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Analysis I
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Optional HW 5

Due: Never.

1. Let f be a smooth function.
   (a) Show that $\frac{f(\Delta x)-2f(0)+f(-\Delta x)}{\Delta x^2} = f''(0) + O(\Delta x^2)$. Find an example function for which the error is on the order of Δx^2.
   (b) Find constants so that $\frac{c_2 f(2\Delta x)+c_1 f(\Delta x)+c_0 f(0)}{\Delta x^2} \to f''(0)$ as $\Delta x \to 0$. Show that the error is $O(\Delta x)$ and find an example where the error is on the order of Δx.