lang’s book. Practice problems are not to be handed in. The HW problems will be graded thoroughly and may be revised once, by the Tuesday after they were returned. Please submit each problem on a detached sheet of paper with your name on it.

Practice problems:

1. II.2.4
2. Let $g(x)$ be a bounded function in a neighborhood of a . Let $\lim_{x \rightarrow a} f(x) = 0$. Show that $\lim_{x \rightarrow a} f(x)g(x)$ exists and equals 0.
3. II.3.8
4. II.4.4

Homework Problems:

P1. II.4.1

P2. Prove that a periodic continuous function on \mathbb{R} is uniformly continuous or find a counterexample.

P3. In this problem you will find examples of functions $f_n(x)$ defined on $(0, 1)$ such that

$$\lim_{x \rightarrow 0} \sum_{n=1}^{\infty} f_n(x) \neq \sum_{n=1}^{\infty} \lim_{x \rightarrow 0} f_n(x).$$

- (a) Find an example where the sum on the left hand side is $+\infty$ for all $x \in (0, 1)$.
- (b) Find an example where all the sums and limits are finite.