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Analysis I
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Day 17 — Summary — Completion of a vector space

97. $\mathbb{R}$ can be defined as the set of equivalence classes of Cauchy sequences of $\mathbb{Q}$. This is called the completion of $\mathbb{Q}$.

98. The completion of a normed vector space is defined as the set of equivalence classes of Cauchy sequences of elements in the space. The completion is a complete normed vector space.

98) The vector space of equivalence classes of Cauchy seq within a normed vector space is complete.

Given a Cauchy seq of equivalence classes of Cauchy sequences

$\{X_n^{(1)}\}_{n=1}^{\infty}$ is a Cauchy seq

$\{X_n^{(2)}\}_{n=1}^{\infty}$ — — —

$\vdots$

$\{[\{X_n^{(1)}\}], [\{X_n^{(2)}\}], \ldots [\{X_n^{(k)}\}] \ldots\}$ is a Cauchy seq of equiv class of Cauchy seq

$\forall \varepsilon \; \exists N$ st $m, \tilde{m} \geq N \Rightarrow \|[\{X_n^{(m)}\}] - [\{X_n^{(\tilde{m})}\}]\|$

How do you build a Cauchy seq that this seq of Cauchy seq converges to

$\{X_n^{(1)}\}_{n=1}^{\infty}$ take 1st elt

$\{X_n^{(2)}\}$ take ~~2nd~~ elt corresponding to N st $n, m \geq N \Rightarrow \|X_n^{(2)} - X_m^{(2)}\| < \frac{1}{2}$

$\{X_n^{(3)}\}$

$\vdots$

$\{X_n^{(k)}\}$ take ~~kth~~ elt — — — — $\|X_n^{(k)} - X_m^{(k)}\| < \frac{1}{k}$