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Analysis I
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Day 20 — Summary — Power Series

118. For any power series $\sum a_n x^n$, there is a radius of convergence R (which may be zero, finite, or infinite), such that the series converges absolutely for all $|x| < R$ and does not converge absolutely for any $|x| > R$.

119. The radius of convergence of $\sum a_n x^n$ is $1/\limsup_{n\to\infty} |a_n|^{1/n}$.

120. Let $f(x) = \sum a_n x^n$ be a power series with radius of convergence $R > 0$. Then, for all $|x| < R$, $f'(x) = \sum_{n=1}^{\infty} n a_n x^{n-1}$ and this sum converges absolutely for all $|x| < R$.

121. Let $\{f_n\}$ be a sequence of functions in $C^1([a, b])$ and assume that $f_n' \to g$ uniformly, and that $f_n(x_0)$ converges for some x_0. Then, there exists a function f such that $f_n \to f$ uniformly, and f is differentiable, and $f' = g$.

122. Let $f(x) = \sum a_n x^n$ be a power series with radius of convergence $R > 0$. Then, an antiderivative of $f(x)$ in $-R < x < R$ is given by $\sum_{n=0}^{\infty} \frac{a_n}{n+1} x^{n+1}$ and this sum converges absolutely for all $|x| < R$.