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Analysis I
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Day 18 — Summary — Compactness

99. The function spaces L^p can be defined as the completion of continuous functions under the L^p norm (for $1 \leq p < \infty$).
100. Definition: A subset S of a normed vector space is (sequentially) compact if every sequence within the set has a subsequence that converges to an element of S.
101. A compact subset of a normed vector space is closed and bounded.
102. A closed subset of a compact set is compact.
103. A subset of $\mathbb{R}^k$ is compact if and only if it is closed and bounded
104. Let S be a compact subset of a normed vector space V, and let f be a continuous function from S to the normed vector space F . Then the image of S under f is compact.
105. A continuous function over a compact set in a normed vector space achieves its maximum and minimum.
106. A continuous function from a compact subset of a normed vector space to a normed vector space is uniformly continuous.
107. Consider a subset S of a normed vector space. S is sequentially compact if and only if any open cover of S has a finite subcover. This provides an alternative definition of compactness.

107) why is this defn of compactness different.

Thm: If V is a normed vector space
(or a metric space)
Sequential compactness $\Longleftrightarrow$ open cover compactness

In a general space,
sequential compactness $\not\Leftarrow$ open cover compactness

Eg Let $I = [0,1]$.

Let $V = $ All functions on $([0,1]) = \prod_{\alpha \in [0,1]} \mathbb{R}$

Let $S = \prod_{\alpha \in [0,1]} I$

S is open cover compact but not sequentially compact
(wrt a certain defn of open sets)

See Wikipedia page on Tychonoff's Thm