

20 October 2015
Analysis I
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HW 7

Due: 27 Oct 2015

The problems are written in the format ‘chapter.section.problem-number’ from Lang’s book. Practice problems must be handed in and will be checked for honest effort. Portfolio problems will be graded thoroughly and may be revised until your solutions are of professional quality. Please submit each portfolio problem on a detached sheet of paper with your name on it.

Practice problems:

1. VII.1.5
2. VII.3.3
3. VII.3.7

Portfolio problems:

P16. Draw and *completely* justify the Venn diagram for the following sets of sequences of functions:

- $\{\{f_n\}_{n=1}^{\infty} \mid f_n \rightarrow 0 \text{ pointwise}, f_n : \mathbb{R} \rightarrow \mathbb{R}\}$
- $\{\{f_n\}_{n=1}^{\infty} \mid f_n \rightarrow 0 \text{ uniformly}, f_n : \mathbb{R} \rightarrow \mathbb{R}\}$
- $\{\{f_n\}_{n=1}^{\infty} \mid f_n \rightarrow 0 \text{ in } L_1, f_n : \mathbb{R} \rightarrow \mathbb{R}\}$
- $\{\{f_n\}_{n=1}^{\infty} \mid f_n \rightarrow 0 \text{ in } L_2, f_n : \mathbb{R} \rightarrow \mathbb{R}\}$

To show that any particular region in the Venn diagram is nonempty, provide an example of a function in that region. It is possible to complete this exercise using only functions that are bounded, have bounded support, and are piecewise constant.

P17. Same as (P16) but with $f_n : [0, 1] \rightarrow \mathbb{R}$.

P18. Same as (P16) but with $f_n : \mathbb{N} \rightarrow \mathbb{R}$. This problem is the same as comparing convergence of a sequence of sequences in the following senses: pointwise, uniform, ℓ_1 , and ℓ_2 .