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Analysis I  
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### Day 5 — Summary — Differentiability, Mean Value Theorem

28. The derivative of  $f$  at  $x$  is  $\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ , if this limit exists. A function is differentiable on a set if it is differentiable at every point in that set.
29. Product rule, quotient rule, chain rule.
30. Differentiability implies continuity.
31. Let  $C^p([a, b])$  be the set of functions defined on  $[a, b]$  that are differentiable  $p$  times, and the  $p$ -th derivative is continuous. Let  $C^\infty$  be the set of functions that are in  $C^p$  for all  $p$ .
32. At a local maximum (or minimum) of a differentiable function, the derivative is zero (provided that this max or min occurs in the interior of the function's domain).
33. Mean value theorem: If  $f$  is continuous on  $[a, b]$  and is differentiable on  $(a, b)$ , then for some  $c \in (a, b)$ ,  
$$f'(c) = \frac{f(b) - f(a)}{b - a}.$$