27 October 2015
Analysis I
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Day 16 — Summary — Equivalence relations

94. A relation, $\sim$, on a set X is an equivalence relation if it is reflexive, symmetric, and transitive. That is, if for all $a, b, c \in X$
    (a) $a \sim a$ (reflexivity)
    (b) $a \sim b \Rightarrow b \sim a$ (symmetry)
    (c) $a \sim b$ and $b \sim c \Rightarrow a \sim c$ (transitivity)

95. Given a set X and an equivalence relation $\sim$, the equivalence class of an element $a \in X$ is the set of elements equivalent to a. The set of equivalence classes is denoted by $X / \sim$. We can define operations (e.g. addition, multiplication) on equivalence classes if the operation is well defined (is independent of which representative is chosen from the equivalence classes).

96. We can define an equivalence relation between two Cauchy sequences of a (not necessarily complete) normed vector space:
$$\{x_n\} \sim \{y_n\} \text{ if and only if } \lim_{n\to\infty}(x_n - y_n) = 0$$
The set of equivalence classes forms a normed vector space.