3 September 2015
Analysis I
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HW 2

Due: 8 Sep 2015

The problems are written in the format 'chapter.section.problem-number' from Lang's book. Practice problems must be handed in and will be checked for honest effort. Portfolio problems will be graded thoroughly and may be revised until your solutions are of professional quality. Please submit each portfolio problem on a detached sheet of paper with your name on it.

Practice problems:

1. II.2.4
2. Let $g(x)$ be a bounded function in a neighborhood of a. Let $\lim_{x\to a} f(x) = 0$. Show that $\lim_{x\to a} f(x)g(x)$ exists and equals 0.
3. II.3.8
4. II.4.4
5. *The track problem*. Here is a claim: if the temperature of a circular running track is given by a continuous function of a single position variable, there are two diametrically opposite points that have equal temperature.
   (a) Write the claim as a formal statement about continuous functions.
   (b) Prove that the claim is true or find a counterexample. Hint: Think about the intermediate value theorem.

Portfolio problems:

P4. II.4.1

P5. Prove that a periodic continuous function on $\mathbb{R}$ is uniformly continuous or find a counterexample.

P6. In this problem you will find examples of functions $f_n(x)$ defined on $(0, 1)$ such that

$$\lim_{x\to 0} \sum_{n=1}^{\infty} f_n(x) \neq \sum_{n=1}^{\infty} \lim_{x\to 0} f_n(x).$$

   (a) Find an example where the sum on the left hand side is $+\infty$ for all $x \in (0, 1)$.
   (b) Find an example where all the sums and limits are finite.