6 October 2015
Analysis I
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Day 11 — Summary — Inner Products and Equivalent Norms

57. An inner product $\langle\cdot,\cdot\rangle$ satisfies the following axioms for all $u, v, w \in V$:
    (a) $\langle v, w\rangle = \langle w, v\rangle$
    (b) $\langle u, v + w\rangle = \langle u, v\rangle + \langle u, w\rangle$
    (c) If $c \in \mathbb{R}$, $\langle cv, w\rangle = c\langle v, w\rangle = \langle v, cw\rangle$
    (d) $\langle v, v\rangle \geq 0\ \forall v$ and $\langle v, v\rangle = 0 \Rightarrow v = 0$.
58. Inner products induce a norm $\|v\| = \sqrt{\langle v, v\rangle}$.
59. Inner products satisfy the Cauchy-Schwarz inequality $\langle v, w\rangle \leq \|v\|\|w\|$.
60. Definition: Two norms $\|\cdot\|_a$ and $\|\cdot\|_b$ are equivalent on a vector space V if there exists $c, C > 0$ such that
$$c\|x\|_b \leq \|x\|_a \leq C\|x\|_b\ \forall x \in V.$$
61. All norms on finite dimensional vectors spaces, e.g. $\mathbb{R}^n$, are equivalent.
62. In infinite dimensional vector spaces, some pairs of norms are not equivalent.